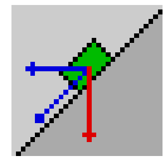
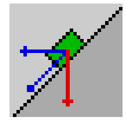
Part III

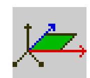
Mechanics



7 Block on Incline

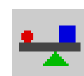


7.1 Prerequisite



A great help to this assignment are the *Vector* simulations.

7.2 Introduction

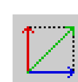


Newton's law tells us that when a force acts on a body it will accelerate (see assignment *Center of Mass*). If more than one force acts on the body we have to make the sum of all the forces to know how it will be accelerating. A familiar force is gravity. More complicated forces are friction forces, e.g. the drag on an airplane, viscosity, etc. In this assignment we are going to study one case of friction, called sliding or dry friction, which occurs when one solid body slides on another.

7.3 Theory

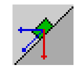
7.3.1 Static and Kinetic Friction Coefficients

A block with mass m sits on an inclined plane. The plane makes an angle α with the horizontal. When this angle raises the block will start sliding at a certain angle. We call the largest angle for which the block is not yet sliding the angle α_s. The forces acting on the block at rest are the weight of the block $\vec{W}$, the normal force from the inclined plane on the block $\vec{N}$ and the static friction $\vec{F_s}$. For ideally frictionless surfaces $\vec{F_s}$ is zero. The static friction is the force that has to be overcome so the block would start moving. As soon as the block starts sliding the kinetic friction will slow it down. The magnitude of the frictonal force (static and kinetic) is proportional to the normal force and reasonably independent of the speed with which the block slides down. These laws are empirical. Have a look at the force diagram (figure 6) for the block.



Newton's law tells us that a body is at rest if no force is acting on it, or if the sum of all forces is zero. (See the assignment *Vector Addition* to find the sum of the forces.) In this case we have:

$$\vec{N} + \vec{F_s} + \vec{W} = 0 \qquad (22)$$



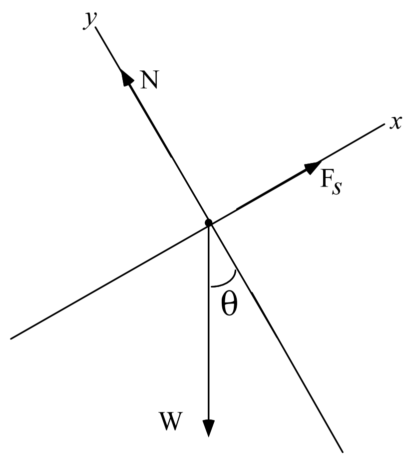
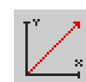


Figure 6: Force diagram for the block



To handle this we have to resolve the forces into their x and y components. (See assignment *Vector Components*.). Let's take the x-axis along the plane and the y axis perpendicular to the plane. We obtain two equations: one with the x components:

$$F_s - W \sin\alpha = 0 \tag{23}$$

and one with the y components:

$$N - W \cos\alpha = 0 \tag{24}$$

Experiment tells us that the magnitude F_s of the static friction $\vec{F_s}$ is proportional to the normal force and is given by:

$$F_s \leq \mu_s N, \tag{25}$$

with μ_s being the coefficient of static friction and N the magnitude of the normal force. μ_s is a dimensionless number and depends on the two surfaces that slide over each other. It is not exactly a constant but when the speed of motion is not too big (so that not too much heat is produced by the friction) it is as good as constant. When $\alpha = \alpha_s$, (α_s the angle above which the block starts sliding), the static friction is maximum and

$$F_s = \mu_s N \tag{26}$$

When the angle is equal to α_s we obtain for equation (24):

$$N = W \cos\alpha_s \tag{27}$$

and in equation (23)

$$\mu_s N = W \sin \alpha_s \tag{28}$$

Dividing equation (28) by (27) gives us:

$$\mu_s = \tan \alpha_s \tag{29}$$

So, when we find the angle α_s above which the block starts sliding we can determine the coefficient of static friction.

In the same way, an angle of inclination α_k will let us determine the kinetic friction coefficient. At this angle the sum of the forces acting on the block (normal, weight and kinetic friction) is zero. The block was sliding already, so it keeps on sliding with constant speed. The magnitude of the kinetic friction F_k is given by:

$$F_k = \mu_k N, \tag{30}$$

with μ_k the coefficient of kinetic friction. This coefficient is always smaller than the coefficient of static friction. Once the block is moving the total force$\vec{F_\Sigma}$ on the block is:

$$\vec{F_\Sigma} = \vec{N} + \vec{W} + \vec{F_k} \tag{31}$$

with $\vec{F_k}$ the kinetic friction. If we change the inclination of the plane so the block slides with a constant speed, we know $\vec{F_\Sigma} = 0$. Again we can write this vector equation into two equations with its components:

- perpendicular to the plane:

$$0 = N - W \cos \alpha \tag{32}$$

- along the plane:

$$0 = W \sin \alpha - F_k \tag{33}$$

what gives us $F_k = \mu_k N = \mu_k W \cos \alpha$.

7.3.2 Conservative and Non-Conservative Forces

Gravity is a so-called *conservative* force. If a block falls free from a height all its potential energy will be transformed into kinetic energy when it reaches the ground if gravity were the only force acting on the block (no friction). The total energy (the sum of potential and kinetic energy) will be constant. If you let the block slide down an inclined plane without friction from the same height, its potential energy will, just the same way as in the free fall, be transformed into kinetic energy and the total energy will be exactly the same. So, it doesn't matter which way you let the block go, as long as the begin and end point are the same and only the conservative force (gravity in this case) is acting on the block.

If you let the block slide down on a incline with friction not all potential energy will be transformed into kinetic energy because some is “lost” to heat. The total energy (not taking into account the “lost energy”) when the block reaches the ground will now be different than when it started. Therefore we call friction a *non-conservative* force. You can argue, of course, that if the dissipated energy is accounted for (the particles inside the block and the plane will have taken up that so called “lost” energy), the total energy of the whole system would be constant.

7.3.3 Gravitational Potential and Kinetic Energy

To calculate the energy of the block we need a reference level for the system where we say the total energy is zero. Very often the ground level is taken as zero energy level. Another sensible choice is the starting point of the block, which is the reference taken in the simulation. At that point the potential, kinetic, and total energy are zero, therefore, the potential energy will become negative as the block slides down. The gravitional potential energy of a mass m is given by:

$$E_p = mgy \tag{34}$$

with g the gravitational acceleration, and y the vertical distance from the chosen reference level. (With our choice of reference level, y is negative as soon as the block slides, and so will be the potential energy.) The kinetic energy of the block with mass m is:

$$E_k = \frac{1}{2}mv^2 \tag{35}$$

with v the speed of the block. The motion of the block is a motion with constant acceleration, (the resulting force on the block is constant—we will go further into this during the laboratory), we know that the speed v of the block is

$$v = at \tag{36}$$

with a the acceleration of the block and t the time past since its starting point, when the initial velocity was zero. The displacement s of the block is:

$$s = \frac{1}{2}at^2 \tag{37}$$

measured along the plane. Substituting v in equation (35) by (36) and (37) gives:

$$E_k = mas \tag{38}$$

7.4 Laboratory

In the following experiments we are going to become familiar with the forces acting on a block on an incline and the effects of these forces. The free-body force diagram in the display shows the forces acting on the block. If the block does not slide down, the static friction is never overcome, so the friction force $\vec{f}$ displayed is the static friction. When the block will slide down, the static friction is overcome, and the force diagram you will see is the one at the moment the block starts sliding. The displayed friction force $\vec{f}$ is then the kinetic friction.

