Effect of notch depth on strain-concentration factor of notched cylindrical bars under static tension

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Abstract

The effect of the notch depth on the strain-concentration factor (SNCF) is studied for circumferentially notched cylindrical bars under static tension. The employed new SNCF is defined under the triaxial stress state at the net section. The elastic SNCF increases as the net-to-gross diameter ratio \( d_0/D_0 \) increases and reaches a maximum at the depth of \( d_0/D_0 \approx 0.7 \). Beyond this value of \( d_0/D_0 \) it rapidly decreases to the unity with \( d_0/D_0 \). The elastic SNCF of the extremely deep notch of \( d_0/D_0 = 0.2 \) is almost equal to that of the shallow notch of \( d_0/D_0 = 0.95 \). Nevertheless, the SNCF of the shallow notch sharply increases with plastic deformation at the notch root, while the SNCF of \( d_0/D_0 = 0.2 \) increases only slightly. The SNCF of the deep notch of \( d_0/D_0 = 0.6 \) also increases with plastic deformation and reaches a peak value. This peak value is nearly equal to or slightly lower than that of the shallow notch and hence the plastic SNCF of shallow notches cannot be neglected. The variations of the SNCF with \( P/P_Y \), the ratio of tensile load to that at yielding at the notch root, depends only on the notch geometry up to general yielding.

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Keywords: Notch; Notch depth; Notched bar; Strain-concentration factor

1. Introduction

Elastic stress-concentration factor (SSCF) has been extensively studied for various types of notch and loading. The results have been published by Pilkey (1997) and Nishida (1974) and used for engineering design. However, the effect of the notch depth on the elastic SSCF has been examined mainly through the Neuber’s analytical results (Pilkey, 1997) which indicate that the SSCF rapidly decreases with decreasing notch depth from a maximum value at an intermediate notch depth. This means that the elastic SSCF of a shallow notch is much lower than that of a notch with an intermediate depth. The stress concentration in elastic deformation has thus been considered to be negligible for shallow notches.

Neuber’s rule (Neuber, 1961) predicts that, as the plastic deformation develops from the notch root, the plastic strain-concentration factor (SNCF) is higher than the elastic value, while the plastic SSCF is lower than the elastic value. Many studies, experimentally and analytically performed under static tension, have confirmed the Neuber’s prediction (Neuber, 1961; Theocaris, 1962; Theocaris and Marketos, 1963; Durelli and Sciammarella, 1963; Theocaris, 1965; Ogura et al., 1981;...
Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>current net-section area</td>
</tr>
<tr>
<td>$d_0, d$</td>
<td>initial and current net-section diameters</td>
</tr>
<tr>
<td>$D_0$</td>
<td>initial gross diameter</td>
</tr>
<tr>
<td>$E$</td>
<td>Young’s modulus</td>
</tr>
<tr>
<td>$K_{\text{con}}^\varepsilon$</td>
<td>conventional strain-concentration factor</td>
</tr>
<tr>
<td>$K_{\text{new}}^\varepsilon$</td>
<td>new strain-concentration factor</td>
</tr>
<tr>
<td>$P$</td>
<td>tensile load</td>
</tr>
<tr>
<td>$P_Y$</td>
<td>tensile load at yielding at the notch root</td>
</tr>
<tr>
<td>$r$</td>
<td>current distance from the center of the net-section ($0 \leq r \leq r_n$)</td>
</tr>
<tr>
<td>$r_n$</td>
<td>current net-section radius ($= d/2$)</td>
</tr>
<tr>
<td>$s$</td>
<td>$r/r_n$ ($0 \leq s \leq 1.0$)</td>
</tr>
<tr>
<td>$2 \ln(d_0/d)$</td>
<td>deformation parameter</td>
</tr>
<tr>
<td>$\varepsilon_z$</td>
<td>axial strain</td>
</tr>
<tr>
<td>$(\varepsilon_z)_{\text{con}}^\varepsilon$</td>
<td>conventional average axial strain</td>
</tr>
<tr>
<td>$(\varepsilon_z)_{\text{new}}^\varepsilon$</td>
<td>new average axial strain</td>
</tr>
<tr>
<td>$(\varepsilon_z)_{\text{max}}$</td>
<td>maximum axial strain at the notch root</td>
</tr>
<tr>
<td>$v$</td>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>initial notch radius</td>
</tr>
<tr>
<td>$\sigma_{\text{eq}}$</td>
<td>equivalent stress $= (\sigma_z - \sigma_\theta)^2 + (\sigma_\theta - \sigma_r)^2 + (\sigma_r - \sigma_z)^2)^{1/2}/\sqrt{2}$</td>
</tr>
<tr>
<td>$(\sigma_{\text{eq}})_{\text{max}}$</td>
<td>equivalent stress at the notch root</td>
</tr>
<tr>
<td>$\sigma_r, \sigma_\theta, \sigma_z$</td>
<td>radial, axial and tangential stresses</td>
</tr>
<tr>
<td>$\sigma_Y$</td>
<td>yield stress</td>
</tr>
<tr>
<td>$(\sigma_z)_{\text{av}}$</td>
<td>average axial stress at the net section ($= P/A$)</td>
</tr>
<tr>
<td>$(\sigma_z)<em>{\text{max}}, (\sigma</em>\theta)_{\text{max}}$</td>
<td>axial and tangential stresses at the notch root in elastic deformation</td>
</tr>
</tbody>
</table>

