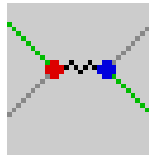
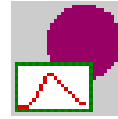


Part IV  
Modern Physics





## 15 Black Body Radiation



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### 15.1 Introduction

If we heat up a body, it will emit light. For example, think of a hot filament inside a light bulb, or the glowing charcoals on your barbecue. This radiation is called thermal radiation. Basically, it comes about because the moving charges in the body act like little antennas. It should thus be possible, in principle, to calculate it by combining thermodynamic theory of thermal equilibrium with the classical theory of electromagnetism.

Classical physics, however, does not do a great job at predicting the spectral radiation distribution curve, i.e. how much light is emitted at a given wavelength, especially in the short wavelength (high frequency) region. In fact, if the classical theory were right we would be in real danger in front of our barbecue, as we would be bombarded with x-rays. Planck's theory of quantized energies explained why this does not happen.

In this assignment we will become familiar with Planck's blackbody theory and how it solved the "ultraviolet catastrophe" predicted by classical physics.

### 15.2 Theory

#### 15.2.1 Fundamentals

As usual in physics, we start by studying an idealized case to make the study of this otherwise rather formidable looking problem tractable. Light colored opaque objects reflect most of the light that falls on them whereas dark ones absorb most of it. This means a dark opaque object will heat up more than a light one. If it is in thermal equilibrium with its surroundings it will also have to emit light to stay in equilibrium. So, let's study light emission by a dark, or ideally, a black body that gets heated by radiation falling upon it. The simplest ideal *black* body, i.e. a body that absorbs *all* incident radiation, is a small hole leading into a cavity. The light can only enter through the hole and cannot escape before it is totally absorbed by the walls of the cavity. The radiation can only escape through the hole. It turns out that the radiation of such a black body does not depend on the material used for the walls of the cavity. It only depends on the temperature. The quantity that one measures and that



we wish to calculate is the radiancy  $R$ , that is the power emitted per unit area. Experimentally it was found that:

$$R \propto \sigma T^4 \quad (109)$$

with  $T$  the temperature in degrees Kelvin, and  $\sigma$  the Stefan-Boltzmann constant:  $\sigma = 5.67 \times 10^{-8}$  Watt/meter<sup>2</sup> Kelvin<sup>4</sup>.

Instead of the total power, one measures the power as function of the wavelength  $\lambda$  of the emitted light. This is the so-called spectral distribution  $R(\lambda)$  of the radiation emitted.

Now, we can measure this whole  $R(\lambda)$  curve at various temperatures and thus obtain  $R(\lambda, T)$ . Precise measurements show that at each temperature  $R(\lambda, T)$  has a maximum at some particular wavelength  $\lambda_{max}$  and that this maximum of the spectral radiancy shifts towards shorter wavelengths with increasing temperature. This is given by Wien's displacement law:

$$\lambda_{max} T = 2.898 \times 10^{-3} \text{meter.Kelvin.} \quad (110)$$

At first, the black body emits mostly in the infrared. At higher temperatures, it emits mostly visible light, gradually moving from red to violet. At any "reasonable" temperature, the radiancy drops fast to zero for wavelengths shorter than  $10^{-6}$  m. Where classical theories were fitting the experimental data well in the long wavelength region, they totally failed for the shorter wavelengths. In fact, they predicted that the curve did not have a maximum at all but continued to go up to for shorter and shorter wavelengths.

Indeed, Rayleigh and Jeans' classical theory led to the following equation:

$$R(\lambda, T) = \frac{2\pi ckT}{\lambda^4} \quad (111)$$

with  $c$  the speed of light,  $k$  the Boltzmann constant,  $T$  the absolute temperature. The results of this equation for small wavelengths is known as the "ultraviolet catastrophe". A catastrophe indeed! Imagine the intensity of UV-light and x-rays!

Planck first fitted a curve to the experimental data. That still did not explain why, of course, but eventually he also came up with an explanation of why the classical theory didn't function. If we think of the atoms in the walls of the cavity as harmonic oscillators, their energy in the classical equipartition theory could be any value  $kT$ . Planck assumed instead that the oscillator energy was quantized. It could only take discrete values. The spectral radiation formula deduced by Planck is:

$$R(\lambda, T) = \frac{2\pi hc^2 \lambda^{-5}}{e^{hc/\lambda kT} - 1} \quad (112)$$

This formula fits the experimental curve both at long and short wavelengths and agrees with the classical theory for large  $\lambda$ . Planck found a value for  $h$  by fitting

his theoretical curve to the experimental one. Later the same value was obtained experimentally in the photoelectric effect:  $h = 6.626 \times 10^{-34}$  Joule.second.

Enjoy your barbecue!

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As we saw, Plank assumed that the oscillator energy was quantized. It could only take discrete values:

$$E_n = nhf \quad (113)$$

with  $n$  an integer and  $h$  Planck's constant. The absorption or emission of energy would then also go by jumps, and not continuously. In this case, the probability that a harmonic oscillator would have an energy  $E_n$  is given by:

$$P_n = Ce^{-E_n/kT} \quad (114)$$

with  $C$  determined by the normalization:

$$\sum_{n=0}^{\infty} P_n = 1 \quad (115)$$

The average energy of one oscillator is then not  $kT$  but:

$$\bar{E} = \sum_{n=0}^{\infty} E_n P_n \quad (116)$$

When Eq. (113) is substituted in here, you obtain after doing the sums that

$$\bar{E} = \frac{hf}{e^{hf/kT} - 1} = \frac{hc/\lambda}{e^{hc/\lambda kT} - 1} \quad (117)$$

This equation agrees with the classical theory for  $f \rightarrow 0$  and gives  $\bar{E} = kT$ .

The classical theory is correct in how to calculate the radiation once we know the average energy per oscillator. So, we can just replace the classical average energy  $kT$  in Eq. (111) by Eq. (117) to get Plank's spectral radiation formula (112).

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## 15.3 Laboratory

### 15.3.1 Experiment A: Introduction and Setup

In this experiment, we will familiarize ourselves with the laboratory, especially with the display.

1. In the PEARLS window, select *Modern Physics*.



2. Now launch the *Black Body Radiation* item.
3. You will now see the experiment window, headed by a toolbar, with a control window at the left and a laboratory display at the right.
4. Adjust the window size as desired.
5. Move the laboratory display to a convenient location on your screen by dragging the display with your mouse.
6. The default settings for this experiment are:

<i>Temperature</i>	$1.100 \times 10^3 \text{ K}$
<i>Planck's Constant</i>	$6.630 \times 10^{-34} \text{ Js}$
<i>Zoom</i>	1.000 pixels/meter

7. You should see a red “black” body on a black background with the calculated message *spectral intensity peaks at  $2.64 \times 10^{-6} \text{ m}$* . As default the calculated graph *Spectral Intensity versus Wavelength* is displayed. The calculated integrated intensity (that is the radiancy) and the peak spectral intensity are written on the display.
8. Change the temperature to its minimum value.
  - (a) What happened to the color of the body?
  - (b) How did the graph change?
  - (c) Did the wavelength at the peak change?
  - (d) In which region of the spectrum does the intensity peak?
  - (e) Did the intensity of the emission change?

### 15.3.2 Experiment B: The wavelength at the peak and the temperature

In this experiment we will study how the wavelength at the peak of the emission relates to the temperature of the body.

1. Start up the PEARLS *Black Body Radiation* experiment, or if it is already running, restore the startup conditions by clicking the  $\leftrightarrow$  button or choosing *Revert* from the *Simulation* menu.
2. Set the temperature to 700K.
  - (a) What is the wavelength at the emission peak?
  - (b) In what region of the electromagnetic spectrum is the emission peak?
  - (c) Multiply the peak wavelength with the temperature. What is the result?

3. Set the temperature to 1400 K, twice its first value.
  - (a) What is the wavelength at the emission peak?
  - (b) How does this wavelength compare to the one in 2a?
  - (c) In what region of the electromagnetic spectrum is the emission peak?
  - (d) Multiply the peak wavelength with the temperature. What is the result?
4. Set the temperature to 2800 K, four times its minimum value.
  - (a) What is the wavelength at the emission peak?
  - (b) How does this wavelength compare to the one in 2a?
  - (c) In what region of the electromagnetic spectrum is the emission peak?
  - (d) Multiply the peak wavelength with the temperature. What is the result?
  - (e) Comparing the results from 2c, 3d, 4d, what might you conclude?
5. Restore the default values.
6. Turn off the automatic scaling in the  $x$ - axis of the plot. (Click the ? button in the graph window. Turn off the *Automatic Scaling in X-axis*).
7. Move the temperature slowly to higher values using the slider. Observe the plot.
  - (a) Does the intensity peak move towards higher or lower wavelengths?

### 15.3.3 Experiment C: Temperature and the color of the body

In this experiment we observe the appearance of the black body while changing its temperature.

1. Start up the PEARLS *Black Body Radiation* experiment, or if it is already running, restore the startup conditions by clicking the  $\leftrightarrow$  button or choose *Revert* from the *Simulation* menu.
2. Set *Temperature* equal to 300 K.
  - (a) What color does the body have?
  - (b) Does it emit any visible light? The visible spectrum is displayed on the plot.
  - (c) Does the graph tell you the same information?
  - (d) What is the value of the peak intensity?
3. Set the temperature equal to 1500 K.



- (a) What color does the body have?
  - (b) Does it emit any visible light?
  - (c) Does the graph tell you the same information?
  - (d) How does the peak intensity compare to the one in 2d?
4. Set the temperature equal to 5000 K.
- (a) What color does the body have?
  - (b) Is the wavelength of the visible emitted light shorter or longer than in 3a?
  - (c) Does the graph tell you the same information?
  - (d) How does the peak intensity compare to the one in 2d and 3d?
5. Set the temperature equal to 10000 K.
- (a) What color does the body have?
  - (b) How does the peak intensity compare to the one in 2d and 3d and 4d?

#### 15.3.4 Experiment D: Radiancy

In this experiment we will study how the radiancy (the total energy radiated per unit of time and per unit of area if the emitter) changes with the temperature.

1. Start up the PEARLS *Black Body Radiation* experiment, or if it is already running, restore the startup conditions by clicking the  $\leftrightarrow$  button or choose *Revert* from the *Simulation* menu.
2. In the graph window click on the ?. Change the maximum  $y$  value to  $4 \times 10^{11} \text{ W/m}^3$ .
3. Set *Temperature* equal to 500 Kelvin.
  - (a) What is the value of the integrated intensity?
4. Set *Temperature* equal to 1000 Kelvin, twice the original value.
  - (a) What is the value of the integrated intensity? Compare to step 3a. Compare to the change in temperature.
  - (b) The integrated intensity is the area under the curve. Compare the area in this step with the one from step 3a.
5. Set *Temperature* equal to 2000 Kelvin.
  - (a) What is the value of the integrated intensity? Compare to step 3a and step 4a. Compare to the change in temperature.
  - (b) The integrated intensity is the area under the curve. Compare the area in this step with the one from step 3a and step 4a.



### 15.3.5 Experiment E: The ultraviolet catastrophe

In this experiment we observe what happens when Planck's quantization is not really taken into account in calculating the radiation emission as function of the wavelength.