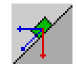
7.4.1 Experiment A: Introduction and Setup

In this experiment, we will familiarize ourselves with the laboratory, especially with the display and the free body force diagram.

1. In the PEARLS window, select *Mechanics*.
2. Launch the *Block on Incline* simulation.
3. You will now see the experiment window, with a control window at the left and a laboratory display at the right.
4. Adjust the window size as desired.
5. Move the laboratory display to a convenient location on your screen by dragging the display with your mouse.
6. The default settings for this experiment are:

Angle	3.5×10^1 degrees
Static Friction Coefficient	5.000×10^{-1}
Kinetic Friction Coefficient	3.000×10^{-1}
Mass	1.000×10^1 kg
Initial Speed	0.000 m/s
Gravity	9.800 m/s^2
Zoom	1.500×10^2 pixels/meter

7. You should see an inclined plane with a block and the angle of incline and the free body force diagram.
   (a) What is the value of the *Angle* of incline?
   (b) What is the value of the *Static Friction* coefficient?
   (c) What is the value of the *Kinetic Friction* coefficient?
   (d) Which forces are acting on the block?
8. Run the experiment, observe and reset.
   (a) Did the block move?
9. Click *Display* and choose *Components*.



(a) What are the components of the weight $\vec{W}$?

(b) Can you tell from the force diagram why the block did move?

10. Set the *Kinetic Friction* coefficient equal to 0.5

11. Run, observe and reset the experiment.

    (a) What happened to the force $\vec{f}$ compared to step 9b?

    (b) Is the force $\vec{f}$ in this step the static or the kinetic friction?

    (c) Did the block move?

    (d) Is the sum of the forces F_Σ zero?

12. Set *Kinetic Friction Coefficient* back to 0.3.

13. Set *Angle* equal to 25 degrees.

14. Compare the length of force $\vec{f}$ with the $\vec{f}$ from step 9b.

    (a) How do they compare?

15. Compare the friction force $\vec{f}$ with $\vec{W}_\|$.

    (a) Do you get the same result as in step 9b?

    (b) Is the sum of the forces F_Σ the same as in step 9b?

16. Run the simulation, stop, and reset.

    (a) Did the block move?

17. Raise the *Mass* to 20 kilogram.

    (a) What happened to the components of the forces?

    (b) Do you expect that the block will move now?

18. Run, observe and reset.

    (a) Did the block move?

19. Observe $\vec{f}$.

20. While observing $\vec{f}$ slide *Kinetic Friction Coefficient* from 0 to 0.5.

    (a) Did $\vec{f}$ change?

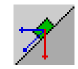
    (b) Is $\vec{f}$ the static or the kinetic friction in this step?

    (c) Is the sum of the forces zero?

7.4.2 Experiment B: The Angle of Inclination and the Static Friction Coefficient μ_s

In this experiment we will study how the static friction coefficient can be determined by the angle of inclination of the plane.

1. Start up the PEARLS *Block on Incline* experiment, or if it is already running, restore the startup conditions by clicking the $\hookleftarrow$ button.
2. From the *Display* menu choose *Components*.
   (a) What is the value of the angle of incline?
   (b) What is the value of the *Static Friction Coefficient*?
   (c) Remember equation(29). For this case: $\tan\alpha_s = 0.5$. What is α_s?
3. Set *Angle* equal to your calculated value.
4. Run, observe and reset the experiment.
   (a) Did the block move?
   (b) Does this result agree with your calculated α_s from step 2c?
5. Set *Angle* equal to 26.6 degrees.
6. Run, observe and reset the experiment.
   (a) Did the block move?
   (b) Compare *Angle* with the calculated α_s. Could you expect that the block would move?
7. Set *Angle* equal to 26.4 degrees.
8. Run, observe and reset the experiment.
   (a) Did the block move?
   (b) Compare *Angle* with the calculated α_s. Could you expect what the block did?
9. Suppose the angle α_s is 30 degrees. Set *Angle* equal to 30 degrees.
   (a) Use your calculator to determine the static friction coefficient μ_s. Remember equation (29).
   (b) Set *Static Friction Coefficient* equal to your calculated value. Does the block slide down?
   (c) Set *Angle* a little higher. Does the block slide down?



7.4.3 Experiment C: Newton's law

In this experiment we will study how the block accelerates due to the forces acting on in.

1. Start up the PEARLS *Block on Incline* experiment, or if it is already running, restore the startup conditions by clicking the $\hookleftarrow$ button.

2. From the *Display* menu choose *Components*.

3. Set *Angle* equal to 30 degrees.

4. Set *Static Friction Coefficient* equal to 0.57.

5. Let's calculate the total force acting on the block. Remember equations (32) and (33).

   (a) Calculate the component of the weight along the plane $W_{\|} = W \sin \alpha$. Remember that $W = mg$ with m the mass of the block, g the gravity acceleration. m is given by the control *Mass* and g is given by the control *Gravity*.

   (b) Calculate the kinetic friction. Remember that $F_k = \mu_k N = \mu_k W \cos \alpha$. μ_k is given by the control *Kinetic Friction*.

   (c) What is the value of the total force on the body along the plane?

6. Click the graph button. Make a graph with *Frictional Force* in the vertical axis and *Time* in the horizontal axis.

7. Run the experiment for 10 seconds and observe the graph.

   (a) How is the frictional force behave with time?

   (b) Does the value in the graph agree with your calulation in step 5b?

8. Close the graph.

9. Knowing the total force working on the block we can calculate its accelaration with Newton's law. Remember $F = ma$ with a the acceleration of the mass m.

   (a) What is the value of the acceleration of the block?

10. Click the graph button and make a graph with *Acceleration* in the vertical axis and *Time* in the horizontal axis.

11. Run the experiment for 10 seconds and reset.

    (a) How does the acceleration behave in time? Is this in accordance with Newton's law?

(b) Is the value of the acceleration in the graph the same as your calculated value of step 9a?

12. Close the graph.

13. Click the graph button. Make a graph with *Velocity* in the vertical axis and *Time* in the horizontal axis.

14. Click the graph button again. Make a graph with *Displacement* in the vertical axis and *Time* in the horizontal axis.

15. Run the experiment for 10 seconds and reset.

    (a) How does the velocity behave in time? Did you expect anything else with a constant acceleration?
    (b) How does the displacement behave in time?