Abbreviations

- GY: general yielding
- SNCF: strain-concentration factor
- SSCF: stress-concentration factor

Hardy and Pipelzadeh, 1996). Note that the plastic SNCF maintains values much greater than the unity, while the plastic SSCF decreases towards the unity (Majima, 1999). The notches employed in the above studies are of intermediate depth, considered to give a strong notch effect. Unfortunately, the effect of plastic deformation on the plastic SNCF of shallow notches has not been evaluated in spite of the fact that such notches are extensively used.

Attempts have been made to predict the axial strain at the notch root under static and cyclic tensile loading. This prediction was made using the SNCF, referred to as the conventional SNCF here, through Neuber’s rule (Sharp and Ward, 1983; Gowhari-anaraki and Hardy, 1991; Sharp, 1995; Hardy and Pipelzadeh, 1996; Zeng and Fatemi, 2001; Harkegard and Mann, 2003; Livieri and Nicoletto, 2003), Glinka’s rule (Glinka, 1985; Sharp, 1995; Zeng and Fatemi, 2001; Livieri and Nicoletto, 2003) or linear rule (Fuchs and Stephens, 1980; Gowhari-anaraki and Hardy, 1991; Hardy and Pipelzadeh, 1996; Zeng and Fatemi, 2001). Comparison of the predicted values with finite element and experimental ones indicates that no rule can accurately predict the magnitude of the axial strain at the notch root. In particular, in notched rectangular bars the accuracy of the prediction decreases with increasing the thickness. The reason for this is that the conventional SNCF is defined under the uniaxial stress state (Majima, 1999), while the axial strain at the notch root occurs under the triaxial stress state. Strain-concentration factor should thus be defined under the triaxial stress state at the net section.

A new definition of elastic-plastic SNCF has been proposed for better understanding of strain concentration under static tension (Majima, 1999). This new SNCF is defined under the triaxial stress state at the net section. This triaxial stress state is completely different from the uniaxial stress state, under which the conventional SNCF is defined. This difference in the stress state produces reasonable values according to the new SNCF and unreasonable values according to the conventional SNCF (Majima, 1999). That is, the conventional SNCF produces values lower than the unity to the concave distribution of the axial strain (Majima, 1999).

This paper deals with the effect of the notch depth on the newly defined SNCF under static tension. Emphasis has been put on the elastic-plastic SNCF of shallow notches. The strain distributions at the net section have been obtained using the finite element method (FEM). The FEM calculations have been performed up to a deformation level close to that when the notch tensile strength is attained.

2. Strain-concentration factor

The new SNCF proposed for tensile loading is given by Majima (1999)

$$K_{\varepsilon}^{\text{new}}(s) = \frac{\varepsilon_{\text{z}}^{\text{max}}(s)}{\varepsilon_{\text{z}}^{\text{new}}(s)}$$

(1)

where $(\varepsilon_{\text{z}})_{\text{max}}$ and $(\varepsilon_{\text{z}})_{\text{new}}$ are the maximum axial strain at the notch root and the new average axial strain or new nominal strain, respectively. The maximum axial strain at the notch root is independent of definition, and hence the new SNCF depends
on a new definition of the average axial strain. For circumferentially notched cylindrical bars \((\varepsilon_z)^{\text{new}}\) is defined as follows (Majima, 1999):

\[
(\varepsilon_z)^{\text{new}}_{\text{av}} = \frac{1}{\pi r_n^2} \int_0^{r_n} \varepsilon_z(r) 2\pi r \, dr = \frac{1}{2} \int_0^1 \varepsilon_z(s) s \, ds
\]

where \(s = r/r_n\).