1. Start up the PEARLS *Black Body Radiation* experiment, or if it is already running, restore the startup conditions by clicking the  $\leftrightarrow$  button or choose *Revert* from the *Simulation* menu.
2. Set the temperature to 700 K.
  - (a) What is the peak emission intensity?
  - (b) What is the peak wavelength?
  - (c) In what region of the electromagnetic spectrum does the emission peak?
3. Set *Planck's Constant* to its minimum value. This value is not zero as the classical limit would be, but we can study very well the tendency of what will happen.
  - (a) What is the peak emission intensity?
  - (b) What is the peak wavelength?
  - (c) In what region of the electromagnetic spectrum does the emission peak theoretically?
  - (d) Does the appearance of the body agree with 3c?
4. Set the temperature to 750 K.
  - (a) How does this peak intensity at this low temperature compare with 2a?
  - (b) In what region of the electromagnetic spectrum does the emission peak theoretically?
  - (c) Does the appearance of the body agree with 4b?
5. Set the temperature to 900 K.
  - (a) How does this peak intensity at this lower temperature compare with 2a?
  - (b) In what region of the electromagnetic spectrum does the emission peak theoretically?
  - (c) Does the appearance of the body agree with 5b?
6. Set the temperature to 1500 K.



- (a) In what region of the electromagnetic spectrum does the emission peak theoretically?
7. Set the temperature to the sun's temperature, 5000 Kelvin.
- (a) In this theory, is the sun emitting visible light?
8. Slide the temperature to higher values. The body would be emitting x-rays as soon as the temperature is higher than 40,000K.

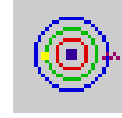
### 15.4 Review

Please circle the correct answers to the following multiple-choice questions. You may use PEARLS to help you find the correct answer.

1. We can think of the sun as a black body radiator. The wavelength at which its radiation emission peaks is in the middle of the visible spectrum:  $5.8 \times 10^{-7}\text{m}$ . The surface temperature of the sun is:
- (a) 1800K  
(b) 5000K  
(c) 7000K
2. If the temperature of a black body is doubled, the total energy radiated per unit time is:
- (a) doubled.  
(b)  $2.9\times$  the original value.  
(c) 16 times the original value.
3. By increasing temperatures, the black body's peak emission will be light :
- (a) with higher frequency  
(b) with longer wavelength  
(c) in the ultraviolet region
4. The appearance of a black body can tell something about its temperature. Increasing the temperature the black body will turn successively: (some intermediate colors might be skipped)
- (a) yellow, blue, white.  
(b) blue, red, white.  
(c) white, blue, red.  
(d) red, white, blue.

5. Planck's radiation law is consistent with the classical laws for:
  - (a) short wavelengths.
  - (b) long wavelengths.
  - (c) the ultraviolet region.
6. If a black body A has a temperature of 3000K and another black body B has a temperature of 6000K, the total power emitted by A can be bigger than the total power emitted by B when:
  - (a) the surface of A < the surface of B.
  - (b) the surface of A > the surface of B.
  - (c) not true.
7. If the peak wavelength of a blackbody at temperature 300 K is  $\lambda$ , at  $T = 3000$  K the peak wavelength will be:
  - (a)  $10\lambda$
  - (b)  $\lambda/10$
  - (c)  $10^4\lambda$
  - (d)  $\lambda/10^4$
8. Planck's theory about quantization of harmonic oscillators is not in contradiction with the results found for large scale oscillators because for these oscillators the quantum numbers are enormous.
  - (a) True.
  - (b) False.
9. A tungsten lamp filament has a temperature of 1800 K. The maximum spectral radiance is in:
  - (a) the visible region.
  - (b) the infrared region.
  - (c) the ultraviolet region.
10. The area under the spectral radiance curve is proportional to:
  - (a) the surface of the body.
  - (b)  $T^4$ , with  $T$  the temperature of the body.

## 16 Bohr Atom



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### 16.1 Prerequisite

A great help to this assignment is the simulation *Circular Motion*.

### 16.2 Introduction

The Bohr model of the hydrogen atom was the first partially successful attempt to describe the newly discovered “quantization” of atomic spectra. Although it was later abandoned in favor of the fully quantum mechanical description, its position in between classical mechanics and quantum mechanics makes it an important model to study.

Classical mechanics fails to describe processes at the atomic level. On the other hand it sounds reasonable to expect that whatever changes have to be made to classical mechanics to describe atomic behavior, the results of this new theory for the macroscopic world must be the same as those obtained with classical mechanics.

In this assignment we will become acquainted with the idea of quantum numbers and discrete states of the hydrogen atom. It is also useful to understand emission and absorption of radiation. We will be able to change the character of the “real” H-atom and see the impact of a differently charged nucleus or a heavier particle (e.g. a muon) in orbit.

### 16.3 Theory

#### 16.3.1 The Hydrogen Emission Spectrum

Classical mechanics predicts that if an electron circles a positively charged nucleus it will radiate light, which means that the electron will be losing energy and its orbit will become smaller and smaller. Finally the electron will end up in the nucleus. This means that all atoms would be radiating for a short time and then collapse.

Balmer observed the radiation of visible light from hydrogen and found that it does not contain all frequencies (or wavelengths) as classical mechanics predicts, but it exists out of a number of discrete frequencies (or wavelengths). The emission spectrum is not continuous but exists out of discrete spectral lines.

Rydberg and Ritz had found a general expression for the wavelength  $\lambda$  of the different spectral lines. They found that :

$$\frac{1}{\lambda} = R \left( \frac{1}{m^2} - \frac{1}{n^2} \right) \quad (118)$$

with  $n > m$  and  $n, m$  integers.  $R$  is the Rydberg constant. Its value slightly changes from element to element. For hydrogen  $R = 1.096776 \times 10^7 m^{-1}$ . The several lines in the spectra are (named as their discoverers)

- $n = 1, m = 2, 3, 4 \dots$  the Lyman series, the lines in the ultraviolet region.
- $n = 2, m = 3, 4, 5 \dots$  the Balmer series, the lines in the visible region.
- $n = 3, m = 4, 5, 6 \dots$  the Paschen series, the lines in the infrared region.

The hydrogen atom does not emit electromagnetic radiation at room temperature but has to be excited by an electrical discharge or by extreme heating before you can measure an emission spectrum. On the other hand when white light (light with all frequencies) is passed through a hydrogen gas at room temperature the gas will absorb light. It will only absorb those frequencies that coincide with the frequencies of the emission spectrum.

### 16.3.2 Bohr's Model

Classical mechanics obviously fails to describe what really goes on. To avoid the collapsing atom, Bohr postulated that the electron could move in certain orbits without radiating. These orbits are not arbitrary but have certain energies (for an analogy: see the assignment *Circular Orbit* in *Mechanics*). If an electron drops from an excited state to a lower energy state it will emit light. The frequency of the emitted light is not the frequency of the motion of the electron as the law of electrodynamics predict. The frequency is determined by the energy conservation. We have to remember that the energy  $E$  of light is connected to its frequency  $f$  by:  $E = hf$  with  $h$  Planck's constant. The frequency of the emitted light will be:

$$hf = E_i - E_f \quad (119)$$

with  $E_i$  the energy of the initial state or orbit of the electron and  $E_f$  the energy of the final state.

Let's consider an electron (with charge  $-e$ ) in a circular orbit with radius  $r$  moving about a nucleus with charge  $Ze$  with  $Z$  the atomic number. The Coulomb force between nucleus and electron is the centripetal force necessary to keep the electron in a circular orbit. After calculating the classical kinetic and potential energy for this model, you will easily find that the total energy  $E$  of the electron is:

$$E = -\frac{1}{2} \frac{kZe^2}{r} = -\frac{1}{2} mv^2 \quad (120)$$



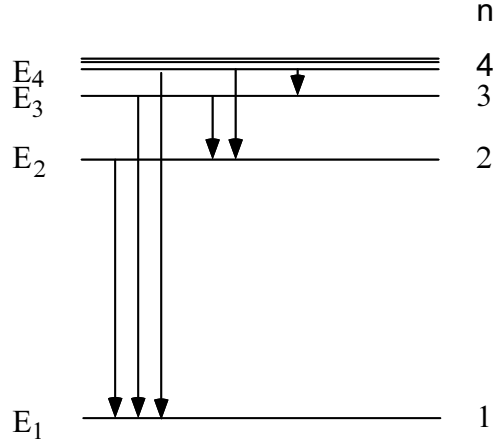


Figure 12: Some Transitions between Energy levels.

with  $k = 8.99 \times 10^9$  Newton meter<sup>2</sup>/C<sup>2</sup>. According to Bohr's postulate, if the electron changes from one initial state  $E_i$  to a final state  $E_f$  the frequency of the radiation will be:

$$f = \frac{E_i - E_f}{h} = -\frac{1}{2} \frac{kZe^2}{h} \left( \frac{1}{r_i} - \frac{1}{r_f} \right) \quad (121)$$

If we compare this with the findings of Balmer, Ritz and Rydberg (Equation (118)) for  $1/\lambda$ , taking into account that  $f = c/\lambda$  we find that Bohr's model could explain the spectra if the radii of the orbits were proportional to squares of integers.

This agrees with Bohr's postulate that the magnitude of the angular momentum of the electron is quantized or:

$$L = mvr = n \frac{h}{2\pi} = n\hbar \quad (122)$$

with  $n$  an integer and  $\hbar = h/2\pi$ . The constant  $\hbar$ , read  $h$  bar, is often more convenient than  $h$ , just as angular frequency  $\omega$  is often more convenient than frequency  $f$ . By eliminating  $v$  from Eqs. (121) and (122), this leads to the expression for the radii of the stable orbits:

$$r = n^2 \frac{\hbar^2}{mkZe^2} = n^2 \frac{a_0}{Z} \quad (123)$$

with

$$a_0 = \frac{\hbar^2}{mke^2} \quad (124)$$

the first ( $n = 1$ ,  $Z = 1$ ) Bohr radius. Knowing that:

- Planck's constant  $h = 6.63 \times 10^{-34}$  Joule.second
- the Coulomb constant  $k = 8.99 \times 10^9$  Newton.meter<sup>2</sup>/Coulomb<sup>2</sup>
- the electron mass  $m = 9.11 \times 10^{-31}$  kg

we can calculate the first Bohr radius:

$$a_0 = 0.529\text{\AA} \quad (125)$$

with one Angström =  $1\text{\AA} = 10^{-10}$  m. Equation (125) means that the diameter of the hydrogen atom predicted by this model is about one Angström, in good agreement with experiment. The possible values of the energy of a hydrogen atom predicted by Bohr are then:

$$E_n = -\frac{mk^2e^4}{2\hbar^2} \frac{Z^2}{n^2} = -Z^2 \frac{E_0}{n^2} \quad (126)$$

with  $E_0 = 13.6$  eV. The unit eV is the abbreviation of electron volt: the unit for energy used in atomic physics.  $1 \text{ eV} = 1.60 \times 10^{-19}$  joule.  $E_0$  is the *ground state energy*, that is the lowest possible total energy of the hydrogen atom. Any state with a quantum number higher than  $n = 1$  will have a higher energy (less negative), the electron will be less bound in the atom, and less energy is needed to free the electron. We call these states *excited states*. The atom will tend to make a transition to a lower stationary state, to obtain a lower total energy. The amount of energy by which the energy of the excited state exceeds that of the ground state is the *excitation energy*.

The *binding energy* of an electron or *ionization energy* is the energy that must be added to the atom on any state to free the bound electron. For example, the binding energy of an electron in a H-atom in the groundstate is:  $E_B = 13.6$  eV. The energy needed to ionize the hydrogen atom is 13.6 eV. If an electron is not bound in the atom the binding energy is zero.