16. Set *Angle* equal to 45 degrees.

17. Procede as in steps 5a, 5b, 5c.

    (a) What is the component of the weight along the plane?
    (b) Compare this with the result in step 5b.
    (c) What is the value of the frictional force?
    (d) Compare this frictional force with the one in step 5a.
    (e) What is the net force on the block?
    (f) Compare this with the result in step 5c.
    (g) What is the value of the acceleration of the block?

7.4.4 Experiment D: The Angle of Inclination and the Kinetic Friction Coefficient μ_k

In this experiment we will study how the kinetic friction coefficient can be determined by the angle of inclination of the plane.

1. Start up the PEARLS *Block on Incline* experiment, or if it is already running, restore the startup conditions by clicking the ↩ button.

2. Set *Angle* equal to 30 degrees.

3. Set *Static Friction Coefficient* equal to 0.57.

4. Click the graph button. Make a graph with *Velocity* in the vertical axis and *Time* in the horizontal axis.

5. Click the graph button again. Make a graph with *Acceleration* in the vertical axis and *Time* in the horizontal axis.

6. Drag the graphs away from each other, so you can see both graphs and the display.
   (a) What is the value of the kinetic friction coefficient?
   (b) Is a net force acting on the block?
7. Run the experiment for 10 seconds and reset.
   (a) How does the velocity behave in time?
   (b) How does the acceleration behave in time?
8. In analogy with experiment B we can find an angle of inclination corresponding to μ_k. Remember $\mu_k = \tan\alpha_k$.
   (a) Calculate the angle corresponding to the *Kinetic Friction Coefficient*.
   (b) Calculate the α_s as in experiment B.
   (c) How does α_s compare to α_k?
9. To verify what happens at the inclination equal to α_k, the block must be sliding down already. Click *Step* on the controlbar.
   (a) From the display, can you tell that the block acquired some velocity?
10. Set *Angle* equal to your calculated value of α_k.
    (a) Is a net force acting on the block?
11. Run the experiment for 10 seconds and reset.
    (a) How is the velocity behaving in time?
    (b) What is the value of the acceleration?
12. Close all graphs.

7.4.5 Experiment E: Conservative and non-conservative forces

In this experiment we will have a closer look at the kinetic, potential and total energy of the block. We will study which forces are conservative and which are not.

1. Start up the PEARLS *Block on Incline* experiment, or if it is already running, restore the startup conditions by clicking the ↩ button.
2. Set *Kinetic Friction Coefficient* equal to 0.
3. Set *Static Friction Coefficient* equal to 0.
4. Click the graph button. Make a graph with *Potential Energy* in the vertical axis and *Time* in the horizontal axis.

5. Click the graph button again. Make a graph with *Kinetic Energy* in the vertical axis and *Time* in the horizontal axis.

6. Click the graph button again. Make a graph with *Total Energy* in the vertical axis and *Time* in the horizontal axis.

7. Drag the graphs so you can see them all.

8. Run the experiment for 10 seconds and reset.

   (a) How does the kinetic energy behave in time? Can you explain why? (use the results from subsection 7.4.3.)

   (b) How does the potential energy behave in time? Can you explain why? (use the results from subsection 7.4.3.)

   (c) How does the total energy, the sum of the potential energy and the kinetic energy behave in time?

   (d) Is energy lost?

   (e) Are all forces acting on the block conservative?

9. Set *Static Friction Coefficient* equal to 0.3.

10. Set *Kinetic Friction Coefficient* equal to 0.3.

11. Run the experiment for 10 seconds and reset.

    (a) How does the total energy behave in time?

    (b) Is energy lost?

    (c) Are all forces working on the block conservative?

12. Set *Static Friction Coefficient* equal to 0.6.

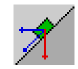
13. Set *Kinetic Friction Coefficient* equal to 0.6.

14. Run the experiment for 10 seconds and reset.

    (a) How does the total energy behave in time?

    (b) Is more energy lost than in step 11b?

15. Close all graphs.



7.4.6 Experiment F: Potential and kinetic energy

In this experiment we will have a closer look at the kinetic, potential and total energy of the block and how they change when the friction changes.

1. Start up the PEARLS *Block on Incline* experiment, or if it is already running, restore the startup conditions by clicking the ↩ button.
2. Set *Kinetic Friction Coefficient* equal to 0.
3. Set *Static Friction Coefficient* equal to 0.
4. Click the graph button. Make a graph with *Potential Energy* in the vertical axis and *Displacement* in the horizontal axis.
5. Click the graph button again. Make a graph with *Kinetic Energy* in the vertical axis and *Displacement* in the horizontal axis.
6. Click the graph button again. Make a graph with *Total Energy* in the vertical axis and *Displacement* in the horizontal axis.
7. Run the experiment for 10 seconds and reset.
   (a) How does the kinetic energy relate to the displacement?
   (b) How does the potential energy relate to the displacement?
   (c) How does the total energy, the sum of the potential energy and the kinetic energy relate to the displacement?
   (d) Is energy lost?
   (e) Are all forces acting on the block conservative?
8. Set *Static Friction Coefficient* equal to 0.3.
9. Set *Kinetic Friction Coefficient* equal to 0.3.
10. Run the experiment for 10 seconds and reset.
    (a) Compare the potential energy of the block with the one in step 7.
    (b) Compare the kinetic energy of the block with the one in step 7.
    (c) Compare the total energy with the one in step 7. Is energy lost?
    (d) Are all forces working on the block conservative?
11. Set *Static Friction Coefficient* equal to 0.6.
12. Set *Kinetic Friction Coefficient* equal to 0.6.
13. Run the experiment for 10 seconds and reset.
    (a) How does the friction compare to step 10?

   (b) How does the energy loss compare to the one in step 10?
   (c) Compare the potential energy of the block with the one in step 10.
   (d) Compare the kinetic energy of the block with the one in step 10.

14. Set *Static Friction Coefficient* equal to 0.9.
15. Set *Kinetic Friction Coefficient* equal to 0.9.
16. Run the experiment for 10 seconds and reset.
   (a) How does the friction compare to step 10 and 16?
   (b) How does the energy loss compare to the one in step 10 and 16?
   (c) Compare the potential energy of the block with the one in step 10 and step 16.
   (d) Compare the kinetic energy of the block with the one in step 10 and step 16.
17. We can calculate the amount of energy that was lost to heat after the block moved 100 m under these conditions.
   (a) Calculate the potential energy $mgy = -mgs \sin\alpha$ for a displacement of 100 m. Compare your value with the graph.
   (b) Calculate the kinetic energy mas for a displacement of 100 m. The acceleration a is given by equation (38) or: $a = g(\sin\alpha - \mu_k \cos\alpha)$. Compare your value with the graph.
   (c) What is the value of the total energy? What is the value of the energy loss? Compare with the graph.
18. Close all graphs.

7.4.7 Experiment G: The Mass of the Block and the Friction Coefficient

In this experiment we will have a closer at what happens when the mass of the block changes.

1. Start up the PEARLS *Block on Incline* experiment, or if it is already running, restore the startup conditions by clicking the ↩ button.
2. Click *Display* and choose *Components*
3. Calculate α_s as in 7.4.2. Remember $\mu_s = \tan\alpha_s$.
   (a) At which angle will the block start sliding?
4. Set *Angle* to α_s degrees.

