Best Simultaneous Approximation in Orlicz Spaces

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Let \( X \) be a Banach space and let \( L^\Phi(I,X) \) denote the space of Orlicz \( X \)-valued integrable functions on the unit interval \( I \) equipped with the Luxemburg norm. In this paper, we present a distance formula \( \text{dist}_\Phi(f_1, f_2, L^\Phi(I,G)) \), where \( G \) is a closed subspace of \( X \), and \( f_1, f_2 \in L^\Phi(I,X) \). Moreover, some related results concerning best simultaneous approximation in \( L^\Phi(I,X) \) are presented.

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1. Introduction

A function \( \Phi : (-\infty, \infty) \rightarrow [0, \infty) \) is called an Orlicz function if it satisfies the following conditions:

1. \( \Phi \) is even, continuous, convex, and \( \Phi(0) = 0 \);
2. \( \Phi(x) > 0 \) for all \( x \neq 0 \);
3. \( \lim_{x \to 0} \Phi(x)/x = 0 \) and \( \lim_{x \to \infty} \Phi(x)/x = \infty \).

We say that a function \( \Phi \) satisfies the \( \Delta_2 \) condition if there are constants \( k > 1 \) and \( x_0 > 0 \) such that \( \Phi(2x) \leq k\Phi(x) \) for \( x > x_0 \). Examples of Orlicz functions that satisfy the \( \Delta_2 \) conditions are widely available such as \( \Phi(x) = |x|^p \), \( 1 \leq p < \infty \), and \( \Phi(x) = (1 + |x|)\log(1 + |x|) - |x| \). In fact, Orlicz functions are considered to be a subclass of Young functions defined in [1].

Let \( X \) be a Banach space and let \((I,\mu)\) be a measure space. For an Orlicz function \( \Phi \), let \( L^\Phi(I,X) \) be the Orlicz-Bochner function space that consists of strongly measurable functions \( f : I \rightarrow X \) with \( \int_I \Phi(\alpha \|f\|)d\mu(t) < \infty \) for some \( \alpha > 0 \). It is known that \( L^\Phi(I,X) \) is a Banach space under the Luxemburg norm.
\[\|f\|_{\Phi} = \inf\left\{ k > 0, \int_I \Phi\left( \frac{1}{k} \|f\| \right) d\mu(t) \leq 1 \right\}. \quad (1.1)\]

It should be remarked that if \(\Phi(x) = |x|^p\), \(1 \leq p < \infty\), the space \(L^{\Phi}(I, X)\) is simply the \(p\)-Lebesgue Bochner function space \(L^p(I, X)\) with \(\|f\|_p = \left( \int_I \|f\|^p \, d\mu(t) \right)^{1/p}\) \(1 \leq p < \infty\).

On the other hand, if \(\Phi(x) = (1 + |x|) \log(1 + |x|) - |x|\), then the space \(L^{\Phi}(I, X)\) is the well-known Zygmund space, \(L \log L^+\). For excellent monographs on \(L^{\Phi}(I, X)\), we refer the readers to [1–3].

For a function \(F = (f_1, f_2) \in (L^{\Phi}(I, X))^2\), we define \(\|F\|\) by

\[\|F\| = \|f_1(\cdot)\| + \|f_2(\cdot)\|_{\Phi}. \quad (1.3)\]

In this paper, for a given closed subspace \(G\) of \(X\) and \(F = (f_1, f_2) \in (L^{\Phi}(I, X))^2\), we show the existence of a pair \(G_0 = (g_0, g_0) \in (L^{\Phi}(I, G))^2\) such that

\[\|F - G_0\| = \inf_{g \in G} \|F - (g, g)\|. \quad (1.4)\]

If such a function \(g\) exists, it is called a best simultaneous approximation of \(F = (f_1, f_2)\). The problem of best simultaneous approximation can be viewed as a special case of vector-valued approximation. Recent results in this area are due to Pinkus [4], where he considered the problem when a finite-dimensional subspace is a unicity space. Characterization results for linear problems were given in [5] based on the derivation of an expression for the directional derivative, and these results generalize the earlier results presented in [6]. Results on best simultaneous approximation in general Banach spaces may be found in [7, 8]. Related results on \(L^p(I, X)\), \(1 \leq p < \infty\), are given in [9]. In [9], it is shown that if \(G\) is a reflexive subspace of a Banach space \(X\), then \(L^p(I, G)\) is simultaneously proximinal in \(L^p(I, X)\). If \(L^{\Phi}(I, X) = L^1(I, X)\), Abu-Sarhan and Khalil [10] proved that if \(G\) is a reflexive subspace of the Banach space \(X\) or \(G\) is a 1-summand subspace of \(X\), then \(L^1(I, G)\) is simultaneously proximinal in \(L^1(I, X)\).