For elastic deformation, Eq. (2) is transformed into:

\[
(\varepsilon_z)^{\text{new}}_{\text{av}} = \frac{1}{\pi r_n^2} \int_0^{r_n} \frac{1}{E} \left\{ \sigma_z - \nu (\sigma_\theta + \sigma_r) \right\} 2\pi r \, dr = \frac{(\sigma_z)_{\text{av}}}{E} - \frac{2\nu}{E} \int_0^1 \left\{ \sigma_\theta(s) + \sigma_r(s) \right\} s \, ds.
\]

(3)

This equation indicates that \((\varepsilon_z)^{\text{new}}_{\text{av}}\) is defined under the triaxial stress state at the net section. It should be noted that \((\varepsilon_z)^{\text{new}}_{\text{av}}\), given by Eq. (2), is defined under the triaxial stress state also in plastically deformed area at the net section. This is due to the following reason: the plastic component of the axial strain is directly related to the triaxial stress state, as is indicated by the theory of plasticity. The definition under the triaxial stress state gives reasonable results consistent with the concave distribution of the axial strain at any deformation level (Majima, 1999).

The conventional SNCF is given by

\[
K^*_c = \frac{(\varepsilon_z)_{\text{max}}}{(\varepsilon_z)^{\text{con}}_{\text{av}}}. \tag{4}
\]

The conventional average axial strain \((\varepsilon_z)^{\text{con}}_{\text{av}}\) is, however, defined under the uniaxial stress state (Majima, 1999). In elastic deformation, \(\sigma_r\) at the notch root \((\sigma_z)_{\text{max}}\) is much greater than \((\sigma_z)_{\text{av}}\), and the equivalent stress at the notch root \((\sigma_{\text{eq}})_{\text{max}}\) is a little lower than \((\sigma_z)_{\text{max}}\) under the biaxial tensile stress state. This indicates that the small plastic deformation occurs around the notch root even in the range \((\sigma_z)_{\text{av}} \leq \sigma_Y\), when \((\sigma_z)_{\text{av}}\) approaches \(\sigma_Y\). Even in this range \((\varepsilon_z)^{\text{con}}_{\text{av}}\) is given by

\[
(\varepsilon_z)^{\text{con}}_{\text{av}} = \frac{(\sigma_z)_{\text{av}}}{E}. \tag{5}
\]

Eqs. (3) and (5) indicate that the conventional definition neglects the effect of tangential and radial stresses. For further development of plastic deformation at the net section, i.e. in the range \((\sigma_z)_{\text{av}} > \sigma_Y\), \((\varepsilon_z)^{\text{con}}_{\text{av}}\) is determined using the uniaxial true stress-total strain curve \(\sigma = f(\varepsilon)\) (Majima, 1999). The reason for using this curve is that \((\varepsilon_z)^{\text{con}}_{\text{av}}\) is defined under the uniaxial stress state and \((\sigma_z)_{\text{av}}\) is based on the instantaneous area of the net section. The conventional average axial strain is therefore given by

\[
(\varepsilon_z)^{\text{con}}_{\text{av}} = f^{-1}\left\{ (\sigma_z)_{\text{av}} \right\}. \tag{6}
\]

3. Specimen geometries and materials

Notched cylindrical specimens employed here have a circumferential U- or arc notch and a constant gross diameter \(D_0\) of 16.7 mm. The net-to-gross diameter ratio \(d_0/D_0\) is varied from 0.2 to 0.98 to evaluate the effect of notch depth on the new SNCF. Three values of the notch radius \(\rho_0\) (0.5, 1 and 2 mm) have been used to vary the notch sharpness \(d_0/2\rho_0\). Three notch depth \((d_0/D_0)\) values (0.2, 0.6 and 0.95) have been chosen in this study to discuss the effect of the notch depth on the elastoplastic SNCF. These typical notch depth values are referred to as the extremely deep notch, the deep notch and the shallow notch, respectively. The length of the notched specimens is 50 mm, which is sufficient for obtaining pure notch effect (Majima et al., 1995).