(a) Does the block slide down?.

5. Set *Angle* to α_s+0.1 degrees, i.e., to your calculated angle plus 0.1 degrees.

   (a) Did the block slide now?

6. Set *Angle* back to your calculated α_s.

7. Set *Mass* equal to 20 kg.

   (a) What happened with the forces in the free body diagram?
   (b) What can you tell about the sum of the forces?

8. Run, observe and reset.

   (a) Did the block slide down, now that it has a bigger mass?

9. Set *Angle* to α_s+0.1 degrees, i.e., to your calculated angle plus 0.1 degrees.

   (a) Did the block slide now?

10. Set *Angle* back to your calculated α_s.

11. Set *Mass* equal to its maximum value.

12. Run and reset.

    (a) Did the block slide?

13. Set *Angle* to α_s+0.1 degrees, i.e., to your calculated angle plus 0.1 degrees.

    (a) Did the block slide now?

7.4.8 Experiment H: Another gravitational field

In this experiment we will change the gravitational acceleration and see its influence on the forces on the block and the energy of the block.

1. Start up the PEARLS *Block on Incline* experiment, or if it is already running, restore the startup conditions by clicking the ↩ button.

2. From the *Display* menu choose *Components*

3. Set *Gravity* equal to double its value.

   (a) Did the body move?
   (b) What happened in the free body diagram?

4. Set *Gravity* back to 9.8 m/s^2.

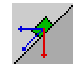
5. Set *Static Friction Coefficient* equal to 0.6.

6. Click the graph button and make a graph with *Friction Force* in the vertical axis and *Time* in the horizontal axis.
7. Click the graph button and make a graph with *Acceleration* in the vertical axis and *Time* in the horizontal axis.
8. Run the experiment for 10 seconds and reset.
9. Set *Gravity* equal to twice its value.
10. Run the experiment for 10 seconds and reset.
    (a) Compare the results in the graphs with what you saw in the free body diagram. Do you see the same?
11. Close all graphs.
12. Set *Gravity* back to 9.8 m/ s^2.
13. Click the graph button and make a graph with *Potential Energy* in the vertical axis and *Displacement* in the horizontal axis.
14. Click the graph button and make a graph with *Kinetic Energy* in the vertical axis and *Displacement* in the horizontal axis.
15. Click the graph button and make a graph with *Total Energy* in the vertical axis and *Displacement* in the horizontal axis.
16. Run the experiment for 10 seconds and reset.
17. Set *Gravity* equal to twice its value.
18. Run the experiment and reset after some time.
    (a) What happened to the potential energy?
    (b) What happened to the kinetic energy?
    (c) What happened to the total energy?

7.5 Review

Please circle the correct answers to the following multiple-choice questions. You may, if you like, use PEARLS to help you find the correct answer.

1. A heavier rock will slide down faster on a hill than a similar lighter rock.
    (a) true
    (b) false



2. The friction force on an object with mass 10 kg sliding down a hill will be 10 times bigger than the friction force on a similar object with mass 1 kg.
   (a) true
   (b) false
3. On the moon where the gravity is 1.67 m/s^2, the friction force on an object sliding down another object will be
   (a) 5.9 times smaller than on earth
   (b) the same as on earth
   (c) 5.9 times bigger than on earth
4. We give a block a push so its starts sliding down a hill making a angle α with the horizontal. The block will slow down
   (a) when the kinetic friction coefficient $\mu_k > \tan\alpha$
   (b) when the kinetic friction coefficient $\mu_k < \tan\alpha$
   (c) when the kinetic friction coefficient $\mu_k = \tan\alpha$
   (d) always.
5. On earth a block starts sliding down an incline when the angle is 45 degrees. On the moon where the gravity is 1.67 m/s^2 the same object on the same incline will start sliding down at an angle:
   (a) $<$ 45 degrees
   (b) $=$ 45 degrees
   (c) $>$ 45 degrees
6. Although the frictional force on a object sliding down an incline on the moon is smaller than on earth the object will have a lower kinetic energy than on earth on the same point on the hill.
   (a) true
   (b) false
7. When the block is just about to slide, the static friction is
   (a) minimum
   (b) maximum
   (c) zero
8. You can go down a snowy hill on your toboggan at a constant speed when
   (a) more people are on the toboggan.

(b) the kinetic friction coefficient is equal to the tangent of the slope angle.

(c) the static friction coefficient is equal to the tangent of the slope angle.

(d) no friction is involved.

9. To pull a sled on a horizontal path less force is needed to overwin the static friction when you pull the sled rope

   (a) horizontally

   (b) in an angle

   (c) either way

10. The kinetic energy of a block sliding down 2 meter on a frictionless hill with a slope angle of 30 degrees, will be the same as when it were falling down in frictionless free fall from the same height over

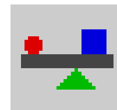
    (a) 1.0 m

    (b) 1.7 m

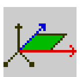
    (c) 2 m

    (d) depends on the mass of the block

8 Center of Mass



8.1 Prerequisite



A great help to this assignment are the *Vector* assignments.

8.1.1 Introduction

In this assignment we will explore the concept of center of mass. It is useful in describing the behavior of isolated systems, i.e. systems which are not subjected to forces from the outside.

8.2 Theory

8.2.1 Newton's Law

When we push on an object, physicists say that there is a force acting on the object. Some experimental facts: if there is no force to balance the push, the object will accelerate (move with changing velocity) as a result. If there is no force acting on an object, or if all the forces cancel each other, the object does not accelerate. These experimental facts are summarized by Newton's Law:

$$\vec{F} = m\vec{a} \tag{39}$$

This states that the acceleration $\vec{a}$ (measured in meters/sec^2) is proportional to the force $\vec{F}$ (measured in Newtons). The constant of proportionality is the mass m, usually measured in grams or kilograms. Note that Newton's law is a vector equation.

8.2.2 Center of Mass Defined

Experiment shows that if we push an object at its center of mass, the object will accelerate, but it will not rotate. Experiments have also found that if no external forces act on a set of masses, then the center of mass of the set does not accelerate. It is for these reasons that the center of mass is a useful quantity in physics.

Imagine a physical system consisting of two bodies. The coordinates of the center of mass of the two-body system are defined as

$$\begin{aligned} x_{cm} &= \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \\ y_{cm} &= \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} \\ z_{cm} &= \frac{m_1 z_1 + m_2 z_2}{m1 + m2} \end{aligned} \tag{40}$$

These three equations can be summarized by a single vector equation, as follows:

$$\vec{R}_{cm} = \frac{m_1 \vec{R}_1 + m_2 \vec{R}_2}{m_1 + m_2} \tag{41}$$

where $\vec{R}_1$, $\vec{R}_2$ and $\vec{R}_{cm}$ are the position vectors of the two masses and the center of mass, respectively.