## 16.4 Laboratory

### 16.4.1 Experiment A: Introduction and Setup

In this experiment, we will familiarize ourselves with the laboratory, especially with the display. (Depending on your computer some aliasing might occur when the electron has a high speed, i.e. in the orbit with quantum number 1. It will appear to run slower and in the opposite direction than in the other orbits).

1. In the PEARLS window, select *Modern Physics*.
2. Now launch the *Bohr Atom* item.
3. You will now see the experiment window, headed by a toolbar, with a control window at the left and a laboratory display at the right.



4. Adjust the window size as desired.
5. Move the laboratory display to a convenient location on your screen by dragging the display with your mouse.
6. The default settings for this experiment are:

<i>Quantum Number</i>	3
<i>Orbits</i>	4
<i>Atomic Number</i>	1
<i>Electron Mass</i>	$9.109 \times 10^{-31}$ kg
<i>Planck Constant</i>	$6.626 \times 10^{-34}$ Js
<i>Zoom</i>	$1.323 \times 10^{11}$ pixels/meter
<i>Time Zoom</i>	$5.702 \times 10^{-16}$

7. You should see a nucleus, an electron on its “green” current orbit (the third orbit from the nucleus), more orbits, and two buttons *Emit* and *Absorb*. By default the energy level diagram is shown, with the energy of the electron in its current orbit. The Bohr Radius of the current orbit is displayed.
8. Choose *Time* from the *Display* menu.
9. Run the simulation and observe the electron completing a whole orbit.
  - (a) Which orbit does the electron occupy?
  - (b) How much time does the electron need to complete an orbit? If you have a hard time measuring the time, let the electron complete several orbits or run the simulation in step mode.
10. Change *Quantum number* to 2 in the control window.
11. Run the simulation and observe the electron completing a whole orbit. *Reset*.
  - (a) Which orbit does the electron occupy?
  - (b) Does the electron need more or less time to complete an orbit?
  - (c) What is the value of the electron energy?
12. Run the experiment again.
13. Click on *Emit*. Observe and *Reset*.
  - (a) What happened to the electron?
  - (b) Look at the energy level graph. Did the energy of the electron change?



14. Run the experiment again.
15. Click on *Absorb*. Observe and reset.
  - (a) What happened to the electron?
  - (b) Look at the energy level graph. Did the energy of the electron change?

#### 16.4.2 Experiment B: Electron Energy Levels and the Quantum Number

In this experiment we will study how the energy levels of the electron vary with the quantum number.

1. Start up the PEARLS *Bohr Atom* experiment, or if it is already running, restore the startup conditions by clicking the defaults button.
  - (a) What is the number of protons in the nucleus?
  - (b) What is the quantum number of the current orbit of the electron?
  - (c) What is the energy of the electron?
  - (d) What is the binding energy of this electron?
2. Run the experiment, let the electron complete one orbit and reset.
3. Set *Quantum Number* equal to 1.
  - (a) What is the energy of the electron?
  - (b) What is the binding energy of the electron?
  - (c) Does a lower energy state exist for this system?
  - (d) Divide this energy by the one found in 1c. What is the result?
  - (e) Recall equation (123). What is the radius of its orbit?
4. Set *Quantum Number* equal to 2.
  - (a) What is the energy of the electron?
  - (b) Divide the ground state energy by this energy. What is the result?

#### 16.4.3 Experiment C: The Emission and Absorption Spectrum of Hydrogen

In this experiment we will study how Bohr's model of the hydrogen atom does help us understand some elements of the emission and absorption spectra of hydrogen.



1. Start up the PEARLS *Bohr Atom* experiment, or if it is already running, restore the startup conditions by clicking the defaults button (the button with the yellow arrow).
2. Set *Quantum Number* equal to 2.
3. Run the experiment and click *Emit*. Reset.
  - (a) How much energy did the electron lose?
  - (b) What is the energy of the emitted light?
  - (c) What is the frequency of the emitted light?
  - (d) Which line of the emission spectrum of hydrogen would this be?
4. Set *Quantum Number* equal to 3.
5. Run the experiment and click *Emit*. Reset.
  - (a) How much energy did the electron lose?
  - (b) What is the energy of the emitted light?
  - (c) What is the frequency of the emitted light?
  - (d) Which line of the emission spectrum of hydrogen would this be? Recall equation (118).
6. Set *Quantum Number* equal to 4.
7. Run the experiment and click *Emit*. Reset.
  - (a) How much energy did the electron lose?
  - (b) What is the energy of the emitted light?
  - (c) What is the frequency of the emitted light?
  - (d) Which line of the emission spectrum of hydrogen would this be?
8. Set *Quantum Number* equal to 1.
9. Run the experiment and click *Absorb*. Reset.
  - (a) How much energy did the electron gain?
  - (b) What is the energy of the absorbed light?
  - (c) What is the frequency of the absorbed light?

#### 16.4.4 Experiment D: “Hydrogenic” Atoms

In this experiment we will study how Bohr’s model of the hydrogen atom does help us understand the energy states of atoms with one electron but more protons in the nucleus (the atomic number is higher). An example of such a “hydrogenic atom” is  $Li^{++}$ .

1. Start up the PEARLS *Bohr Atom* experiment, or if it is already running, restore the startup conditions by clicking the defaults button (the button with the yellow arrow).
  - (a) What is the electron energy for  $n = 3$ ?
  - (b) Observe the radii of the orbits.
2. Set *Atomic Number* equal to 3.
  - (a) How do the orbit radii compare to the radii in step 1b?
  - (b) What is the electron energy for  $n = 3$ ?
  - (c) Divide this energy by the energy from step 1a. What is the result?
3. Set *Atomic Number* equal to 4.
  - (a) How do the orbit radii compare to the radii in step 1b and step 2a?
  - (b) What is the electron energy for  $n = 3$ ?
  - (c) Divide this energy by the energy from step 1a. What is the result?
4. Run the experiment and click *Emit*.
  - (a) How much energy did the electron lose?
  - (b) What is the frequency of the emitted photon?
5. Set *Quantum Number* equal to 1.
  - (a) What is the groundstate energy of this hydrogenic atom?
  - (b) How does this groundstate energy compare to the H-atom one?

#### 16.4.5 Experiment E: Another Particle in Orbit

In this experiment we will study how Bohr’s model of the hydrogen atom does help us understand what would happen if another particle, other than an electron, is in orbit.

1. Start up the PEARLS *Bohr Atom* experiment, or if it is already running, restore the startup conditions by clicking the defaults button.
2. Set the *Quantum Number* equal to 1.



- (a) What is the energy of the electron?
  - (b) What is the value of the bohr radius?
3. Set the mass of the electron to its minimum value, that is ten times smaller than its real mass.
- (a) What happened to the orbit radii?
  - (b) What is the ground state energy of the particle?
  - (c) How does this energy compare to the one in step 2a?
  - (d) How does the "new Bohr Radius" compare to the one in step 2b?
4. Set the mass of the electron to its maximum value, that is ten times bigger than its real mass.
- (a) What happened to the orbit radii?
  - (b) What is the ground energy of the particle?
  - (c) How does this energy compare to the one in step 2a?
  - (d) How does the "new Bohr Radius" compare to the one in step 2b?

#### 16.4.6 Experiment F: The Classical Theory

In this experiment we will see what the classical limit is of the energy of the bound electron, i.e. if we do not believe in quantization. We also will see that the classical theory cannot explain the measured emission spectra.

1. Start up the PEARLS *Bohr Atom* experiment, or if it is already running, restore the startup conditions by clicking the defaults button (the button with the yellow arrow).
2. Set *Planck's Constant* equal to its minimum value.
3. Set *Zoom* to its maximum value. This will give you a better view of what happened.
  - (a) What happened to the orbits?
  - (b) If Planck's Constant would be zero, in other words, in the classical theory where would the electron end up?

#### 16.5 Review

Please circle the correct answers to the following multiple-choice questions. You may use PEARLS to help you find the correct answer.

1. A muon has the same charge as an electron, but its mass is  $105.7 \text{ MeV}/c^2$ , while the electron's mass is  $0.511 \text{ MeV}/c^2$ . The ground state energy of a muonic atom (one proton and a muon in orbit) will be:

- (a)  $-2813 \text{ eV}$
  - (b)  $-0.065 \text{ eV}$
2. The longest wavelength in the Lyman series ( $n_f = 1$ ) of the emission spectra of the hydrogen atom is the wavelength of the light emitted by an electron making a transition from
- (a) the first excited state ( $n = 2$ ) to the groundstate.
  - (b) the highest excited state to the groundstate.
3. To ionize a hydrogen atom the electron has to be removed. If the hydrogen atom is in its groundstate, the atom has to absorb light with minimum frequency:
- (a)  $3.29 \times 10^{15} \text{ Hz}$
  - (b)  $9.10 \times 10^{12} \text{ Hz}$
4. If the light to ionize a hydrogen atom in its groundstate has as minimum frequency  $f$ , the light to ionize a hydrogen atom not in its groundstate, but in the state  $n = 3$ , need to have a minimum frequency of:
- (a)  $f$
  - (b)  $f/3$
  - (c)  $f/9$
  - (d)  $3f$
  - (e)  $9f$
5.  $\text{He}^+$  (once ionized helium) is a “hydrogenic atom”. If we compare the first Bohr radius of hydrogen  $a_0$  with the one of  $\text{He}^+$ , the latter will be (Helium’s atomic number is 2 ( $Z=2$ )):
- (a)  $a_0/2$
  - (b)  $a_0/4$
  - (c)  $2a_0$
  - (d)  $4a_0$
  - (e)  $a_0$
6. The groundstate energy of the  $\text{He}^+$  ion, compared to the hydrogen groundstate energy  $E_0$  is (hydrogen’s atomic number  $Z$  is 1, helium’s atomic number  $Z$  is 2):
- (a)  $E_0/2$
  - (b)  $E_0/4$



- (c)  $2E_0$
  - (d)  $4E_0$
  - (e)  $E_0$
7. In Bohr's model the frequencies of the spectral lines in the emission spectra of  $He^+$  are easily found, knowing the hydrogen (H) spectral lines by (remember: hydrogen's atomic number  $Z$  is 1, helium's atomic number  $Z$  is 2):
- (a) dividing the H frequencies by 4.
  - (b) dividing the H frequencies by 2.
  - (c) multiplying the H frequencies by 4.
  - (d) multiplying the H frequencies by 2.
  - (e) neither of the above.
8. In Bohr's model of the hydrogen atom we can compare the speed of the electron in the groundstate  $v_0$  and the one in the first excited state  $v_1$ :
- (a)  $v_1 = v_0$
  - (b)  $v_1 = 2v_0$
  - (c)  $v_1 = 4v_0$
  - (d)  $v_1 = v_0/2$
  - (e)  $v_1 = v_0/4$
9. An electron and a proton are separated by a large distance. The electron approaches the proton with a kinetic energy of 1eV. The electron is captured by the proton to form a hydrogen atom in the ground state. The energy of the emitted photon will be:
- (a) 13.6 eV
  - (b) 14.6 eV
  - (c) 12.6 eV
10. The excitation energy is defined as the difference in energy of the electron in the excited state and in the groundstate. A hydrogen atom is in an excited state, for which the electron has an energy  $-3.40$  eV. The atom makes a transition to a state which excitation energy is 12.75 eV. A photon with energy:
- (a) 2.55 eV was absorbed.
  - (b) 2.55 eV was emitted.
  - (c) 12.75 eV was absorbed.
  - (d) 12.75 eV was emitted.

## 17 Brownian Motion



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### 17.1 Introduction

Perhaps no phenomenon illustrates quite so clearly the fundamentally probabilistic nature of the universe at the microscopic level as does Brownian Motion. This effect was discovered by the botanist Robert Brown while studying particles of pollen immersed in a liquid under the microscope.