The materials employed are an austenitic stainless steel and a Ni–Cr–Mo steel. Young’s modulus \(E\), Poisson’s ratio \(\nu\) and yield stress \(\sigma_Y\) of these materials are listed in Table 1. The true stress-plastic strain relation has been obtained from tension tests. The results are divided into a few ranges of plastic strain to accurately fit the following fifth-degree polynomial:

\[
\sigma = C_0 + C_1\varepsilon_p + C_2\varepsilon_p^2 + C_3\varepsilon_p^3 + C_4\varepsilon_p^4 + C_5\varepsilon_p^5. \tag{7}
\]

The values of the coefficients in each range of plastic strain are listed in Table 1. The true stress-plastic strain curves used in the calculations are shown in Fig. 1.
Table 1
Mechanical properties and polynomial coefficients

<table>
<thead>
<tr>
<th>Material</th>
<th>Plastic strain range</th>
<th>$C_0$</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austenitic stainless steel</td>
<td>$\varepsilon_p \leq 0.2$</td>
<td>$2.459 \times 10^2$</td>
<td>$4.389 \times 10^3$</td>
<td>$-3.265 \times 10^4$</td>
<td>$2.402 \times 10^5$</td>
<td>$-8.899 \times 10^5$</td>
<td>$1.258 \times 10^6$</td>
</tr>
<tr>
<td>Ni–Cr–Mo steel</td>
<td>$0.2 &lt; \varepsilon_p \leq 0.5$</td>
<td>$3.789 \times 10^2$</td>
<td>$1.535 \times 10^3$</td>
<td>$1.173 \times 10^4$</td>
<td>$-1.874 \times 10^5$</td>
<td>$0.0$</td>
<td>$0.0$</td>
</tr>
<tr>
<td>Ni–Cr–Mo steel</td>
<td>$\varepsilon_p \leq 0.1$</td>
<td>$5.250 \times 10^2$</td>
<td>$7.644 \times 10^3$</td>
<td>$-7.377 \times 10^4$</td>
<td>$2.596 \times 10^5$</td>
<td>$0.0$</td>
<td>$0.0$</td>
</tr>
<tr>
<td>Ni–Cr–Mo steel</td>
<td>$0.1 &lt; \varepsilon_p$</td>
<td>$7.426 \times 10^2$</td>
<td>$6.945 \times 10^3$</td>
<td>$-8.143 \times 10^4$</td>
<td>$0.0$</td>
<td>$0.0$</td>
<td>$0.0$</td>
</tr>
</tbody>
</table>

Fig. 1. True stress–plastic strain curves.
Fig. 2. Finite element model.

4. FEM calculation

A finite element mesh of one quarter of a notched specimen is shown in Fig. 2. An eight-node axisymmetric ring element is chosen to model the specimens. The numbers of elements and nodes are 540 and 1717, respectively, and the degrees of freedom are 3434.

FEM calculations have been performed under the application of displacement increments at the end of the unnotched part. The magnitude of the displacement increment is small enough to provide an elastic solution for the first few increments in each specimen. All the calculations have been performed using MARC K6.2 on an APOLLO workstation (MARC analysis research corporation, 1994).

5. Deformation parameter

The parameter $2 \ln(d_0/d)$ is used in this study to express the degree of deformation of the notched specimens. This measure of deformation is obtained on the assumptions of the uniform deformation in the immediate vicinity of the net section and the volume constancy in this portion. The second assumption can be applied only to plastic strain. The parameter $2 \ln(d_0/d)$ thus plays the roles of an average axial strain as well as a measure of deformation at the deformation levels where the elastic strain component is negligible compared with the plastic one. Moreover, under tensile deformation, the ratio $d_0/d$ represents the degree of decrease in the net diameter irrespective of deformation level. This means that $2 \ln(d_0/d)$ can be used as a measure of deformation from infinitesimal to large deformation (Majima, 1999).

The two assumptions above indicate that $2 \ln(d_0/d)$ approaches $(\varepsilon_z)_{\text{new}}$, given by Eq. (2), after adequate development of plastic deformation from the notch root (Majima, 1999). This indicates that the definition of $(\varepsilon_z)_{\text{new}}$ is reasonable.
Table 2

<table>
<thead>
<tr>
<th>ρ₀ [mm]</th>
<th>d₀/D₀</th>
<th>Austenitic stainless steel</th>
<th>Ni–Cr–Mo steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.20</td>
<td>1.14</td>
<td>4.02</td>
</tr>
<tr>
<td></td>
<td>0.60</td>
<td>6.37</td>
<td>38.84</td>
</tr>
<tr>
<td>0.95</td>
<td>20.10</td>
<td>53.52</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>0.20</td>
<td>1.48</td>
<td>3.29</td>
</tr>
<tr>
<td></td>
<td>0.60</td>
<td>8.55</td>
<td>35.24</td>
</tr>
<tr>
<td>0.95</td>
<td>24.99</td>
<td>53.43</td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td>0.20</td>
<td>1.80</td>
<td>2.81</td>
</tr>
<tr>
<td></td>
<td>0.60</td>
<td>11.81</td>
<td>31.05</td>
</tr>
<tr>
<td>0.95</td>
<td>29.97</td>
<td>53.22</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 3. Effect of the notch depth on the relation between tensile load and deformation for notch radius of 1.0 mm.