For simplicity, let us say that the center of mass of a set of bodies is stationary. If no external forces are pushing on the bodies, then we say that the system is isolated, and the center of mass remains at rest. This conclusion holds even if the two bodies are exerting forces on one another. These are called internal forces, and they have no effect on the motion of the center of mass.

In the laboratory, we will explore the consequences of these equations for a two dimensional case, involving motion in X and Y only.

8.3 Laboratory

8.3.1 Experiment A: Introduction

In this experiment, we will examine how the center of mass position changes as the masses of the alligator and the canoe are varied.

1. In the PEARLS window, select *Mechanics*.

2. Now launch the *Center of Mass* entry.

3. You will now see the laboratory window, with a Control Window at the left and a laboratory display at the right.

4. Adjust the window size as desired.

5. Move the laboratory display to a convenient location on your screen by dragging the display with your mouse.

6. The default settings for this experiment are:

Gator Position Rel. Canoe	0 m
Mass of Gator	1.000×10^2 kg
Mass of Canoe	1.000×10^2 kg
System CM Rel. Shore	0 m
Zoom	5.000×10^1 pixels/meter

7. Observe the three cross-hairs which indicate the center of mass of the alligator, the canoe, and the system (alligator plus canoe). Note that the system center of mass is halfway between the canoe's center of mass and the alligator's.

8. Change the *Gator Mass* control to 200 kg using the slider.

   (a) In which direction did the system center of mass move?

9. Change the *Canoe Mass* control to 200 kg using the slider.

   (a) In which direction did the system center of mass move?

   (b) Where is the system center of mass located now?

10. Change the *Gator Mass* control to 20 kg using the slider.

    (a) In which direction did the system center of mass move?

11. Change the *Canoe Mass* control to 20 kg using the slider.

    (a) In which direction did the system center of mass move?

    (b) Where is the system center of mass located now?

    (c) Where is the system center of mass located for a pair of equal masses?

12. Change the *Canoe Mass* control to 0 kg.

    (a) In which direction did the system center of mass move?

    (b) Where is the system center of mass located now?

8.3.2 Experiment B: The Case of Equal Masses

In this experiment, we will investigate the effect of the alligator pedaling her unicycle, thus changing her position within the canoe. In general, this will change her position relative to the shore, as well as the canoe's position relative to the shore.

1. Start up the PEARLS *Center of Mass* experiment, or if it is already running, restore the default conditions by clicking the ↩.

2. Pull down the *Display Menu* and turn off the *Center of Mass* option, to help clarify the diagram.

3. Note the value of the *Gator Pos. Rel. Canoe* control. It should read zero.
4. Also note the values of the Gator Pos. Rel. Shore, Canoe Pos. Rel. Shore and System CM Rel. Shore meters. They should also read zero initially.
5. Let the alligator pedal forward on her unicycle. Do this by reducing the *Gator Pos. Rel. Canoe* control to a value of −2 meters.
   (a) Did the system center of mass change?
   (b) What is the alligator's position relative to the shore now?
   (c) What is the canoe's position relative to the shore now?
6. Let the alligator pedal backward on her unicycle. Do this by increasing the *Gator Pos. Rel. Canoe* control to a value of 2 meters.
   (a) Did the system center of mass change?
   (b) What is the alligator's position relative to the shore?
   (c) What is the canoe's position relative to the shore?

8.3.3 Experiment C: The Case of the Heavy Alligator

In this experiment, we again investigate the effect of the alligator pedaling her unicycle, thus changing her position within the canoe. In experiment A, we did this for the case of equal masses. Here we will first make the alligator heavy compared with the canoe.

1. Start up the PEARLS *Center of Mass* experiment, or if it is already running, restore the default conditions by clicking the ↩.
2. Pull down the *Display Menu* and turn off the *Center of Mass Labels* option, to help clarify the diagram.
3. Set the *Gator Mass* control to 200 kg.
4. Set the *Canoe Mass* control to 2 kg. The alligator now weighs 100 times more than the canoe.
5. Note the value of the *Gator Pos. Rel. Canoe* control. It should read zero.
6. Also note the values of the *Gator Pos. Rel. Shore, Gator Pos. Rel. Shore* and *System CM Rel. Shore* meters. They should also read zero initially.
7. Let the alligator pedal forward on her unicycle. Do this by reducing the *Gator Pos. Rel. Canoe* control to a value of −2 meters.
   (a) Did the system center of mass change?
   (b) What is the alligator's position relative to the shore?

   (c) What is the canoe's position relative to the shore?
   (d) Which moved further relative to the shore: the alligator or the canoe?

8. Let the alligator pedal backward on her unicycle. Do this by increasing the *Gator Pos. Rel. Canoe* control to a value of 2 meters.
   (a) Did the system center of mass change?
   (b) What is the alligator's position relative to the shore?
   (c) What is the canoe's position relative to the shore?
   (d) Which moved further relative to the shore: the alligator or the canoe?
9. Click the graph button and make a graph with *System C.M. (center of mass) Pos.(position) Rel.(relative to the) Shore* in the vertical axis and *Time* in the horizontal axis. Click *OK*.
10. Click the graph button again and make a graph with *Canoe Pos. Rel. Shore* in the vertical axis and *Time* in the horizontal axis. Click *OK*.
11. Click the graph button again and make a graph with *Gator Pos. Rel. Shore* in the vertical axis and *Time* in the horizontal axis. Click *OK*.
12. Run the simulation for 10 seconds. Stop and reset the simulation.
13. Compare the three graphs.
    (a) In between what values does the gator position relative to the shore vary?
    (b) In between what values the canoe position relative to the shore vary?
    (c) Does the position of the center of mass relative to the shore vary?
    (d) Is the position relative to the shore of the canoe zero at the same time the position of the gator is zero relative to the shore?
    (e) When the position of the canoe is positive, is the position of the gator positive or negative? In other words when the gator moves to the left, do the canoe move to the left or the right relative to the shore?

8.3.4 Experiment D: Alligator on an Aircraft Carrier

In this experiment, we investigate the effect of the alligator pedaling her unicycle one last time. In experiment C, we arranged matters such that the alligator was very heavy compared with her boat. Now we do the opposite: we imagine that the alligator is on an aircraft carrier, which is much heavier than she is.

1. Start up the PEARLS *Center of Mass* experiment, or if it is already running, restore the default conditions by clicking the ↩.