Einstein was the first to develop a proper theory to describe this phenomenon. Perrin closely observed the wanderings of small suspended particles under the microscope and calculated Avogadro's Number, finding that Einstein's theory fit the experiment.

In this assignment we will observe the motions of simulated brownian particles. In so doing we will become acquainted with some important statistical methods in physics.

### 17.2 Theory

Einstein concluded that the irregular motions of the tiny particles in a fluid result from continual collisions with the molecules comprising the fluid. The molecules are in perpetual motion, constantly colliding with each other and with the Brownian particles. Experiments have found that the movement increases with the temperature of the fluid and decreases with the viscosity of the fluid. The movement also decreases as the size of the particles is increased. The density of the individual particles generally has little effect.

It is not possible to calculate the trajectory of a brownian particle in detail, owing to the huge number of molecules with which it is interacting. We can, however, calculate the *average* behavior of a large number of particles. For example, it is possible to calculate how far particles move on average from a set starting point as a function of time.

Imagine that we carry out a large number of brownian motion experiments, keeping track of the distance of each particle from its starting point as a function of time. If we square this number and then average over all the particles, we obtain the *mean-square displacement*, denoted by  $\langle r^2 \rangle$ . In mathematical terms this may be written as

$$\langle r^2 \rangle = (r_1^2 + r_2^2 + r_3^2 + \dots + r_n^2)/n \quad (127)$$



for  $n$  particles.

The movement of the brownian particles may be described by a model from the science of statistics known as the "random walk". On the basis of this model, one can derive the interesting result that  $\langle r^2 \rangle$  is proportional to time. This remarkable result shows that even a completely random process is susceptible to mathematical analysis.

### Optional Theory

What is the proportion factor? We make the assumption that no external forces act on the particles, but only internal ones. These forces are the drag  $\vec{F}_{drag}$  of the fluid on the particles (see assignment: *Viscous flow* in *Fluid Mechanics*) and the fluctuating force  $\vec{F}_{collisions}(t)$  due to the collisions. This means that Newton's law becomes:

$$m\vec{a} = \vec{F}_{drag} + \vec{F}_{collisions}(t) \quad (128)$$

In this equation:

$$\vec{F}_{drag} = -6\pi\eta r\vec{v} \quad (129)$$

with  $\eta$  the viscosity of the fluid and  $r$  the radius of the particle.

Equation (128) gives us a solution for the mean square of the displacement of the particle. It is normally solved first for one dimension, let's say  $\langle x^2 \rangle$ . The equations for the other components are identical, so it is easy to generalize and find  $\langle r^2 \rangle$ . Solving the equation is not too hard because of the probabilistic terms it contains. We also have to remember the equipartition theory: for a system containing a large number of molecules the average energy is only dependent on the temperature  $T$  and is distributed in equal shares to each independent way in which the molecules can absorb the energy. Here we consider the translational motion of the particles. There are three independent ways that the particles can absorb energy: the three dimensions. The mean translational kinetic energy of the particle for the one dimensional case is  $\frac{1}{2}kT$  with  $k$  the Boltzmann's Constant and  $T$  the absolute temperature.

The proportion factor depends on the viscosity, the temperature and the radius of the particle. For times of the order of seconds and a large number of particles, the mean square displacement in one dimension of a particle undergoing Brownian motion in a fluid with viscosity  $\eta$  is:

$$\langle x^2 \rangle = \frac{2kT}{6\pi\eta r} t \quad (130)$$

with  $k$  the Boltzmann's constant given by:  $k = 1.381 \times 10^{-23}$  Joule/Kelvin.

The radius of the spherical particle is of microscopic magnitude, which is large in comparison with the size of a molecule. In two dimensions:

$$\langle x^2 \rangle + \langle y^2 \rangle = \frac{4kT}{6\pi\eta r} t \quad (131)$$

The meaning of this equation will become clearer during the laboratory.



## 17.3 Laboratory

### 17.3.1 Experiment A: Introduction and Setup

In this experiment, we will familiarize ourselves with the laboratory, especially with the display.

1. In the PEARLS window, select *Modern Physics*.
2. Now launch the *Brownian Motion* item.
3. You will now see the experiment window, headed by a toolbar, with a control window at the left and a laboratory display at the right.
4. Adjust the window size as desired.
5. Move the laboratory display to a convenient location on your screen by dragging the display with your mouse.
6. The default settings for this experiment are:

<i>Particle Radius</i>	$5.000 \times 10^{-7}$ m
<i>Population</i>	10 particles
<i>Viscosity</i>	$1.000 \times 10^{-3}$ Ns/m <sup>2</sup>
<i>Temperature</i>	$3.000 \times 10^2$ Kelvin
<i>Boltzman Constant</i>	$1.380 \times 10^{-23}$ J/K
<i>Zoom</i>	$1.000 \times 10^7$ pixels/meter
<i>Time Zoom</i>	1.000

7. You should see one particle (hiding the other 9 particles) in the origin of the  $X - Y$  axes and two graphs. The actual running time of the experiment is displayed. Drag the graphs to a convenient location so you can see both. The graphs show the mean square displacement  $\langle r^2 \rangle$  of the particles as function of time. One graph is the theoretical one, the other the experimental (that is the simulation) one.
8. *Run* the experiment and observe the movement of the 10 brownian particles. They are all the same size. They are colored differently to facilitate the observation of the trajectory of each particle. (In real life it is impossible to put 10 brownian particles on top of each other and run an experiment like this simulation. This simulation is equivalent to ten real experiments running in parallel.)
9. Let the experiment run for 20 seconds, then *Stop* the experiment.
10. Follow one particular particle of the set.
  - (a) Is the particle walking randomly?



- (b) Does the particle make headway?
- 11. *Reset* to return the experiment to its initial state.
- 12. Have a look at the two graphs.
  - (a) How does  $\langle r^2 \rangle$  *theory* vary in time?
  - (b) How does the experimental graph differ from the theoretical one?
- 13. Run the previous experiment a couple of times for 20 seconds. *Reset* after each experiment.
  - (a) Are the experimental curves the same?
  - (b) How could you change the experiment to have a less jagged experimental curve?

### 17.3.2 Experiment B: Particle Size

In this experiment we will study how the size of the Brownian particles influences their behavior.

1. Start up the PEARLS *Brownian Motion* experiment, or if it is already running, restore the startup conditions by clicking the defaults button (the button with the yellow arrow).
2. *Run* the experiment for 20 seconds. *Stop*. Then *Reset*.
3. Set the particle size to twice its original size  $1.10^{-6}$  m. You can use the slider or type the value in, by clicking in the little window under *Particle Radius* where the value is shown.
4. Run the experiment again for 20 seconds. Then *Reset*.
  - (a) Did the particles behave differently and in what way?
  - (b) How is the second theoretical graph different from the first one?
5. Set the particle size to half its original size  $2.510^{-7}$  m and run the experiment again for 20 seconds. Then *Reset*.
  - (a) Did the particles behave differently and in what way?
  - (b) What is the difference in slope between the three theoretical graphs?

### 17.3.3 Experiment C: The Effect of Viscosity and Temperature

This experiment will show us how the motion of the Brownian particles change when the temperature and the viscosity of the liquid change.

1. Start up the PEARLS *Brownian Motion* experiment, or if it is already running, restore the startup conditions by clicking the defaults button (the button with the yellow arrow).
2. *Run* the experiment for 20 seconds. *Stop* and *Reset*.
3. Set the temperature to double its original value, so  $T = 600K$ .
4. *Run* the experiment for 20 seconds. *Stop* and *Reset*.
  - (a) Did you see a difference in the motion of the particles?
  - (b) How did the slope of  $\langle r^2 \rangle$  *Theory* change?
5. Set the viscosity to double its original value, so  $\eta = 0.002 \text{ Ns/m}^2$ .
6. *Run* the experiment for 20 seconds. *Stop* and *Reset*.
  - (a) How does the  $\langle r^2 \rangle$  *Theory* graph from 2 compare to this one?
7. Set the temperature back to  $300K$ , and the viscosity to half its original value, so  $\eta = 0.0005 \text{ Ns/m}^2$ .
8. *Run* the experiment for 20 seconds. *Stop* and *Reset*.
  - (a) Did you see a difference in the motion of the particles?
9. Set the temperature to half its original value, so  $T = 150K$ .
10. *Run* the experiment for 20 seconds. *Stop* and *Reset*.
  - (a) Did you see a difference in the motion of the particles?
  - (b) Can you compare the  $\langle r^2 \rangle$  *Theory* graph from 2 and this one?
11. Set the viscosity to very low values. Look at the motion of the particles. Set the viscosity to very high values. Look at the motion of the particles.

### 17.3.4 Experiment D: Numbers in Statistics

We will study the influence of the number of particles on the measured  $\langle r^2 \rangle$ .

1. Start up the PEARLS *Brownian Motion* experiment, or if it is already running, restore the startup conditions by clicking the defaults button (the button with the yellow arrow).



2. Click the *Graph* button on the toolbar and create a graph with  $\langle r^2 \rangle / \langle r^2 \rangle_{\text{Theory}}$  on the vertical axis and *Time* on the horizontal axis. Click *OK*.
3. Drag the new graph so you can see the three graphs and the display.
4. *Run* the experiment for 20 seconds. Then *Reset*.
5. Set the population to its maximum value.
6. *Run* the experiment for 20 seconds. Then *Reset*.
  - (a) To how many real life experiments is this simulated one equal to?
  - (b) How did the experimental  $\langle r^2 \rangle$  change?
  - (c) Did the theoretical  $\langle r^2 \rangle$  change?
  - (d) What do you see in the  $\langle r^2 \rangle / \langle r^2 \rangle_{\text{Theory}}$  graph?
7. Set the population to 1. *Run* the experiment for 20 seconds. Then *Reset*.
  - (a) To how many real life experiments is this simulated one equal to?
  - (b) How did the experimental  $\langle r^2 \rangle$  change?

### 17.3.5 Experiment E: The Average Displacement in a Random Walk

In this experiment we look at another statistical quantity: the average displacement  $\langle x \rangle$  and  $\langle y \rangle$  of a Brownian particle.

1. Start up the PEARLS *Brownian Motion* experiment, or if it is already running, restore the startup conditions by clicking the defaults button (the button with the yellow arrow).
2. Close all present graphs.
3. Click the graph button. Make a graph with  $\langle x \rangle$  in the vertical axis and time in the horizontal axis. Click *OK*.
4. Click the graph button again. Make a graph with  $\langle y \rangle$  in the vertical axis and time in the horizontal axis. Click *OK*.
5. Drag the graphs to a convenient spot on the screen.
6. *Run* the experiment run for 20 seconds. Then *Reset*.
  - (a) Do  $\langle x \rangle$  and  $\langle y \rangle$  vary in function of time?
7. Set the population to its maximum value.
8. *Run* the experiment for 20 seconds. Then *Reset*.

- (a) How are the graphs different from the ones in 6a?
- 9. Set the population to 1. *Run* the experiment for 20 seconds. Then *Reset*.
  - (a) How are the graphs different from the ones in 6a and 8a?

### 17.3.6 Experiment F: A Different World

This experiment is a real virtual lab experiment in which we consider another Boltzmann's constant.