Fig. 4. New SNCF and conventional SNCF in elastic deformation.

6. Results

6.1. Relation between tensile load and deformation

The effect of the notch depth on the relation between the tensile load \( P \) and deformation \( 2 \ln \left( \frac{d_0}{d} \right) \) is shown in Fig. 3. At the same value of \( 2 \ln \left( \frac{d_0}{d} \right) \), \( P \) increases with decreasing the notch depth because the initial net diameter increases with decreasing the notch depth. Moreover, \( P \) of Ni–Cr–Mo steel is greater than that of austenitic stainless steel. This occurs because the flow stress of Ni–Cr–Mo steel is greater than that of austenitic stainless steel in the deformation range shown in Fig. 3. The values of the tensile loads at which yielding at the notch root and general yielding start are given in Table 2. The former tensile load, denoted by \( P_Y \), is described later. The notch radii of 0.5 and 2.0 mm also give approximately the same result.

6.2. Effect of the notch depth on elastic \( K_{new}^{SNCF} \) and \( K_{con}^{SNCF} \)

Fig. 4 shows the effect of the notch depth on the elastic new and conventional SNCFs. The elastic value of the new SNCF is greater than that of the conventional SNCF. This is independent of notch depth and notch radius. The elastic new SNCF increases with increasing net-to-gross diameter ratio \( d_0/D_0 \) and reaches its maximum at \( d_0/D_0 \approx 0.7 \). By further increasing \( d_0/D_0 \), the elastic new SNCF decreases with \( d_0/D_0 \). It rapidly decreases especially in the range \( d_0/D_0 > 0.85 \). The elastic new and conventional SNCFs are approximately equal for shallow notch. It should be noted that the elastic new SNCF of the
extremely deep notch of $d_0/D_0 = 0.2$ is nearly equal to that of the shallow notch of $d_0/D_0 = 0.95$. Moreover, the elastic new SNCF of the deep notch of $d_0/D_0 = 0.6$ is much greater than that of the shallow notch.

6.3. Effect of the notch depth on the variation of $K_{\varepsilon}^{\text{new}}$ with deformation

Figs. 5(a) and 5(b) show the variations of the new SNCF with deformation. Approximately the same result is obtained for the notch radius of 2 mm. The initial constant value is that of the elastic new SNCF. The range of $2 \ln (d_0/d)$ for this constant value increases with increasing notch radius. The new SNCF rapidly increases from its elastic value to a peak value as the plastic deformation develops from the notch root for both shallow ($d_0/D_0 = 0.95$) and deep ($d_0/D_0 = 0.60$) notches. This peak occurs at general yielding for the shallow notch. For the deep notch, its general yielding occurs at a deformation level greater than $2 \ln (d_0/d) = 0.006$. It is apparent that the peak value of the shallow notch is greater than (Fig. 5(b)) or nearly equal to (Fig. 5(a)) that of the deep notch. The peak value is much greater than the elastic value for the shallow notch. On the other hand, the new SNCF of $d_0/D_0 = 0.2$ increases a little from its elastic value and then, for further plastic deformation, decreases to values lower than the elastic value. The peak value is very low compared to those of the shallow and deep notches. The effect of plastic deformation on the new SNCF is thus negligible for the extremely deep notch.

It is important to note that, for shallow notch, a small plastic deformation rapidly increases the SNCF approximately up to the peak value of the deep notch even if the elastic SNCF is much lower than that of the deep notch. Moreover, the peak value is attained at a small value of $2 \ln (d_0/d)$. This indicates that a shallow notch has a strong effect on strength and fracture of notched bars.

For shallow notch, the rapid increase from the elastic to peak value becomes greater with decreasing the notch radius. This occurs because the increase in $(\varepsilon_z)_{\max}$ with plastic deformation at the notch root increases with decreasing the notch radius, while $(\varepsilon_z)_{\text{newav}}$ is independent of the notch radius, as is shown in Fig. 6. This independence is due to a restriction of the plastic region in an immediate vicinity of the notch root, and this restriction is caused by both a shallow notch depth and a small notch radius.