2. Pull down the *Display Menu* and turn off the *Center of Mass Labels* option, to help clarify the diagram.
3. Set the *Gator Mass* control to 2 kg.
4. Set the *Canoe Mass* control to 200 kg. The boat now weighs 100 times more than the alligator.
5. Note the value of the *Gator Pos. Rel. Canoe* control. It should read zero.
6. Also note the values of the *Gator Pos. Rel. Shore*, *Canoe Pos. Rel. Shore* and *System CM Rel. Shore* meters. They should also read zero initially.
7. Let the alligator pedal forward on her unicycle. Do this by reducing the *Gator Pos. Rel. Canoe* control to a value of -2 meters.
   (a) Did the system center of mass change?
   (b) What is the alligator's position relative to the shore?
   (c) What is the canoe's position relative to the shore?
   (d) Which moved further relative to the shore: the alligator or the canoe?
8. Let the alligator pedal backward on her unicycle. Do this by increasing the *Gator Pos. Rel. Canoe* control to a value of 2 meters.
   (a) Did the system center of mass change?
   (b) What is the alligator's position relative to the shore?
   (c) What is the canoe's position relative to the shore?
   (d) Which moved further relative to the shore: the alligator or the canoe?
9. Click the graph button and make a graph with *System C.M. (center of mass) Pos.(position) Rel.(relative to the) Shore* in the vertical axis and *Time* in the horizontal axis. Click *OK*.
10. Click the graph button again and make a graph with *Canoe Pos. Rel. Shore* in the vertical axis and *Time* in the horizontal axis. Click *OK*.
11. Click the graph button again and make a graph with *Gator Pos. Rel. Shore* in the vertical axis and *Time* in the horizontal axis. Click *OK*.
12. Run the simulation for 10 seconds. Stop and reset the simulation.
13. Compare the three graphs.
    (a) In between what values does the gator position relative to the shore vary?
    (b) In between what values the canoe position relative to the shore vary?

(c) Does the position of the center of mass relative to the shore vary?

(d) Is the position relative to the shore of the canoe zero at the same time the position of the gator is zero relative to the shore?

(e) When the position of the canoe is positive, is the position of the gator positive or negative? In other words when the gator moves to the left, do the canoe move to the left or the right relative to the shore?

8.4 Review

Please circle the correct answers to the following multiple-choice questions. You may, if you like, use PEARLS to help you find the correct answer.

1. Consider two objects, A and B, of equal mass. Where is the center of mass of the two-mass system?

   (a) at A

   (b) at B

   (c) midway between A and B

2. Is the combined center of mass of two particles always on a straight line connecting the two?

   (a) yes

   (b) no

3. What is the state of motion of the center of mass of a set of particles, if no external forces act on them?

   (a) the center of mass accelerates

   (b) the center of mass moves in a circle

   (c) the velocity of the center of mass is constant

4. Say that particle A has a mass of 4 kg and is located at $x = -1$ m, and particle B has a mass of 1 kg and is located at $x = 1$ m. Where is the combined center of mass of A and B?

   (a) -1.0 meters

   (b) -0.6 meters

   (c) 0 meters

   (d) 0.25 meters

5. Is the center of mass of a two particle system always closer to the heavier particle?

(a) yes
(b) no

6. Can internal forces among a system of particles change the velocity of the combined center of mass?

   (a) yes
   (b) no

7. Consider the alligator on a unicycle from the laboratory section. Say her mass is the same as the mass of the canoe. If she moves 0.4 meters relative to the canoe, how far does she move relative to the shore?

   (a) 0.1 meters
   (b) 0.2 meters
   (c) 0.4 meters
   (d) 0.8 meters

8. In the previous question, how far does the canoe move relative to the shore?

   (a) -0.4 meters
   (b) -0.2 meters
   (c) 0.4 meters
   (d) -0.8 meters

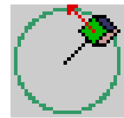
9. Consider two masses, A and B. If both masses are quadrupled, does the center of mass location change?

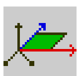
   (a) yes
   (b) no

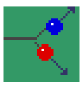
10. If there are no external forces, is it possible for the alligator and the canoe to move in the same direction relative to the shore?

    (a) yes
    (b) no

9 Circular Motion







9.1 Prerequisite

A great help to this assignment are the assignments *Vectors* and *Collision*.

9.2 Introduction

Our planet is continously moving in a circular motion. So does the moon around earth, or the tether ball around the tether pole. So do the particles in a centrifuge, once it has reached it constant speed or the boleadores from the South American cowboy before he throws them at the unlucky cow.

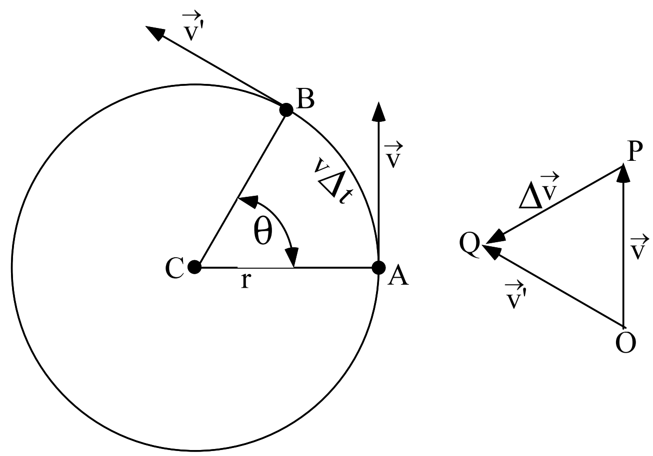


Figure 7: Uniform Circular Motion

Curves rounded by bike or car are parts of circular motions. In this assignment, we will study all the components needed for a uniform circular motion. Velocity, acceleration, force, angular momentum and their relations will become clear.

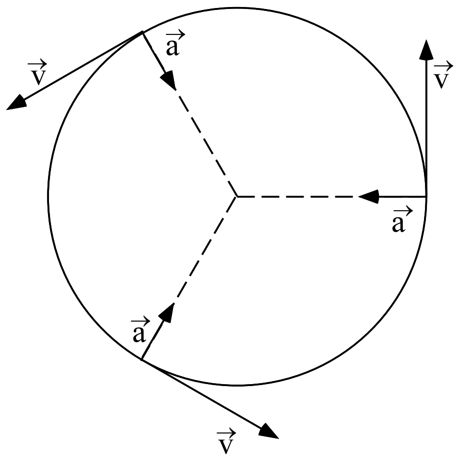
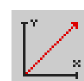


Figure 8: Acceleration and Velocity in the Uniform Circular Motion

9.3 Theory

9.3.1 Centripetal Acceleration



A uniform motion means that the speed of the motion is constant. This doesn't really mean that the velocity vector is constant (see assignment *Vector Components*). The speed is only the magnitude of the velocity. The vector can still change in direction. This is exactly what happens in the uniform circular motion.

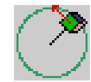
A mass moving with a constant speed in a circle has a velocity that continuously changes direction, but doesn't change in magnitude. Figure (7) shows this clearly. If the velocity changes the acceleration is not zero. It can be shown using geometry and some algebra that the vector with magnitude:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{v^2}{r} \tag{42}$$

is the instantanuous acceleration of the particle. For each position on the circle you can draw the acceleration and velocity vector continuously changing in direction but constant in magnitude. Remark that the acceleration is at all times perpendicular to the velocity. (See figure (8)). Because the acceleration is pointing towards the center we call it the *centripetal* acceleration.