It is the aim of this work to prove a distance formula \(\text{dist}_{\Phi}(f_1, f_2, L^{\Phi}(I, G))\), where \(f_1, f_2 \in L^{\Phi}(I, X)\), similar to that of best approximation. This will allow us to generalize some recent results on \(L^1(I, X)\) to \(L^{\Phi}(I, X)\).

Throughout this paper, \(X\) is a Banach space, \(\Phi\) is an Orlicz function, and \(L^{\Phi}(I, X)\) is the Orlicz-Bochner function space equipped with the Luxemburg norm.

2. Distance formula

Let \(G\) be a closed subspace of \(X\). For \(x, y \in X\), define

\[\text{dist}(x, y, G) = \inf_{z \in G} \|x - z\| + \|y - z\|. \quad (2.1)\]
For $f_1, f_2 \in L^\Phi(I,X)$, we define $\text{dist}_\Phi(f_1, f_2, L^\Phi(I,G))$ by
\begin{equation}
\text{dist}_\Phi(f_1, f_2, L^\Phi(I,G)) = \inf_{g \in L^\Phi(I,G)} \|(f_1, f_2) - (g,g)\|
= \inf_{g \in L^\Phi(I,G)} \|\|f_1(\cdot) - g(\cdot)\| + \|f_2(\cdot) - g(\cdot)\|\|_\Phi. \tag{2.2}
\end{equation}

Our main result is the following.

**Theorem 2.1.** Let $G$ be a subspace of the Banach space $X$ and let $\Phi$ be an Orlicz function that satisfies the $\Delta_2$ condition. If $f_1, f_2 \in L^\Phi(I,X)$, then the function $\text{dist}(f_1(\cdot), f_2(\cdot), G)$ belongs to $L^\Phi(I)$ and
\begin{equation}
\|\text{dist}(f_1(\cdot), f_2(\cdot), G)\|_\Phi = \text{dist}_\Phi(f_1, f_2, L^\Phi(I,G)). \tag{2.3}
\end{equation}

**Proof.** Let $f_1, f_2 \in L^\Phi(I,X)$. Then there exist two sequences $(f_{n,1})$, $(f_{n,2})$ of simple functions in $L^\Phi(I,X)$ such that
\begin{equation}
\|f_{n,1}(t) - f_1(t)\| \longrightarrow 0, \quad \|f_{n,2}(t) - f_2(t)\| \longrightarrow 0, \quad \text{as } n \longrightarrow \infty \tag{2.4}
\end{equation}
for almost all $t$ in $I$. The continuity of $\text{dist}(x,y,G)$ implies that
\begin{equation}
|\text{dist}(f_{n,1}(t), f_{n,2}(t), G) - \text{dist}(f_1(t), f_2(t), G) | \longrightarrow 0, \quad \text{as } n \longrightarrow \infty. \tag{2.5}
\end{equation}

Set $H_n(t) = \text{dist}(f_{n,1}(t), f_{n,2}(t), G)$. Then each $H_n$ is a measurable function. Thus $\text{dist}(f_1(\cdot), f_2(\cdot), G)$ is measurable and
\begin{equation}
\text{dist}(f_1(t), f_2(t), G) \leq \|f_1(t) - z\| + \|f_2(t) - z\| \tag{2.6}
\end{equation}
for all $z$ in $G$. Therefore,
\begin{equation}
\text{dist}(f_1(t), f_2(t), G) \leq \|f_1(t) - g(t)\| + \|f_2(t) - g(t)\| \tag{2.7}
\end{equation}
for all $g \in L^\Phi(I,G)$. Thus
\begin{equation}
\|\text{dist}(f_1(\cdot), f_2(\cdot), G)\|_\Phi \leq \|\|f_1(\cdot) - g(\cdot)\| + \|f_2(\cdot) - g(\cdot)\|\|_\Phi \tag{2.8}
\end{equation}
for all $g \in L^\Phi(I,G)$. Hence $\text{dist}(f_1(\cdot), f_2(\cdot), G) \in L^\Phi(I)$ and
\begin{equation}
\|\text{dist}(f_1(\cdot), f_2(\cdot), G)\|_\Phi \leq \text{dist}_\Phi(f_1, f_2, L^\Phi(I,G)). \tag{2.9}
\end{equation}