Fig. 7 shows the distribution of the normalized axial strain $\varepsilon_z/(\varepsilon_z)_{\max}$ at the net section at the deformation level very close to the peak displayed in Fig. 5. For the shallow notch ($d_0/D_0 = 0.95$), the distribution is almost constant or slightly convex in the range $0 \leq r/r_n \leq 0.8$, and is sharply concave in the range $r/r_n > 0.8$. This indicates that the deformed region with large plastic strain is restricted in an immediate vicinity of the notch root. On the other hand, the distribution in the deep notch ($d_0/D_0 = 0.60$) is less concave than that of the shallow notch. This occurs because the deep notch produces the larger percentage of the deformed region with large plastic strain than the shallow notch does. Fig. 7 also indicates that the distribution of $\varepsilon_z/(\varepsilon_z)_{\max}$ at the net section is independent of the stress-strain curve. This indicates that the deformation level is approximately the same even if the value of $2 \ln (d_0/d)$ is different. The values of $(\varepsilon_z)_{\max}$ are listed in Table 3 for the deformation level being considered. The same result is obtained for the notch radii of 0.5 and 2 mm.
6.4. Comparison between new and conventional SNCFs

The variations of $K_{\text{new}}^\varepsilon$ and $K_{\text{con}}^\varepsilon$ with deformation are compared in Fig. 8 for the shallow notch. The conventional SNCF also increases from its elastic value to the maximum at $(\sigma_z)_{av} = \sigma_Y$ (Majima, 1999). The new SNCF is a little greater than the conventional SNCF in the range $(\sigma_z)_{av} \leq \sigma_Y$. The maximum of the conventional SNCF is nearly equal to the new SNCF at $(\sigma_z)_{av} = \sigma_Y$. For further deformation, i.e. $(\sigma_z)_{av} > \sigma_Y$, the conventional SNCF sharply drops and becomes lower than the unity (Majima, 1999). On the other hand, the new SNCF continues to increase to a peak value and then becomes nearly constant up to $2\ln(d_0/d) = 0.002$. The peak value of the new SNCF is much greater than the maximum of the conventional SNCF. It should be noted that the new SNCF shows a strong dependence on the notch radius in the range $(\sigma_z)_{av} > \sigma_Y$, while the conventional one shows a little dependence. Fig. 6 indicates a strong effect of the notch radius on the distribution of the axial strain. This means that the conventional SNCF does not express the accurate effect of the notch radius.

The rapid decrease in $K_{\text{con}}^\varepsilon$ to values lower than the unity is caused by the rapid increase in $(\varepsilon_z)_{av}^{\text{con}}$. The rate of increase in $(\varepsilon_z)_{av}^{\text{con}}$ becomes greater than that in $(\varepsilon_z)_{max}$. This high rate of increase in $(\varepsilon_z)_{av}^{\text{con}}$ is due to the definition of $(\varepsilon_z)_{av}^{\text{con}}$ under the uniaxial stress state (Majima, 1999). Fig. 7, however, shows that the distributions of $\varepsilon_z$ are concave even at the deformation level around the peak for the shallow notch. These concave distributions require that strain-concentration factor should be greater than the unity regardless of definition. The definition of the conventional SNCF is thus unreasonable for the shallow notch as well as for the deep notch (Majima, 1999). The extremely deep notch also shows the rapid decrease in $K_{\text{con}}^\varepsilon$ to values lower than the unity.

6.5. Variations of $K_{\text{new}}^\varepsilon$ with tensile load

It is also important to evaluate the variation of $K_{\text{new}}^\varepsilon$ with the tensile load $P$. Figs. 9(a) and 9(b) show the variations of $K_{\text{new}}^\varepsilon$ with $P$. The elastic $K_{\text{new}}^\varepsilon$ is constant, and the range of this constant value in Ni-Cr-Mo steel is larger than that in austenitic stainless steel. This occurs because the tensile load at yielding at the notch root $P_Y$ is proportional to yield stress $\sigma_Y$, as is discussed later. For the deep notch (Fig. 9(a)) $K_{\text{new}}^\varepsilon$ increases from its elastic value to the maximum as $P$ increases. For further deformation, it decreases with $P$. The degree of this increase and decrease becomes greater with decreasing the notch radius.
Fig. 8. Comparison between new and conventional SNCFs.

Fig. 9. (a) Variation of the new SNCF with tensile load for the deep notch \( \frac{d_0}{D_0} = 0.60 \). (b) Variation of the new SNCF with tensile load for the shallow notch \( \frac{d_0}{D_0} = 0.95 \).