9.3.2 Angular Displacement, Angular Velocity and Angular Acceleration

In this section we are looking at a more practical way to define rotational motion than the one in the previous section. We want to be able to describe a rotational



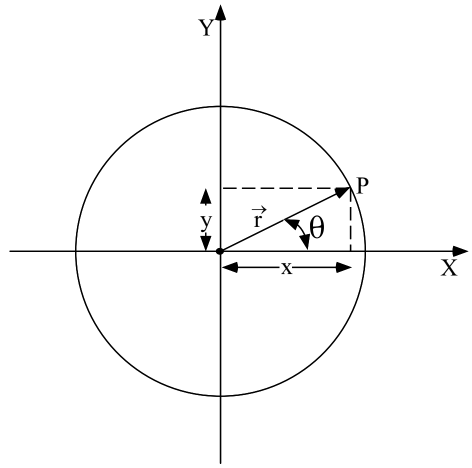


Figure 9: Angular Position

motion the way we describe a translational motion. For example, a uniform linear motion, means that the linear velocity is constant and thus its acceleration zero. The same way we would like to say that a uniform circular motion has constant angular velocity and thus zero angular acceleration. In figure (9) you can see that the position of the particle is at all times determined by the angle θ if we agree upon the reference frame and the sense of rotation.

We call the rotation sense positive if it is counterclockwise (θ increases). The rotation is negative when the particle rotates clockwise (θ decreases). As unit for the angle θ mostly *radians* are used. If the particle has completed a whole circle, we say it has gone 2π radians or 360 degrees. So 1 radian = 57.5 degrees. Instead of measuring the displacement along the circle, we can measure it in the difference in angles. The angular displacement is:

$$\Delta\theta = \theta_1 - \theta_2 \tag{43}$$

This displacement happened in a time Δt. The angular speed (averaged over the interval Δt) is the:

$$\omega_{av} = \frac{\Delta\theta}{\Delta t} \tag{44}$$

In the limit for (Δt aproaching zero) we become the instantaneous angular speed:

$$\omega = \frac{d\theta}{dt} \tag{45}$$

The angular speed is constant in a uniform circular motion. The same way we can define angular acceleration:

$$\alpha = \frac{d\omega}{dt} \tag{46}$$

which is of course zero in the uniform circular motion ($\omega = $ cte.)

The components of the position vector $\vec{r}$ relate to the angular position in the following way:

$$\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned} \tag{47}$$

The linear speed v relates to the angular speed ω in the following way:

$$v = \omega r \tag{48}$$

The linear acceleration a for the uniform circular motion becomes with equation (42)

$$a = \omega^2 r \tag{49}$$

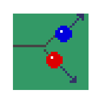
9.3.3 Centripetal Force

Newton's law $\vec{F} = m\vec{a}$ tells us that a body cannot have an acceleration if no force is acting on the body. Therefore a particle going around in a circle with constant speed must have a force acting on it. We know that the acceleration is centripetal and constant in magnitude $a = \frac{v^2}{r}$, so will be the force and its magnitude will be:

$$F = ma = \frac{mv^2}{r} \tag{50}$$

with m the mass of the particle that is moving in a circle, v the speed of the particle, r the radius of the circle. This force is the centripetal force. This means that to keep a mass moving in een circle with constant speed you have to apply a force. In the tether ball this force will be the tension in the string. In an atom the force that keeps the electron circling around the nucleus is an electrostatic force. The moon stays in orbit around the earth because of the gravitational centripetal force.

9.3.4 Angular momentum



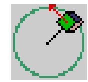
In the assignment *Collision* we became familiar with the linear momentum of a body. The linear momentum of a body with mass m moving with a velocity v is:

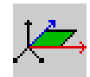
$$\vec{p} = m\vec{v} \tag{51}$$

In analogy to the linear momentum in translational motion we can define a angular momentum for the rotational motion:

$$\vec{L} = \vec{r} \times \vec{p} \tag{52}$$

with $\vec{r}$ the position vector of the particle (the origin is the center of the circle) and $\vec{p}$ the linear momentum. The product is a vector cross product. (See

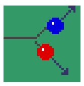




assignment *Vector Cross Product*). The angular momentum will be normal to the plane made by $\vec{r}$ and $\vec{p}$ (use the right hand rule). The magnitude of this angular momentum is:

$$L = rp\sin\phi \tag{53}$$

with ϕ the angle between $\vec{r}$ and $\vec{p}$. p and r are the magnitude of the linear momentum and the position vector, resp. In the uniform circular motion ϕ is 90 degrees because $\vec{v}$ is always normal to $\vec{r}$.



9.3.5 Conservation of Angular Momentum

In the assignment *Collision* we learned about the importance of the law conservation of momentum. In a rotational motion a similar law is valid: conservation of angular momentum. We saw that the law of conservation of linear momentum is very practical when we consider what happens when there are a large numbers of particles with forces in between them and forces on them from outside. If no external force is acting on the system the linear momentum stays constant. In the rotational motion the law translates to: if no external torques act upon the system the angular momentum stays constant. A torque is the force times the lever of that force.

9.4 Laboratory

In the laboratory we will become familiar with all vectors that influence the uniform circular motion.

9.4.1 Experiment A: Introduction and Setup

In this experiment, we will familiarize ourselves with the laboratory, especially with the display and the free body force diagram.

1. In the PEARLS window, select *Mechanics*.
2. Launch *Circular Motion* item.
3. You will now see the experiment window, with a control window at the left, and a laboratory display at the right.
4. Adjust the window size as desired.
5. Move the laboratory display to a convenient location on your screen by dragging the display with your mouse.
6. The default settings for this experiment are:

Radius	7.500×10^1 m
Angular Velocity	1.400 rad/s
Tangential Velocity	1.050×10^2 m/s
Mass	1.000 kg
Zoom	1.000 pixels/meter

7. You should see X, Y and Z axis, a block with its velocity vector $\vec{v}$, a dashed circle, the path of the block.

8. Run the simulation, let the block complete a whole circle, stop, and reset.

   (a) Is the block running clockwise or counterclockwise?

   (b) Is *Angle* always positive or negative? Look at the meter.

   (c) Is *Angular Velocity* positive or negative?

9. Set *Angular Velocity* equal to -1.4 rad/s. Click in the small window under the control, type in the number and press return on your keyboard.

10. Run the experiment, let the block complete a whole circle, stop, and reset.

    (a) Is the block running clockwise or counterclockwise?

    (b) Is *Angle* always positive or negative? Look at the meter.

11. Run the experiment.

12. Turn the rotation control to have a look at all sides.

    (a) In which plane is the block moving?

13. Reset the experiment.

    (a) What is the *Angle* for this position of the block?

    (b) Where is the block positioned?

14. Click *Display*. Turn on the option *Components*. The components of the position vector will be displayed.

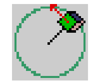
15. Run the experiment and observe the components.

9.4.2 Experiment B: The Angular and Linear Velocity

In this experiment we will study how the angular and the linear velocity relate.

1. Start up the PEARLS *Circular Motion* experiment, or if it is already running, restore the startup conditions by clicking the $\hookleftarrow$ button.