Fix $\epsilon > 0$. Since the set of simple functions are dense in $L^\Phi(I,X)$, there exist simple functions $f_i^*$ in $L^\Phi(I,X)$ such that $\|f_i - f_i^*\|_\Phi \leq \epsilon/6$ for $i = 1,2$. Assume that $f_i^*(t) = \sum_{k=1}^n x_k^i \chi_{A_k}(t)$ with $A_k$’s are measurable sets, $x_k^i \in X$, $k = 1,2,\ldots,n$, $i = 1,2$, $A_k \cap A_j = \phi$, $k \neq j$, and $\bigcup_{k=1}^n A_k = I$. We can assume that $\mu(A_k) > 0$ and $\Phi(1) \leq 1$. For each $k = 1,2,\ldots,n$, let $y_k \in G$ be such that
\begin{equation}
\|x_k^1 - y_k\| + \|x_k^2 - y_k\| \leq \text{dist}(x_k^1, x_k^2, G) + \frac{\epsilon}{3}. \tag{2.10}
\end{equation}
Set $g(t) = \sum_{k=1}^{n} y_k \chi_{A_k}(t)$ and

$$F(t) = \text{dist}(f_1(t), f_2(t), G) + \|f_1(t) - f_1^*(t)\| + \|f_2(t) - f_2^*(t)\| + \frac{\epsilon}{3}. \tag{2.11}$$

Then

$$\int_{I} \Phi \left( \frac{||f_1^*(t) - g(t)|| + ||f_2^*(t) - g(t)||}{\|F\|_\Phi} \right) d\mu(t)$$

$$= \sum_{k=1}^{n} \int_{A_k} \Phi \left( \frac{||x_k^1 - y_k|| + ||x_k^2 - y_k||}{\|F\|_\Phi} \right) d\mu(t)$$

$$= \sum_{k=1}^{n} \int_{A_k} \Phi \left( \frac{\text{dist}(x_k^1, x_k^2, G) + \epsilon/3}{\|F\|_\Phi} \right) d\mu(t)$$

$$< \sum_{k=1}^{n} \int_{A_k} \Phi \left( \frac{\text{dist}(f_1^*(t), f_2^*(t), G) + \epsilon/3}{\|F\|_\Phi} \right) d\mu(t)$$

$$\leq \int_{I} \Phi \left( \frac{||f_1(t) - f_1^*(t)|| + ||f_2(t) - f_2^*(t)|| + \text{dist}(f_1(t), f_2(t), G) + \epsilon/3}{\|F\|_\Phi} \right) d\mu(t)$$

$$= \int_{I} \Phi \left( \frac{F(t)}{\|F\|_\Phi} \right) d\mu(t) \leq 1. \tag{2.12}$$

Consequently,

$$|||f_1^*(\cdot) - g(\cdot)|| + ||f_2^*(\cdot) - g(\cdot)||_\Phi \leq \left\| \left\| f_1(\cdot) - f_1^*(\cdot) \right\| + \left\| f_2(\cdot) - f_2^*(\cdot) \right\| \right\|_\Phi + \text{dist}(f_1(\cdot), f_2(\cdot), G) + \frac{\epsilon}{3}. \tag{2.13}$$

Notice that

$$\text{dist}_\Phi (f_1, f_2, L^\Phi(I, G)) \leq \text{dist}_\Phi (f_1^*, f_2^*, L^\Phi(I, G)) + \|f_1 - f_1^*\|_\Phi + \|f_2 - f_2^*\|_\Phi$$

$$< \frac{\epsilon}{3} + \left\| \left\| f_1^*(\cdot) - g(\cdot) \right\| + \left\| f_2^*(\cdot) - g(\cdot) \right\| \right\|_\Phi$$

$$\leq \frac{\epsilon}{3} + \left\| \text{dist}(f_1(\cdot), f_2(\cdot), G) + \left\| f_1(\cdot) - f_1^*(\cdot) \right\| \right\|_\Phi$$

$$\leq \frac{2\epsilon}{3} + \| \text{dist}(f_1(\cdot), f_2(\cdot), G) \|_\Phi$$

$$+ \|f_1(\cdot) - f_1^*(\cdot)\|_\Phi + \|f_2(\cdot) - f_2^*(\cdot)\|_\Phi$$

$$\leq \epsilon + \| \text{dist}(f_1(\cdot), f_2(\cdot), G) \|_\Phi,$$
which (since $\epsilon$ is arbitrary) implies that
\[ \text{dist}_\Phi(f_1, f_2, L^\Phi(I, G)) \leq \|\text{dist}(f_1(\cdot), f_2(\cdot), G)\|_\Phi. \] (2.15)

Hence by (2.9) and (2.15) the proof is complete. \hfill \square

A direct consequence of Theorem 2.1 is the following result.