On the other hand, \( K_{\text{new}}^\varepsilon \) increases at an increasing rate from its elastic value and reaches the peak value, for the shallow notch (Fig. 9(b)). The peak value is also the maximum \( K_{\text{new}}^\varepsilon \) and occurs at general yielding. The tensile load strongly depends on the value of flow stress of material, so that the tensile load where \( K_{\text{new}}^\varepsilon \) becomes maximum in Ni–Cr–Mo steel is larger than that in austenitic stainless steel. The reason for this is that the flow stress in Ni–Cr–Mo steel is larger than that in austenitic stainless steel in the deformation range shown in Fig. 5. The variation of \( K_{\text{new}}^\varepsilon \) with \( P \) is thus strongly affected by the stress-strain curve. The same is applied to the extremely deep notch.

6.6. Distribution of the axial stress at the net section

It is obvious that the magnitude of the axial stress at the net section increases with increasing the flow stress in a stress-strain curve. Fig. 10, however, shows that the distribution of the normalized axial stress \( \sigma_z / \sigma_Y \) is nearly independent of the stress-strain curve even at general yielding. At yielding at the notch root, i.e. at elastic limit of notched bars, \( \sigma_z / \sigma_Y \) also gives the
same result. These indicate that if the deformation level is the same, $\sigma_z/\sigma_Y$ at the net section gives the distribution independent of material at least up to general yielding. Fig. 10 also indicates that the stress concentration at the notch root disappears as plastic deformation develops around the notch root. These are independent of the notch depth.

7. Discussion

Fig. 9 shows that the variation of the new SNCF strongly depends on the material as well as the notch geometry. Therefore, the effect of mechanical properties of a material on the variation of the new SNCF seems to be complex. However, this effect can be almost eliminated using a method described below, and the variation of the new SNCF can be evaluated only by considering the notch geometry.

The tensile load $P$ depends on the magnitude and distribution of the axial stress $\sigma_z$. The magnitude of $\sigma_z$ depends on the flow stress in a stress-strain curve. As a result, the stress-strain curve has a strong effect on the variation of $K_{new}^{\varepsilon}$ with $P$, as is shown in Fig. 9. However, the independence of $\sigma_z/\sigma_Y$ from the material indicates that the ratio $P/\sigma_Y$, given by the following expression, is independent of the stress-strain curve;

$$\int_A \frac{\sigma_z}{\sigma_Y} dA = \frac{1}{\sigma_Y} \int_A \sigma_z dA = \frac{P}{\sigma_Y}.$$  \hspace{1cm} (8)

In elastic deformation the distributions of stress components depend only on notch geometry, and the magnitude of stress components is proportional to $P$. The equivalent stress at the notch root $(\sigma_{eq})_{\text{max}}$, given by the following expression, is thus proportional to $P$:

$$(\sigma_{eq})_{\text{max}} = (\sigma_z)_{\text{max}} \left\{ 1 - \left( \frac{(\sigma_\theta)_{\text{max}}}{(\sigma_z)_{\text{max}}} \right) + \left( \frac{(\sigma_\theta)_{\text{max}}}{(\sigma_z)_{\text{max}}} \right)^2 \right\}^{1/2} \hspace{1cm} (9)$$

where the ratio $(\sigma_\theta)_{\text{max}}/(\sigma_z)_{\text{max}}$ is held constant during elastic deformation. At the start of yielding at the notch root, $(\sigma_{eq})_{\text{max}}$ is equal to $\sigma_Y$, and hence the tensile load at yielding at the notch root $P_Y$ is proportional to $\sigma_Y$. The ratio $P/P_Y$ is thus independent of the stress-strain curve at least up to general yielding because of the independence of $P/\sigma_Y$. This indicates that the same value of $P/P_Y$ represents the same level of deformation even if stress-strain curves are different.

Figs. 11(a) and 11(b) show that the variation of $K_{new}^{\varepsilon}$ with $P/P_Y$ is almost independent of the stress-strain curve at least up to general yielding. These results indicate that $P/P_Y$ is effective for estimating the deformation level between different materials. For the deep notch (Fig. 11(a)), $K_{new}^{\varepsilon}$ increases from its elastic value to a first peak at a decreasing rate, and then increases to the maximum at a decreasing rate. The maximum occurs a little before general yielding. The first peak and the maximum of both materials occur at approximately the same value of $P/P_Y$. For the shallow notch (Fig. 11(b)), $K_{new}^{\varepsilon}$ increases at an increasing rate to a peak value, which occurs at approximately the same value of $P/P_Y$ regardless of material.
to general yielding. These values of $P/P_Y$ at the peak and at the maximum decrease with increasing the notch radius. The same result in Fig. 11 is also obtained for the extremely deep notch except that the variation of $K_{\text{new}}^{\varepsilon}$ is small.