1. Start up the PEARLS *Brownian Motion* experiment, or if it is already running, restore the startup conditions by clicking the defaults button (the button with the yellow arrow).
2. *Run* the experiment for 20 seconds. Then *Reset*.
3. Increase Boltzmann's constant to the maximum value possible in the experiment. *Run* the experiment and *Reset*.
4. Decrease Boltzmann's constant to the minimum value possible in the experiment. *Run* the experiment and *Reset*.
  - (a) What difference did you see in the behavior of the particles in step 3 and step 4?
  - (b) What difference do you see between the theoretical graphs from step 2, step 3, and step 4?

## 17.4 Review

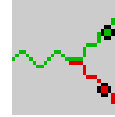
Please circle the correct answers to the following multiple-choice questions. You may use PEARLS to help you find the correct answer.

1. The mean square displacement of the Brownian particles is independent of the liquid in which they move.
  - (a) True.
  - (b) False.
2. The mean square displacement of a Brownian particle varies linearly with time for observation times in the order of seconds:
  - (a) True.
  - (b) False.
3. If you raise the temperature of the liquid in which Brownian particles are suspended to double its value, the mean square displacement of the particles will be:



- (a) the same.
  - (b) doubled.
  - (c) halved.
4. The movement of the Brownian particles is a result of the collision of the particles with the molecules of the liquid, which are continuously moving.
- (a) True.
  - (b) False.
5. The Brownian particles are of microscopic magnitude which is big in comparison to the molecules of the liquid.
- (a) True.
  - (b) False.
6. Knowing the temperature and the viscosity of the liquid, the radius of the Brownian particles, the number of molecules per mole could be calculated by measuring the mean square displacement of Brownian particles over some time.
- (a) True.
  - (b) False.
7. The mean square displacement of a Brownian particle is . . . . . for smaller than for bigger particles.
- (a) smaller.
  - (b) bigger.
  - (c) the same.

## 18 Compton Effect



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### 18.1 Introduction

The description of light as electromagnetic waves can explain several effects (e.g. doppler effect, interference, diffraction ...) but not all of them. Some effects, e.g. the photoelectric and the Compton effect could not be explained by this electromagnetic wave description and led to a new theory. In this theory light appears to be made up by particles, called photons. Actually, for a complete description of light's behavior, one sometimes needs to think of it as waves and as particles in the very same experiment. This is called: particle-wave duality.

In the Compton effect x-rays (light with short wavelengths between  $10^{-8}$  and  $10^{-16}$  m) are scattered by electrons. This scattering could only be explained as a collision between the photons of the x-rays and the electron. Although photons and electrons aren't exactly like pool balls, the collision analysis we made in the assignment: *Collisions* still applies.



### 18.2 Theory

When an x-ray beam falls on a crystal it is scattered by the electrons in the crystal. This means the x-rays are coming out in other directions besides the incident one. Usually, this is described as diffraction of x-ray waves. When Compton measured the scattering of an x-ray beam, with a very well determined wavelength  $\lambda$ , at different scattering angles, he found something unexpected. At each angle ( $\neq 0$ ) of the scattered x-rays he detected x-rays with the same wavelength  $\lambda$  but also x-rays with a longer wavelength. The light "wave" theory cannot really explain this. Think of a x-ray with a certain wavelength  $\lambda$  or frequency  $f$ . This electromagnetic wave will make the electrons in the atoms of the crystal oscillate with the same frequency. These moving charges will emit electromagnetic waves (just like antennas) of the same frequency  $f$  or wavelength  $\lambda$ . So where do the electromagnetic waves with longer wavelength come from?

Compton explained this using the then new photon description of light and collision theory. Think of a photon of the incident x-ray beam colliding with a valence electron of an atom in the crystal (Figure 13). A photon and an electron on the pool table! Let's recall two important conservation theorems for the elastic collision: the momentum and the kinetic energy of the colliding

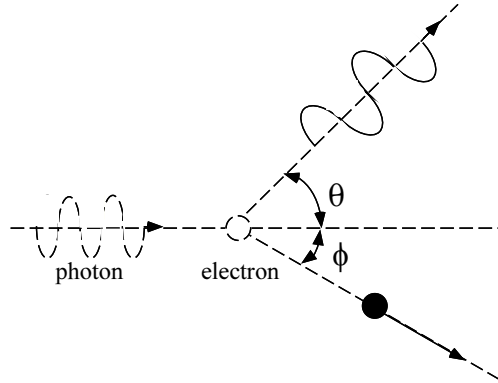


Figure 13: Compton Scattering

balls are conserved. Because the colliders here are a photon and an electron, we have to apply the relativistic theory. An electron at rest has a certain energy ( $E = m_0c^2$  with  $m_0$  its restmass) and that the mass of that electron changes when its velocity changes. The conservation theorems stay valid if we take this changes into account.

Let us first consider the photon. The energy of a photon is given by:

$$E = hf \quad (132)$$

where  $E$  is the energy of the photon,  $f$  the frequency and  $h$  Planck's Constant,  $h = 6.63 \times 10^{-34}$  Joule.second. This equation tells us that a photon with lower energy will have a lower frequency or a longer wavelength.

The photon's momentum is a vector whose direction is parallel to the photon's velocity and whose magnitude is given by

$$p = E/c = hf/c = h/\lambda \quad (133)$$

with  $c$  the speed of light.

Now we consider the electron: Since the energy of the incident x-ray (e.g. 17.4 keV for a ray with wavelength  $\lambda = 7.11 \times 10^{-9}$  m is much larger than the binding energy of the valence electrons in carbon ( $\approx 11eV$ ) (the atoms making up graphite, the crystal used in the experiments by Compton) we can neglect any effects of the binding of the electrons to the nuclei in the crystal. (1 keV is one kilo electron volt, or  $1 \text{ keV} = 10^3 \text{ eV}$ , with  $1 \text{ eV} = 1.602 \times 10^{-19}$  Joule). We therefore say we consider the electrons as "free". Since their kinetic energy is very small with respect to that of the x-rays, we can also neglect it. Thus, we simply consider the electron at rest. Its energy before the collision is thus its rest energy ( $E_0 = m_0c^2$ ).

The momentum of the electron before the collision is zero because we considered it at rest. After the collision it will have picked up speed and the momentum



will be  $\vec{p}_e$ .

After the collision the electron will have picked up energy from the photon that loses some of its energy in the collision.

The relativistic theory tells us that the energy of the electron after collision will be:

$$E_e = \sqrt{E_0^2 + p_e^2 c^2} \quad (134)$$

The conservation of momentum gives:

$$\vec{p}_1 = \vec{p}_2 + \vec{p}_e \quad (135)$$

with  $\vec{p}_1$  and  $\vec{p}_2$  the momenta of the photon before, resp. after the collision.

The conservation of energy gives:

$$p_1 c + E_0 = p_2 c + \sqrt{E_0^2 + p_e^2 c^2} \quad (136)$$

with  $p_1$  and  $p_2$  the magnitude of the momenta of the photon before and after the collision.

If we consider the angle between the scattered and incoming x-ray  $\theta$  as given, then one can derive from equation (135) and (136) the following equation known as the Compton equation:

$$\lambda_2 - \lambda_1 = \frac{hc}{m_0 c^2} (1 - \cos\theta) \quad (137)$$

with  $\lambda_1, \lambda_2$ , the two measured wavelengths of the photons after the collision,  $\lambda_1$  being the wavelength of the incident photon,  $m_0$  the restmass of the electron,  $c$  the speed of light and  $\theta$  the scattering angle of the photon. The factor  $h/m_0 c$  is called the Compton wavelength of the electron. This is the wavelength of a photon with an energy equal to the rest energy of the electron.

## 18.3 Laboratory

### 18.3.1 Experiment A: Introduction and Setup

In this experiment, we will familiarize ourselves with the laboratory, especially with the display and the controls. We will create a Compton Scattering experiment and observe the scattered photon and electron.

1. In the PEARLS window, select *Modern Physics*.
2. Now launch the *Compton Effect* item.



3. You will now see the experiment window, headed by a toolbar, with a control window at the left and a laboratory display at the right.
4. Adjust the window size as desired.
5. Move the laboratory display to a convenient location on your screen by dragging the display with your mouse.
6. You see an  $X$  and  $Y$  axis, a “green” electron with its green path after collision and a “yellow” photon and its path. The scattering angles of the photon and electron are shown.
7. The default settings for this experiment are:

<i>Photon Deflection</i>	$5.400 \times 10^1$ degrees
<i>Initial Wavelength</i>	$2.426 \times 10^{-12}$ m
<i>Particle Mass</i>	$9.109 \times 10^{-31}$ kg
<i>Wavelength Change</i>	$1.000 \times 10^{-12}$ m
<i>Compton Wavelength</i>	$2.426 \times 10^{-12}$ m
<i>Particle Recoil Angle</i>	$4.446 \times 10^1$ degrees
<i>Zoom</i>	$1.200 \times 10^{13}$ pixels/meter
<i>Time Zoom</i>	$3.000 \times 10^{-20}$

8. *Run* the experiment. Observe the motion of the electron and photon, then *Stop* and *Reset*.
9. From the *Display* menu, located above the toolbar, choose *Photon Wave*.
10. *Run* the experiment. Observe, then *Stop* and *Reset*.
  - (a) What difference do you see between the incoming x-rays and the scattered x-rays?
  - (b) Is the default wavelength chosen in a special way?
11. Set *Initial Wavelength*, the wavelength of the incoming x-ray, to  $1 \times 10^{-12}$  m.
12. *Run* the experiment. Observe, then *Stop* and *Reset*.
  - (a) Did the path of the electron change?
13. Slide *Photon Deflection*, the scattering angle of the photon, to higher values, and observe the display and the control window.
  - (a) Did the path of the electron change?
  - (b) Which controls or meters in the control window changed when you changed *Photon Deflection*?

### 18.3.2 Experiment B: The Relation between Wavelength and Energy

In this experiment we will study how wavelength and energy are related.

1. Start up the PEARLS *Compton Effect* experiment, or if it is already running, restore the startup conditions by clicking the defaults button  $\leftrightarrow$ .
2. From the *Display* menu, choose *Photon Wave*.
3. Click the graph button. Make a graph with the photon energy in the vertical axis and time in the horizontal axis. Change the maximum value of the energy to 1.5 MeV. Click *OK*.
4. Drag the graph so you can see as well the display as the graph.
5. *Run* the experiment and observe the scattering. *Stop* and *Reset*.
  - (a) Can you tell by looking at the waves in the display that the energy of the photon after collision is smaller?
6. Set the wavelength *Initial Wavelength* of the incoming x-ray to half its original value, so  $\lambda = 1.213 \times 10^{-12}$  m .
7. *Run* the experiment and observe the scattering. *Stop* and *Reset*.
  - (a) Looking at the display can you tell if the scattered photon will have a larger or smaller energy than in the initial step?
  - (b) Look at the graph. How does the energy of the incoming photon in this step compare to the one in the step 5?
8. Set the wavelength *Initial Wavelength* of the incoming x-ray to twice its original value, so  $\lambda = 4.86 \times 10^{-12}$  m.
9. *Run* the experiment and observe the scattering. *Stop* and *Reset*.
  - (a) By looking at the graph can you tell how the energy of the incoming photon changed relative to step 5?

### 18.3.3 Experiment C: The Observed Compton Shift and the Scattering Angle

In this experiment we will study how the Compton shift changes with the angle  $\theta$ .

1. Start up the PEARLS *Compton Effect* experiment, or if it is already running, restore the startup conditions by clicking the defaults button  $\leftrightarrow$ .
2. From the *Display* menu choose *Photon Wave*.