**Theorem 2.2.** Let $G$ be a closed subspace of the Banach space $X$ and let $\Phi$ be an Orlicz function that satisfies the $\Delta_2$ condition. For $g \in L^\Phi(I, G)$ to be a best simultaneous approximation of a pair of elements $(f_1, f_2)$ in $L^\Phi(I, G)$, it is necessary and sufficient that $g(t)$ is a best simultaneous approximation of $(f_1(t), f_2(t))$ in $G$ for almost all $t \in I$.

### 3. Proximinality of $L^\Phi(I, G)$ in $L^\Phi(I, X)$

A closed subspace $G$ of $X$ is called 1-summand in $X$ if there exists a closed subspace $Y$ such that $X = G \bigoplus_1 Y$, that is, any element $x \in X$ can be written as $x = g + y$, $g \in G$, $y \in Y$, and $\|x\| = \|g\| + \|y\|$. It is known that a 1-summand subspace $G$ of $X$ is proximinal in $X$, and $L^1(I, G)$ is proximinal in $L^1(I, X)$, [11].

Our first result in this section is the following.

**Theorem 3.1.** If $G$ is simultaneously proximinal in $X$, then every pair of simple functions admits a best simultaneous approximation in $L^\Phi(I, G)$.

**Proof.** Let $f_1, f_2$ be two simple functions in $L^\Phi(I, X)$. Then $f_1, f_2$ can be written as $f_1(s) = \sum_{k=1}^{n} u_k^1 \chi_k(s)$, $f_2(s) = \sum_{k=1}^{n} u_k^2 \chi_k(s)$, where $I_k$’s are disjoint measurable subsets of $I$ satisfying $\bigcup_{k=1}^{n} I_k = I$, and $\chi_k$ is the characteristic function of $I_k$. Since $f_1$ and $f_2$ represent classes of functions, we may assume that $\mu(I_k) > 0$ for each $1 \leq k \leq n$. By assumption, we know that for each $1 \leq k \leq n$ there exists a best simultaneous approximation $w_k$ in $G$ of the pair of elements $(u_k^1, u_k^2) \in X^2$ such that
\[ \text{dist}(u_k^1, u_k^2, G) = \|u_k^1 - w_k\| + \|u_k^2 - w_k\|. \] (3.1)

Set $g = \sum_{k=1}^{n} w_k \chi_k(s)$. Then, for any $\alpha > 0$ and $h \in L^\Phi(I, G)$, we obtain that
\begin{align*}
\int_I \Phi\left(\frac{\|f_1(t) - h(t)\| + \|f_2(t) - h(t)\|}{\alpha}\right) d\mu(t) &\geq \sum_{k=1}^{n} \int_{I_k} \Phi\left(\frac{\|u_k^1 - h(t)\| + \|u_k^2 - h(t)\|}{\alpha}\right) d\mu(t) \\
&\geq \sum_{k=1}^{n} \int_{I_k} \Phi\left(\frac{\|u_k^1 - w_k\| + \|u_k^2 - w_k\|}{\alpha}\right) d\mu(t) \\
&= \int_I \Phi\left(\frac{\|f_1(t) - g(t)\| + \|f_2(t) - g(t)\|}{\alpha}\right) d\mu(t).
\end{align*}
(3.2)

Taking the infimum over all such $\alpha$’s, we have that
\[ \|\|f_1(\cdot) - h(\cdot)\| + \|f_2(t) - h(\cdot)\||_\Phi \geq \|\|f_1(\cdot) - g(\cdot)\| + \|f_2(t) - g(\cdot)\||_\Phi \] (3.3)
for all \( h \in L^\Phi(I, G) \). Hence
\[
\text{dist}_\Phi(f_1, f_2, L^\Phi(I, G)) = \| \| f_1(\cdot) - g(\cdot) \| + \| f_2(\cdot) - g(\cdot) \| \|_\Phi \\
\geq \| \| f_1(\cdot) - h(\cdot) \| + \| f_2(\cdot) - h(\cdot) \| \|_\Phi.
\] (3.4)

Now we prove the following 2-dimensional analogous of [12, Theorem 4].

**Theorem 3.2.** Let \( G \) be a closed subspace of the Banach space \( X \) and let \( \Phi \) be an Orlicz function that satisfies the \( \Delta_2 \) condition. If \( L^1(I, G) \) is simultaneously proximinal in \( L^1(I, X) \), then \( L^\Phi(I, G) \) is simultaneously proximinal in \( L^\Phi(I, X) \).