The stress-strain curves of the ferrous materials used here are very different from each other. However, the axial strain at the notch root is very small even at general yielding, as shown in Fig. 6. For Ni–Cr–Mo steel the magnitude of the axial strain is about twice as large as that in Fig. 6. The effect of the rate of strain hardening is thus almost negligible on the variation of $K_{\text{new}}^{\varepsilon}$ with $P$. This indicates that the main factor affecting the variation of $K_{\text{new}}^{\varepsilon}$ with $P$ is the magnitude of yield stress. It follows that $K_{\text{new}}^{\varepsilon}$ versus $P/P_Y$ relation is almost independent of the stress-strain curve. The only factor that affects the $K_{\text{new}}^{\varepsilon}$ versus $P/P_Y$ curve up to general yielding is the notch geometry.

The sharp peak around general yielding of the shallow notch is unique. The sharp decrease after general yielding is attributed to the rapid increase in the new average axial strain $(\varepsilon_z)_{\text{newav}}$. Fig. 12 shows that $(\varepsilon_z)_{\text{newav}}$ increases very rapidly after general yielding. This rapid increase is followed by a rapid decrease after general yielding. These values of $P/P_Y$ at the peak and at the maximum decrease with increasing the notch radius. The same result in Fig. 11 is also obtained for the extremely deep notch except that the variation of $K_{\text{new}}^{\varepsilon}$ is small.
yielding for the shallow notch (solid symbols). On the other hand, the rate of increase in \((\varepsilon_z)_{new}^{\text{new}}\) is milder after general yielding for the deep notch (open symbols). This difference in the rate of increase in \((\varepsilon_z)_{new}^{\text{new}}\) brings about the difference in the shape of the \(K_\text{P}^{\text{new}}\) versus \(P/P_Y\) curve. Ni–Cr–Mo steel gives approximately the same result because Fig. 12 is displayed using \(P/P_Y\).

Fig. 13 shows the variation of the distribution of the axial strain at the net section with \(P/P_Y\) for the shallow notch \((d_0/D_0 = 0.95)\). All the curves are mildly concave for the deep notch (Majima, 1999). On the other hand, the axial strain is constant or slightly convex in the range \(0 < r/r_0 < 0.8\) and sharply concave in the range \(r/r_0 > 0.8\) for the shallow notch. In particular, the rate of increase in the axial strain in the range \(r/r_0 > 0.8\) is greater after general yielding than the rate of increase in \((\varepsilon_z)_{max}\). This causes the sharp increase in \((\varepsilon_z)_{new}^{\text{new}}\) after general yielding (Fig. 12), and then the sharp decrease in the relation between \(K_\text{P}^{\text{new}}\) and \(P/P_Y\) (Fig. 11(b)). Approximately the same result is obtained for Ni–Cr–Mo steel because the deformation level is given by \(P/P_Y\). The result in Fig. 13 is also independent of the notch radius.

8. Conclusions

The effect of the notch depth on the new SNCF, defined under the triaxial stress state at the net section, has been studied for circumferentially notched cylindrical bars under static tension. The elastic SNCF versus net-to-gross diameter ratio \((d_0/D_0)\) curve is convex with a maximum value at approximately \(d_0/D_0 = 0.7\). Beyond this value of \(d_0/D_0\) the elastic SNCF rapidly decreases towards the unity at \(d_0/D_0 = 1.0\). The elastic SNCF of the extremely deep notch \((d_0/D_0 = 0.2)\) is almost equal to that of the shallow notch \((d_0/D_0 = 0.95)\).

For elastic-plastic deformation, the SNCF of the shallow notch \((d_0/D_0 = 0.95)\) sharply increases with plastic deformation at the notch root, while the SNCF of \((d_0/D_0 = 0.2)\) increases only slightly. The SNCF of the deep notch \((d_0/D_0 = 0.6)\) also increases with plastic deformation and reaches a peak value. The peak value of the shallow notch is nearly equal to or greater than that of the deep notch. The shallow notch produces a high value of the SNCF under an infinitesimal plastic deformation even if its elastic SNCF is low. The variation of \(K_\text{P}^{\text{new}}\) with \(P/P_Y\), the ratio of tensile load to that at yielding at the notch root, depends only on the notch geometry up to general yielding.

References