3. Set *Initial Wavelength* equal to  $3.10^{-12}$  m.
  - (a) What is the value of the *Wavelength Change*?
4. Run the experiment and *Reset*.
5. Set *Photon Deflection* to 90 degrees.
6. Run the experiment and reset.
  - (a) Is *Wavelength Change* (or the Compton shift) smaller or larger than in 3a?
  - (b) Comparing *Wavelength Change* and *Compton Wavelength*, what do you see?
  - (c) What is the wavelength of the photons scattered at this angle?
7. Set *Photon Deflection* equal to 180 degrees.
8. Run the experiment and reset.
  - (a) What is the recoil angle for the electron?
  - (b) Comparing *Wavelength Change* and *Compton Wavelength*, what do you see?
9. Set *Photon Deflection* equal to zero.
10. Run the experiment and reset.
  - (a) What happened to the electron?
  - (b) In the display, do you see a difference between the incoming wave and the observed wave?
  - (c) What is the value of *Wavelength Change*?

#### 18.3.4 Experiment D: The Compton Shift and the Wavelength of the Incoming Photon

In this experiment we will change the wavelength of the incoming x-ray and observe the wavelength of the scattered x-ray.

1. Start up the PEARLS *Compton Effect* experiment, or if it is already running, restore the startup conditions by clicking the defaults button  $\leftrightarrow$ .
2. Choose *Photon Wave* from the *Display* menu.
3. Run the experiment and reset.
  - (a) How is the outgoing wave different from the incoming wave?

- (b) What is the value of the *Wavelength Change*?
- 4. Set *Initial Wavelength* to  $5.10^{-13}$  m.
- 5. Run the experiment and reset.
  - (a) Is the difference in wavelength between incoming photon and scattered photon more or less noticeable than in step 3b?
  - (b) Did *Wavelength Change* change?
  - (c) Compare the order of magnitude of *Wavelength Change* and *Initial Wavelength*.
- 6. Set *Initial Wavelength* to  $1.10^{-11}$  m.
- 7. Run the experiment and reset.
  - (a) Is the difference in wavelength between incoming photon and scattered photon more or less noticeable than in step 3b?
  - (b) Did *Wavelength Change* change?
  - (c) Compare the order of magnitude of *Wavelength Change* and *Initial Wavelength*.

### 18.3.5 Experiment E: The Recoiled Electron Speed

In this experiment we will demonstrate a relativistic effect by observing the speed of the recoiled electron.

1. Start up the PEARLS *Compton Effect* experiment, or if it is already running, restore the startup conditions by clicking the defaults button (the button with the yellow arrow).
2. Click the graph button and make a graph with *Electron Velocity/c* in the vertical axis and *Time* in the horizontal axis. The *Electron Velocity/c* is actually the electron speed relative to the speed of light. Set the maximum value of *Electron Velocity/c* to 2.
3. Drag the graph so you can see the display.
4. Set *Initial Wavelength* to  $2 \times 10^{-12}$  m.
5. Run the experiment and reset.
6. Set *Initial Wavelength* to half its first value, so  $\lambda = 1 \times 10^{-12}$  m.
7. Run the experiment and reset.
8. If you do not see a difference in the speed of the electron with the previous step, set *Time Zoom* to a smaller value at which you see a difference in speed. Restart the experiment at step 4.



- (a) Did the electron speed increase or decrease?
- 9. Set *Initial Wavelength* to  $0.5 \times 10^{-12}$  m.
- 10. Run the experiment and reset.
- 11. Set *Initial Wavelength* to its minimum value.
- 12. Run the experiment and reset.
  - (a) What is the maximum value of the electron speed in these steps?
- 13. Set *Initial Wavelength* to  $2 \times 10^{-12}$  m.
- 14. Set *Photon Deflection* to 180 degrees.
- 15. Run the experiment and reset.
  - (a) Is the “electron velocity/c” higher or lower than in step 4?
- 16. Set *Initial Wavelength* to its minimum value.
- 17. Run the experiment and reset.
  - (a) What is the maximum electron speed in these steps?

### 18.3.6 Experiment F: A Different Particle

In this experiment we will change the mass of the particle and observe how the collision changes.

1. Start up the PEARLS *Compton Effect* experiment, or if it is already running, restore the startup conditions by clicking the defaults button (the button with the yellow arrow).
2. Click the graph button and make a graph with *Electron Velocity/c* in the vertical axis and *Time* in the horizontal axis.
3. Set the maximum value of the *Electron Velocity/c* in the graph equal to 2. Click *OK*. The *Electron Velocity/c* is actually the electron speed relative to the speed of light.
4. Choose *Photon Wave* and *Electron Wave* from the *Display* menu.
5. Run the experiment and reset.
  - (a) What is the value of the *Compton Wavelength*?
  - (b) What is the value of the *Particle Recoil Angle*?
  - (c) What is the value of the *Wavelength Change* or Compton shift?

6. Set the *Particle Mass* to its minimum value (a particle with a rest mass 10 times smaller than the electron).
7. Run the experiment and reset.
8. Look at the display and compare with the previous step 5.
  - (a) Is the recoil angle of this particle different from the electron recoil angle in step 5b?
  - (b) Did the Compton Shift change, compared to step 5c?
  - (c) Did the Compton Wavelength change, compared to step 5a?
  - (d) Is the speed of this particle different from the electron speed?
9. Decrease *Initial Wavelength* to its minimum value.
10. Run the experiment and reset.
  - (a) Is the recoil angle different from the one in step 8a?
  - (b) Did the Compton Wavelength change?
  - (c) What do you observe in the wavelength of the scattered photon?
  - (d) Did the Compton Shift change?
  - (e) What happened to the speed of the particle?
11. Set *Initial Wavelength* equal to the Compton Wavelength of the “light” particle.
12. Run the experiment and reset.
  - (a) How does the recoil angle of the particle compare to the experiment 5?
13. Restore the start-up conditions, turn *Photon Wave* and *Electron Wave* on in the *Display Menu*.
14. Set the *Particle Mass* to 10 times the restmass of the electron.
15. Run the experiment and reset.
16. Look at the display and compare with the experiment 5.
  - (a) Is the recoil angle of this particle different from the electron recoil angle in step 5b?
  - (b) Did the Compton Shift change, compared to step 5c?
  - (c) Did the Compton Wavelength change, compared to step 5a?
  - (d) What happened to the speed of the particle?



17. Set *Initial Wavelength* equal to the Compton Wavelength of the “heavy” particle.
18. Run the experiment and reset.
  - (a) How does the recoil angle of the particle compare to the experiment 5?

### 18.3.7 Experiment G: The Electron and Photon as Pool Balls

In this experiment we will study the classical collision laws: conservation of momentum and conservation of total energy.

1. Start up the PEARLS *Compton Effect* experiment, or if it is already running, restore the startup conditions by clicking the defaults button  $\leftrightarrow$ .
2. Make a graph with *Electron Momentum X* in the vertical axis and *Time* in the horizontal axis.
3. Click the graph button and make a graph with *Photon Momentum X* in the vertical axis and *Time* in the horizontal axis.
4. Both graphs are on top of each other. Drag one away so you can see both.
5. Run the experiment and reset.
6. Look at the plots.
  - (a) Does the electron have momentum before the collision?
  - (b) What is the total  $x$ -momentum before collision?
  - (c) What is the total  $x$ -momentum after collision?
7. Click the graph button and make a graph with *Electron Momentum Y* in the vertical axis and *Time* in the horizontal axis.
8. Click the graph button and make a graph with *Photon Momentum Y* in the vertical axis and *Time* in the horizontal axis.
9. Both graphs are on top of each other. Drag one away so you can see both.
10. Run the experiment and reset.
11. Look at the plots.
  - (a) Does the electron have momentum before the collision?
  - (b) What is the total  $y$ -momentum before collision?
  - (c) What is the total  $y$ -momentum after collision?



12. Close all the graphs.
13. Click the graph button and make a graph with *Photon Energy* in the vertical axis and *Time* in the horizontal axis.
14. Click the graph button and make a graph with *Total Energy* in the vertical axis and *Time* in the horizontal axis.
15. Both graphs are on top of each other. Drag one away so you can see both.
16. Run the experiment and reset.
17. Look at the plots.
  - (a) Does the total energy change after collision?
  - (b) Does the electron have energy before the collision?

## 18.4 Review

Please circle the correct answers to the following multiple-choice questions. You may use PEARLS to help you find the correct answer.

1. Consider a photon with a wavelength  $\lambda = 10^{-12}$  m, scattered by an electron. The Compton Shift will be maximum for  $\theta$  equal to:
  - (a) 0 degrees
  - (b) 90 degrees
  - (c) 180 degrees
2. Consider a Compton Scattering effect with visible light. Will the Compton Shift be noticeable? (The wavelength of visible light ranges from  $4 \times 10^{-7}$  m to  $7 \times 10^{-7}$  m.)
  - (a) no
  - (b) yes
3. The energy of the photons is larger if their wavelength is
  - (a) larger
  - (b) smaller
4. If the incoming photon has a wavelength equal to the Compton Wavelength, and the scattered photon is measured at  $\theta = 60$  degrees, the scattered photon will have a wavelength equal to:
  - (a) Twice the Compton Wavelength
  - (b) 3/2 of the Compton Wavelength



- (c) depends on the recoil angle of the electron
5. In Compton Scattering of an x-ray and a free electron the electron must recoil. The recoil angle will be determined by:
- (a) the law of energy conservation.
  - (b) the law of momentum conservation.
  - (c) neither one of these laws.
6. If we compare the direction of the incident photon beam and the direction of scattering of a Compton Scattered electron with maximum kinetic energy they will be:
- (a) the same
  - (b) opposite
  - (c) make an angle  $\theta + \phi \neq 0$  and  $\neq 180$  degrees.
7. The energy of the photon associated with a wavelength equal to the Compton Wavelength of the electron is the rest energy of the electron:
- (a) true
  - (b) false
8. The conservation laws for energy and momentum are valid in the Compton Scattering although this is a relativistic process:
- (a) true
  - (b) false
9. The Compton Shift depends on the wavelength of the incoming x-ray:
- (a) true
  - (b) false
10. The wavelength of Compton Scattered electrons is measured at  $\theta = 90$  degrees. If  $\Delta\lambda/\lambda = 1\%$  the wavelength  $\lambda$  of the incident photons should be:
- (a)  $\lambda_{Compton}/100$
  - (b)  $\lambda_{Compton} \times 100$
  - (c)  $\lambda_{Compton} \times 200$
  - (d)  $\lambda_{Compton}/50$

## 19 Radioactive Decay

$$\frac{dN}{dt} = -\lambda N$$

### 19.1 Introduction

Radioactive decay is a statistical process (see assignment *Brownian Motion* for another statistical process). In this process we are not looking at how one particular nucleus is decaying, but at a number of nuclei. The decaying of any individual nucleus is a random event. Rutherford measured that the rate at which the radioactive particles are emitted from a substance is not constant but decreases exponentially in time. This is a characteristic of statistical behavior.

In this assignment we will become familiar with the concept of half-time of a radioactive substance. We will also be able to explore how theory and experiment in statistical processes agree well with each other for large numbers and how we cannot predict the behavior of one single atom.

### 19.2 Theory

The number of nuclei decaying in a certain time interval  $dt$  is proportional to the number of nuclei  $N(t)$  present in the sample at time  $t$  and the length of the time interval  $dt$ . Or, after a time interval  $dt$  the number of nuclei that remain to decay has decreased by:

$$-dN(t) = \lambda N(t)dt \quad (138)$$

The proportion factor  $\lambda$  is called the decay constant and is the probability per time unit of the decay of a nucleus.