**Proof.** Let \( f_1, f_2 \in L^\Phi(I, X) \). Then \( f_1, f_2 \in L^1(I, X) \); see [13]. By assumption, there exists \( g \in L^1(I, G) \) such that
\[
\| \| f_1(\cdot) - g(\cdot) \| + \| f_2(\cdot) - g(\cdot) \| \|_1 \leq \| \| f_1(\cdot) - h(\cdot) \| + \| f_2(\cdot) - h(\cdot) \| \|_1
\] (3.5)
for every \( h \in L^1(I, G) \). By Theorem 2.2 [10],
\[
\| f_1(t) - g(t) \| + \| f_2(t) - g(t) \| \leq \| f_1(t) - h(t) \| + \| f_2(t) - h(t) \|
\] (3.6)
for almost all \( t \in I \). But \( 0 \in G \). Thus
\[
\| f_1(t) - g(t) \| + \| f_2(t) - g(t) \| \leq \| f_1(t) \| + \| f_2(t) \|
\] (3.7)
or
\[
\| g(t) \| \leq \| f_1(t) \| + \| f_2(t) \|.
\] (3.8)
Hence \( g \in L^\Phi(I, G) \) and
\[
\| \| f_1(\cdot) - g(\cdot) \| + \| f_2(\cdot) - g(\cdot) \| \|_\Phi \leq \| \| f_1(\cdot) - h(\cdot) \| + \| f_2(\cdot) - h(\cdot) \| \|_\Phi
\] (3.9)
for all \( h \in L^1(I, G) \). \( \Box \)

**Theorem 3.3.** Let \( G \) be a 1-summand subspace of the Banach space \( X \). Then \( L^\Phi(I, G) \) is simultaneously proximinal in \( L^\Phi(I, X) \).

The proof follows from Theorem 3.2 and [10, Theorem 2.4].

**Theorem 3.4.** Let \( G \) be a reflexive subspace of the Banach space \( X \). Then \( L^\Phi(I, G) \) is simultaneously proximinal in \( L^\Phi(I, X) \).

The proof follows from Theorem 3.2 and [10, Theorem 3.2].

**References**


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Space dynamics is a very general title that can accommodate a long list of activities. This kind of research started with the study of the motion of the stars and the planets back to the origin of astronomy, and nowadays it has a large list of topics. It is possible to make a division in two main categories: astronomy and astrodynamics. By astronomy, we can relate topics that deal with the motion of the planets, natural satellites, comets, and so forth. Many important topics of research nowadays are related to those subjects. By astrodynamics, we mean topics related to spaceflight dynamics.

It means topics where a satellite, a rocket, or any kind of man-made object is travelling in space governed by the gravitational forces of celestial bodies and/or forces generated by propulsion systems that are available in those objects. Many topics are related to orbit determination, propagation, and orbital maneuvers related to those spacecrafts. Several other topics that are related to this subject are numerical methods, nonlinear dynamics, chaos, and control.

The main objective of this Special Issue is to publish topics that are under study in one of those lines. The idea is to get the most recent researches and published them in a very short time, so we can give a step in order to help scientists and engineers that work in this field to be aware of actual research. All the published papers have to be peer reviewed, but in a fast and accurate way so that the topics are not outdated by the large speed that the information flows nowadays.

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Since the current literature on dynamics and control is scattered over a range of journals, books, chapters of books, and a large number of conference proceedings, which are often difficult to obtain, the goal of this special issue of MPE is to present papers, containing complete reviews on dynamics models, available in the current literature, to classify them, and to discuss their applications and limitations. In this special issue, the authors could recommend appropriate models and control criteria for various applications on engineering and sciences and suggest directions for further works. It is also open for critical reviews, open problems, and future developments. Discussions on MEMS and NEMS are also encouraged.

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Special Issue on
Nonlinear Vibrations, Stability Analysis and Control

Call for Papers

Important advances in mathematics, physics, biology, and engineering science have shown the importance of the analysis of instabilities and strongly coupled dynamical behavior. New investigation tools enable us to better understand the dynamical behavior of more complex structures. However, the increasing interest in mechanical structures with extreme performances has propelled the scientific community toward the search for solution of hard problems exhibiting strong nonlinearities. As a consequence, there is an increasing demand both for nonlinear structural components and for advanced multidisciplinary and multiscale mathematical models and methods. In dealing with the phenomena involving a great number of coupled oscillators, the classical linear dynamic methods have to be replaced by new specific mathematical tools.

This special issue aims to assess the current state of nonlinear structural models in vibration analysis, to review and improve the already known methods for analysis of nonlinear and oscillating systems at a macroscopic scale, and to highlight also some of the new techniques which have been applied to complex structures.

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