The solution of this differential equation is:

$$N(t) = N_0 e^{-\lambda t} \quad (139)$$

with  $N_0$  the number of nuclei at  $t = 0$ . In the real lab not the number of nuclei that remain to decay  $N$  is measured but the rate of decay  $R(t)$ . This is given by:

$$R(t) = -\frac{dN(t)}{dt} = \lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t} \quad (140)$$

with  $R_0$  the decay rate at  $t = 0$ .





The half-life  $t_{1/2}$  is the time after which the number of radioactive nuclei has decreased to half its original value. In this case  $N = N_0/2$  or

$$\frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}} \quad (141)$$

which gives:

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} \quad (142)$$

The average life time is:

$$t_{av} = \frac{1}{\lambda} = \frac{t_{1/2}}{\ln 2} \quad (143)$$

The common unit for the activity ( $\lambda N$ ) of unstable nuclei, i.e. the radioactivity is “curie” defined as:

1 curie =  $3.7 \times 10^{10}$  disintegrations per second. The SI unit of activity is the becquerel defined as: 1 Bq = 1 disintegration per second.

In the assignment the concept “normalized population” appears. This is defined by  $N_0 = 1$ .

## 19.3 Laboratory

### 19.3.1 Experiment A: Introduction and Setup

In this experiment, we will familiarize ourselves with the laboratory, especially with the display and the controls, and observe the decay of nuclei.

1. In the PEARLS window, select *Modern Physics*.
2. Now launch *Radioactive Decay* item.
3. You will now see the experiment window, headed by a toolbar, with a control window at the left and a laboratory display at the right.
4. Adjust the window size as desired.
5. Move the laboratory display to a convenient location on your screen by dragging the display with your mouse.
6. The default settings for this experiment are:

<i>Half Life</i>	5.000 s
<i>Initial Population</i>	$1.000 \times 10^2$ particles
<i>Zoom</i>	1.000 pixels/meter
<i>Time Zoom</i>	1.000

7. You should see 100 atoms, a counter *Population* = 100, a clock, and two graphs. One graph is the theoretical population versus time. The other one is the experimental (that is the simulation) population versus time.
8. Drag the graphs with your mouse so you can see both and the laboratory display and adjust their size to a convenient one for you.
9. Run the experiment for 30 seconds. Observe the decaying atoms (red atoms are the ones that decayed already) then *Stop* and *Reset*.
  - (a) Did you see a difference between the number of nuclei that decayed in the first seconds and the number that decayed after some time has lapsed?
  - (b) Does the experimental graph show your observation in the laboratory display?
10. Set the *Half-Life* to 2 seconds.
11. Run the experiment for 20 seconds and reset.
  - (a) Does the population decrease slower or faster than in the first experiment?
  - (b) How is this observation expressed in the graph?
12. Set the *Half-Life* back to 5 seconds.
13. Set the *Population* to 30.
14. Run the experiment for 20 seconds and reset.
  - (a) How does the experimental graph compare with the theoretical one?

### 19.3.2 Experiment B: Half-Life

In this experiment we will study the the meaning of the concept of half-life.

1. Start up the PEARLS *Radioactive Decay* experiment, or if it is already running, restore the startup conditions by clicking the defaults button (the button with the yellow arrow).
2. Adjust the size of the graph windows so you can see them clearly, but are still able to see the display.
3. Run the experiment for 20 seconds. Observe the clock and the population. Reset. If the clock is running too fast for you, set *Time Zoom* to a smaller value.
  - (a) Watching the clock and *Population* can you tell how large the population was after 5 seconds?



- (b) Do you come to the same result looking at the graphs?
4. Set *Half Life* to 2 seconds.
    5. Run the experiment until all nuclei are decayed and reset.
      - (a) Watching the clock and *Population* can you tell how large the population was after 2 seconds?
      - (b) What is the difference between the new graphs and the graphs made in step 3?
  6. Set *Half Life* to 20 seconds.
    7. Run the experiment for 20 seconds and reset.
      - (a) Watching the clock and *Population* can you tell how large the population was after 20 seconds?
      - (b) What is the difference between the new graphs and the graphs made in experiment 3?
      - (c) What do you think happened to the decay constant?
  8. Set *Half Life* to its maximum value, using the slider.
    9. Run the experiment for 20 seconds and reset.
      - (a) What happened in the display?
      - (b) What happened to the curves?
      - (c) What do you think happened to the decay constant?

### 19.3.3 Experiment C: Numbers in statistics

In this experiment we will see how the size of the population affects the experimental results.

1. Start up the PEARLS *Radiactive Decay* experiment, or if it is already running, restore the startup conditions by clicking the defaults button (the button with the yellow arrow).
2. Click the graph button on the toolbar.
3. Make a graph with *Population/Theory* in the vertical axis and *Time* in the horizontal axis. *Population/Theory* means the experimentally measured population relative to the population determined by the theory.
4. Do not close the default graphs. If you lack space just put the newly made graph window on top of the other ones.

5. Run the experiment for 20 s. *Stop*. Then *Reset*.
  - (a) How does the graph experiment/theory behave in time?
  - (b) Is there a difference in the graph shortly after starting the experiment and after  $\approx 18$  s?
6. Set *Population* equal to 50.
7. Run the experiment for 20 s and reset.
  - (a) How does the new graph compare to the one in step 5?
  - (b) After how much time does the graph start to look differently?
  - (c) What was the population at that time?
8. Set *Population* equal to 25.
9. Run the experiment for 20 s and reset.
  - (a) How does the new graph compare to the one in step 5 and 6?
  - (b) After how much time does the graph start to look differently?
  - (c) What was the population at that time?
10. Set *Population* equal to 12.
11. Run the experiment for 20 s and reset.
  - (a) How does the new graph compare to the other graphs?
  - (b) After how much time does the graph start to look differently?
12. Set *Population* equal to 1.
13. Run the experiment for 20 s and reset.
  - (a) How does the new graph compare to the previous ones?
14. Set *Population* equal to 200.
15. Run the experiment for 20 s and reset.
  - (a) How does the new graph compare to the other graphs?
  - (b) After how much time does the graph start to look differently?
  - (c) What is the population at that time?
16. Uncover the graphs *Population vs. Time* and *Theory vs. Time*.
  - (a) Comparing the equally colored graphs what can you conclude about the influence of the population on the results?



### 19.3.4 Experiment D: Semi-Logarithmic Plots.

In this experiment we will become familiar with semi-logarithmic graphs.

1. Start up the PEARLS *Radiactive Decay* experiment, or if it is already running, restore the startup conditions by clicking the defaults button (the button with the yellow arrow).
2. Click the graph button on the toolbar.
3. Make a graph with *Log Population (Theory)* in the vertical axis and *Time* in the horizontal axis. *Log Population (Theory)* means the logarithm of the theoretically calculates population.
4. Make a graph with *Log Population* in the vertical axis and *Time* in the horizontal axis.
5. Run the experiment for 10 seconds. Stop and reset.
  - (a) How does the logarithm of the polulation vary in time?
6. Set *Half Life* equal to 2 seconds.
7. Run the experiment for 10 seconds. Stop and reset.
  - (a) Did the slope of the line change compared to step 5a?
8. Set *Half Life* equal to 10 seconds.
9. Run the experiment for 10 seconds. Stop and reset.
  - (a) Did the slope of the line change compared to step 5a and 7a?
10. Restore the default values by clicking  $\leftrightarrow$ .
11. Erase all graphs.
12. Run the experiment for 10 seconds.
13. Change the initial population to 50 particles.
14. Run the experiment for 10 seconds.
  - (a) Did the slope of the plot change, compared to step 12?
  - (b) What is the difference between both plots?
15. Change the initial population to 25 particles.
16. Run the experiment for 10 seconds.
  - (a) Did the slope of the plot change, compared to step 12 and step 14b?
  - (b) What is the difference between all three plots?



## 19.4 Review

Please circle the correct answers to the following multiple-choice questions. You may use PEARLS to help you find the correct answer.

- The half-life time for a certain material does not depend on the number of nuclei that can decay.
  - true
  - false
- If the half-life of a radioactive material is  $t_{1/2} = 60$  s and 2000 nuclei decayed in that time, then at  $t = 30$  s 1000 nuclei had decayed.
  - true
  - false
- The average or mean lifetime of unstable nuclei is:
  - equal to the half-life.
  - greater than the half-life.
  - smaller than the half-life.
- If the activity of a radioactive material drops by a factor 10 in 1 minute, the decay constant will be:
  - 2.30/ s
  - 0.038/ s
  - 0.017/ s
- The factor by which an initial number of unstable atoms is decreased for a certain time interval is:
  - dependent of the starting time.
  - independent of the starting time.
- The half-life of  $^{14}\text{C}$  (a radioactive isotope of carbon) is 5732 years. The measurement of the intensity of radioactivity of  $^{14}\text{C}$  could be used to measure the age of materials:
  - a few years old.
  - a few thousand years old.
  - a few million years old.
- If a radioactive element desintegrates for a period of time equal to its average life, the percentage of the original amount that remains will be:



- (a) 37%
- (b) 63%
- (c) 50%
8. A certain radioactive element has a half-life of 20 days.  $3/4$  of the originally present atoms will have disintegrated after:
- (a) 40 days.
- (b) 30 days.
- (c) 28 days.
- (d) 8 days.
- (e) 6 days.
9. The half-life of a radioactive material A is  $t_{1/2}$ . This half-life is half the half-life of a radioactive material B. After a time equal to  $t_{1/2}$  the fraction that remains of A will of course be 0.50. The fraction that remains of B after that time will be:
- (a) 0.75
- (b) 0.71
- (c) 0.25
- (d) 0.29
10. The half-life of a radioactive substance is  $100 \times 10^{-6}$  s. If its initial activity is  $1.16 \times 10^{23}$  Bq, its activity after  $200 \times 10^{-6}$  s is:
- (a) 0 Bq.
- (b)  $0.58 \times 10^{23}$  Bq.
- (c)  $2.9 \times 10^{22}$  Bq.

**Part IX**  
**Answers**

### Answers: Black Body Radiation

- 15.3.1 8a The body turned black.
- 8b The graph shape did not change, but the maximum values of spectral intensity and the width of the curve become smaller.
- 8c Yes, the wavelength at which the intensity peaks is  $2.90 \times 10^{-3}$  m.
- 8d The microwave region.
- 8e Yes, the peak spectral intensity number is lower:  $1.28 \times 10^{-5}$  W/m<sup>3</sup>.
- 15.3.2 2a  $\lambda_m = 4.15 \times 10^{-6}$  m
- 2b the infrared region
- 2c  $2.9 \times 10^{-3}$  mK
- 3a  $\lambda_m = 2.07 \times 10^{-6}$  m
- 3b The wavelength is halved.
- 3c The infrared region.
- 3d  $2.9 \times 10^{-3}$  mK
- 4a  $\lambda_m = 1.04 \times 10^{-6}$  m
- 4b The wavelength is a 1/4 the wavelength of 2a.
- 4c the infrared region
- 4d  $2.9 \times 10^{-3}$  mK
- 4e The product of the peak wavelength and the temperature is a constant equal to  $2.9 \times 10^{-3}$  mK.
- 7a The intensity peak moves towards lower wavelengths or higher frequencies.
- 15.3.3 2a Black.
- 2b No.
- 2c Yes. The intensity is zero for the wavelengths of visible light. (The visible spectrum has lower wavelengths).
- 2d  $3.11 \times 10^7$  W/m<sup>3</sup>
- 3a Red.
- 3b Red light.
- 3c Yes. The spectral intensity is not zero in the red part of the visible spectrum.
- 3d The peak intensity is higher and shifted to lower wavelength.
- 4a Yellow.
- 4b Shorter.

- 4c Yes.
- 4d The peak intensity is higher and shifted to lower wavelength.
- 5a White.
- 5b The peak intensity is higher and shifted to lower wavelengths.
- 15.3.4 3a The integrated intensity is  $1.42 \times 10^1 \text{ W/m}^2$ .
- 4a The integrated intensity is  $2.27 \times 10^2 \text{ W/m}^2$ . This value is 16 times higher than the value of step 3a. The temperature doubled. The change in radiancy is proportional to the fourth power of the temperature.
- 15.3.4 The integrated intensity is  $1.42 \times 10^1 \text{ W/m}^2$ . This value is 16 times higher than the value of step 4a and 256 times higher than the value of step 3a.
- 15.3.5 2a  $2.15 \times 10^9 \text{ W/m}^3$
- 2b  $4.15 \times 10^{-6} \text{ m}$
- 2c Infrared.
- 3a  $4.15 \times 10^{12} \text{ W/m}^3$
- 3b  $6.25 \times 10^{-7} \text{ m}$ .
- 3c Red light.
- 3d No, the body is darker.
- 4a The intensity is  $5.86 \times 10^{12} \text{ W.m}^3$ . For a small increase of 50 degrees the emission is 1.5 times higher.
- 4b Yellow light.
- 4c No, the body is dark red.
- 5a The intensity is  $1.46 \times 10^{13} \text{ W/m}^3$ .
- 5b Blue light.
- 5c No, the body is dark red.
- 6a Ultraviolet light.
- 7a No visible light is emitted by the sun if we believe this theory! The emission peaks in the ultraviolet region.

### Answers: Bohr Atom

- 16.4.1 9a The electron is in the third orbit, the quantum number is three.
- 9b The electron needs approx.  $4.2 \times 10^{-15}$  seconds to complete a whole orbit.

- 11a The electron occupies the next orbit closer to the nucleus, the second orbit from the nucleus.
- 11b Less time.
- 11c The electron energy is  $-3.4$  eV.
- 13a The electron emitted a photon, and jumped to the next orbit closer to the nucleus.
- 13b The energy of the electron became more negative,  $-13.6$  eV.
- 15a The electron absorbed a photon, and jumped to the next orbit further from the nucleus.
- 15b The energy of the electron became less negative,  $-3.4$  eV.
- 16.4.2 1a  $Z = 1$
- 1b  $n = 3$
- 1c  $E = -1.51$  eV.
- 1d  $E_B = 1.51$  eV.
- 3a  $E = -13.6$  eV.
- 3b  $E_B = 13.6$  eV.
- 3c No, this is the ground state.
- 3d 9, the quantum number squared.
- 3e The first Bohr radius:  $a_0 = 0.529 \times 10^{-10}$  m.
- 4a  $E = -3.4$  eV.
- 4b 4, the quantum number squared.
- 16.4.3 3a 10.2 eV.
- 3b 10.2 eV.
- 3c  $E = hf$ , so  $f = E/h = 10.2/(4.138 \times 10^{-15})$  eV/(eV.second). or  $f = 2.46 \times 10^{15}$  Hz
- 3d The first line in the Lyman series.
- 5a 1.89 eV
- 5b 1.89 eV
- 5c  $4.57 \times 10^{14}$  Hz
- 5d The first line in the Balmer series.
- 7a 0.66 eV
- 7b 0.66 eV
- 7c  $1.59 \times 10^{14}$  Hz.
- 7d The first line of the Paschen series.

- 9a 10.2 eV.  
 9b 10.2 eV.  
 9c  $2.46 \times 10^{15}$  Hz.
- 16.4.4 1a  $E_3 = -1.51$  eV  
 2a The radii became smaller.  
 2b  $E_3 = -13.6$  eV.  
 2c 9, the proton number squared.  
 3a The radii became smaller.  
 3b  $E_3 = -24.18$  eV  
 3c 16, the proton number squared.  
 4a 30.23 eV  
 4b  $f = E/h$  or  $f = 30.23\text{eV}/4.138 \times 10^{-15}$  eV.second or  $f = 7.30 \times 10^{15}$  Hz.  
 5a  $E_0 = -217.62$  eV.  
 5b This groundstate energy is 16 times the one of the hydrogen atom.
- 16.4.5 2a  $-13.6$  eV  
 2b  $a_0 = 0.529\text{\AA}$   
 3a Became larger.  
 3b  $-1.36$  eV  
 3c The “light electron” is 10 times less bound.  
 3d The new bohr radius is 10 times larger.  
 4a The “heavy electron” sits “in the nucleus”.  
 4b  $-136.0$  eV  
 4c The “heavy electron” is 10 times more bound.  
 4d The new Bohr radius is 10 times smaller.
- 16.4.6 3a The orbit radii became very small. disappeared.  
 3b The electron is on its way to smash into the nucleus.

### Answers: Brownian Motion

- 17.3.1 10a The particle walks randomly, making many steps up, down, forward, and backward.  
 10b The particle makes headway.  
 12a Linear.

- 12b The experimental graph is qualitatively the same as the theoretical graph, but considerably more jagged.
  - 13a The experimental curves are different.
  - 13b Augment the population.
- 17.3.2
- 4a The motion of the bigger particles is very slow.
  - 4b The theoretical  $\langle r^2 \rangle$  graph of the bigger particles has a half the slope of the one of the smaller particles.
  - 5a The motion of the smaller particles is a lot more violent. They get away from their starting point faster.
  - 5b The theoretical  $\langle r^2 \rangle$  graph of the smaller particles has a bigger slope than the one of the bigger particles.
- 17.3.3
- 4a The particles move faster than in experiment 2.
  - 4b The slope of the new graph is bigger.
  - 6a The graphs are the same.
  - 8a The particles move faster than in experiment 2.
  - 10a The particles move slower.
  - 10b The graphs are the same.
- 17.3.4
- 6a 100 experiments.
  - 6b The higher population experimental curve is a lot less jagged.
  - 6c The theoretical graph does not change at all.
  - 6d The graph corresponding to the higher population stays closer to one over time.
  - 7a 1 experiment.
  - 7b The new graph is very jagged.
- 17.3.5
- 6a  $\langle x \rangle$  and  $\langle y \rangle$  stay close to zero over time.
  - 8a  $\langle x \rangle$  and  $\langle y \rangle$  stay closer to zero over time for the larger population.
  - 9a The graphs are very jagged for this small population.
- 17.3.6
- 4a The particles move faster when the Boltzmann constant is higher. A higher Boltzmann constant means that the average kinetic energy of the particles would be higher.
  - 4b The graph corresponding with the smallest Boltzmann's constant has the smallest slope.



### Answers: Compton Scattering

- 18.3.1 10a The wavelength of the scattered x-rays is larger.  
 10b The wavelength is the Compton wavelength of the electron.  
 12a Yes, the recoil angle is smaller.  
 13a Yes.  
 13b The *Particle Recoil Angle*,  $\phi$ , and the *Wavelength Change*,  $\Delta\lambda$  changed.
- 18.3.2 5a The wavelength of the scattered photon is larger than in the initial setting, so its energy is smaller.  
 7a The wavelength of the associated wave is smaller, so the energy of the photon will be higher in this experiment.  
 7b The energy doubled.  
 9a The energy halved.
- 18.3.3 3a  $\Delta\lambda = 1.000 \times 10^{-12}$  m.  
 6a  $\Delta\lambda = 2.426 \times 10^{-12}$  m, larger than the  $\Delta\lambda$  in step 3a.  
 6b They are equal.  
 6c  $\lambda_2 = 5.43 \cdot 10^{-12} m$   
 8a 0 degrees.  
 8b  $\Delta\lambda$  is twice the Compton wavelength.  
 10a The electron doesn't undergo any recoil.  
 10b No  
 10c  $\Delta\lambda = 0$
- 18.3.4 3a The wavelength of the scattered wave is larger.  
 3b  $\Delta\lambda = 1.10^{-12} m$   
 5a The difference in wavelength is more noticeable.  
 5b No.  
 5c  $\Delta\lambda$  is an order of magnitude bigger than  $\lambda$ .  
 7a Less.  
 7b No.  
 7c  $\Delta\lambda$  is an order of magnitude smaller than  $\lambda$ .
- 18.3.5 8a Increase.  
 12a The speed of light.  
 15a Higher.  
 17a The speed of light.

- 18.3.6 5a The compton wavelength is  $2.426 \times 10^{-12}$  m.  
 5b The recoil angle for the electron is 44.46 degrees.  
 5c  $\Delta\lambda = 1.000 \times 10^{-12}$  m.  
 8a Decreased, it became 10.12 degrees.  
 8b Increased, it became ten times higher.  
 8c Increased 10 times, because the particle has a smaller rest mass.  
 8d Increased to its maximum value,  $c$ .  
 10a Became very small.  
 10b Didn't change, the same particle is observed.  
 10c The wavelength of the scattered photon is larger than the incoming photon's wavelength.  
 10d Didn't change.  
 10e Stayed at its maximum value,  $c$ .  
 12a The recoil angles are the same.  
 16a Increased.  
 16b Decreased.  
 16c Decreased, ten times smaller than the electron compton wavelength.  
 16d Decreased.  
 18a The recoil angles are the same.
- 18.3.7 6a No.  
 6b 0.51 MeV/c (The momentum of the photon).  
 6c 0.51 MeV/c (The sum of the  $x$ -momenta of photon and electron)  
 11a No.  
 11b The total  $y$ - momentum is zero.  
 11c The total  $y$ -momentum is zero.  
 17a No. The total energy is constsnt.  
 17b Yes, its rest energy.

### Answers: Radioactivity

- 19.3.1 9a In the first seconds more nuclei decay than after some time has lapsed.  
 9b Yes (exponential graph).  
 11a The number of nuclei that decayed in the first seconds was higher than in the first experiment.  
 11b The slope of the graph is steeper.

- 14a The experimental graph is more jagged than the theoretical one.
- 19.3.2
  - 3a Approximately 50 nuclei.
  - 3b yes
  - 5a Approximately 50 nuclei.
  - 5b The curves drop faster to zero.
  - 7a Approximately 50 nuclei.
  - 7b The curves drop slower to zero.
  - 7c The decay constant decreased.
  - 9a No nuclei decayed during the experiment.
  - 9b The population stayed constant in time.
  - 9c The decay constant became very small.
- 19.3.3
  - 5a The graph is approximately constant in time.
  - 5b The graph starts to be jagged, the value of Population/Theory differs more and more from one.
  - 7a The graph is linear for a shorter time.
  - 7b After approximately 12 s.
  - 7c The population is of the order of magnitude of 10.
  - 9a The graph is linear for a shorter time.
  - 9b After approximately 8 s.
  - 9c The population is of the order of magnitude of 10.
  - 11a The graph becomes very jagged.
  - 11b Almost immediately.
  - 13a This graph is just jag!
  - 15a The graph is the smoothest one of all.
  - 15b After approximately 18 s the graph starts to look jagged.
  - 15c The population is of the order of magnitude of 10.
  - 16a Theory can best predict the experimental results for a high population.
- 19.3.4
  - 5a The plot is a line. The logarithm of the population varies linear in time.
  - 7a Yes. The slope is steeper.
  - 9a Yes, the slope is less steep than in both steps.
  - 14a The slope does not change when the population changes.
  - 14b The second plot has a lower starting point.
  - 16a The slope is exactly the same for all three plots, because it is determined by the half life.
  - 16b The starting point changes.