2. Click *Display* and choose *Position Vector*.



3. Click *Display* and choose *Time*.
4. Set *Tangential Velocity* equal to 100 m/s.
5. Run the experiment, observe, stop, and reset.
   (a) In which plane is the position vector located at all times?
   (b) In which plane is the velocity vector located at all times?
6. Turn the *Rotation Control* so the Z-axis is pointing upwards. This view should let you check your answers.
7. Have a look at the *Control Window*.
   (a) What is the value of the *Radius* r?
   (b) What is the value of the *Angular Velocity* ω?
   (c) What is the value v of the *Tangential Velocity* ?
   (d) Check the relation $v = \omega r$.
8. Set *Radius* equal to twice its value, so $r = 150$ m.
9. Run the experiment, observe, stop and reset.
   (a) What is the value of the *Angular Velocity*?
   (b) How did the *Tangential Velocity* change?
   (c) Did you see a change in the display?
10. Set the *Radius* to half its original value, so $r = 37.5$ m.
11. Run the experiment, observe, stop, and reset.
    (a) What is the *Angular Velocity*?
    (b) How did the *Tangential Velocity* change?
    (c) Did you see a change in the display?
12. Set *Radius* back to 75 m.
13. Set *Angular Velocity* equal to twice its value, so $\omega = 2.667$ rad/s.
    (a) What is the value of the tangential velocity?
14. Set *Angular Velocity* equal to -2.667 rad/s.
15. Set *Radius* equal to 75 m.
16. Run, observe, stop and reset the experiment.
    (a) What is the value of the tangential velocity?
    (b) In which direction is the block turning?
    (c) Compare the direction in which the velocity vector is pointing with step 5.

9.4.3 Experiment C: The Angular Velocity and the Angle θ

In this experiment we will study how the angular velocity and the angular position relate.

1. Start up the PEARLS *Circular Motion* experiment, or if it is already running, restore the startup conditions by clicking the $\hookleftarrow$ button.
2. From *Display* choose *Time*.
3. Set *Angular Velocity* equal to 1.57 rad/s.
4. Run and observe.
   (a) How much time (approximately) did the block need to complete a whole circle?
5. Reset the simulation.
6. Click the graph button.
7. Make a graph with *Angle* in the vertical axis and *Time* in the horizontal axis.
8. Click *OK*.
9. Run the experiment for 10 seconds, stop, and reset.
10. Look at the plot.
    (a) What is the maximum value of θ?
    (b) How much time does it take the block to complete a whole circle? Does your observation from step 4a agree with this result?
    (c) In between $\theta = 0$ and its maximum value, how does the angle behave in time?
11. Set *Angular Velocity* equal to -1.57.
12. Run the simulation and observe.
    (a) How much time does the block need to complete a whole circle?
13. Reset the simulation.
14. Run the experiment for 10 seconds, stop and reset.
15. Look at the plot.
    (a) Why is the θ negative?
    (b) What is the minimum value of θ?

(c) How much time does it take the block to complete a whole circle? Does your observation from step 12a agree with this result?

(d) In between $\theta = 0$ and its minimum value, how does the angle behave in time?

16. Set *Radius* equal to 150 m.

17. Run the experiment for 10 seconds, and reset.

    (a) Did *Angular Velocity* change?

    (b) How does the plot for this experiment compare with the one of step 15?

18. Set *Angular Velocity* equal to 3.14 rad/s.

    (a) How long will it take before the block has completed a circle?

19. Run the experiment for 10 seconds, and reset.

    (a) Does the plot agree with your calculation?

20. Close all graphs.

9.4.4 Experiment D: The Components of the Position Vector

In this experiment we will study how the X and Y component of the position vector $\vec{R}$ of the block change when the block moves in a circle with constant speed.

1. Start up the PEARLS *Circular Motion* experiment, or if it is already running, restore the startup conditions by clicking the $\hookleftarrow$ button.

2. From *Display* choose *Position Components*.

3. Rotate the display with the *Rotation Control* so you have a clear view at the components X and Y of the position vector R.

4. Set *Angular Velocity* equal to 1.26 rad/s.

5. Run the experiment until the block has completed a whole circle and reset.

    (a) At what position on the circle is X zero?

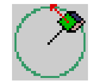
    (b) At what position on the circle is Y zero?

    (c) At what position on the circle is X maximum?

    (d) At what position on the circle is Y maximum?

    (e) What value does the Z-component have at all times?

6. Click the graph button. Make a graph with *X-component* in the vertical axis and *Time* in the horizontal axis.
7. Click the graph button again. Make a graph with *Y-component* in the vertical axis and *Time* in the horizontal axis.
8. Run the experiment for 10 seconds and reset.
   (a) How does the X-component X change in time?
   (b) How does the Y-component Y change in time?
   (c) Can you tell from the plot how long it took the block to complete a whole circle?
9. Click the graph button. Make a graph with *X-component* in the vertical axis and *Angle* in the horizontal axis.
10. Set the minimum of X-component equal to -150 meters and the maximum equal to 150 meters. Click *OK*.
11. Click the graph button again. Make a graph with *Y-component* in the vertical axis and *Angle* in the horizontal axis.
12. Set the minimum of Y-component equal to -150 meters and the maximum equal to 150 meters. Click *OK*.
13. Run the experiment for 10 seconds and reset.
    (a) At which value of θ is X zero? Compare with step 5a.
    (b) At which value of θ is Y zero? Compare with step 5b.
    (c) At which value of θ is X maximum? Compare with step 5c.
    (d) At which value of θ is Y maximum? Compare with step 5d.
    (e) At which value of θ are both components the same?
14. Set *Radius* equal to 150 m.
    (a) How do the plots of the components of the position vector in function of the time change compared to step 8a and 8b?
    (b) How do the plots of the components of the position vector in function of θ change compared to step 13?
15. Set *Radius* back to 75 m.
16. Set *Angular Velocity* equal to 2.52 rad/s.
17. Run the experiment for 10 seconds and reset.



(a) How do the plots of the components of the position vector in function of the time change compared to step 8a and 8b?

(b) How do the plots of the components of the position vector in function of θ change compared to step 13?

18. Close all graphs.

9.4.5 Experiment E: The Angular Momentum

In this experiment we will become familiar with the concept of angular momentum.

1. Start up the PEARLS *Circular Motion* experiment, or if it is already running, restore the startup conditions by clicking the ↩ button.

2. From *Display* choose *Force Vector*, *Angular Momentum Vector*, *Position Vector*, *Velocity Vector*.

3. Set *Angular Velocity* equal to 2 rad/s.

4. Rotate the display with the *Rotation Control* so you have a clear view at all the vectors.

5. Run the experiment so the block completes a whole circle and reset.

   (a) In which plane does the force vector lay at all times?

   (b) In which plane does the position vector lay at all times?

   (c) Along which axis does the angular momentum vector lay at all times?

6. Set *Angular Velocity* equal to -2 rad/s.

7. Run the experiment so the block completes a whole circle and reset.

   (a) How is the angular momentum different from step 5?

   (b) Can you explain this difference using the right hand rule and the definition of angular momentum?

8. Set *Angular Velocity* equal to 4 rad/s.

   (a) How did the force change?

   (b) How did the angular momentum change?

9.4.6 Experiment F: Conservation of Angular Momentum

In this experiment we will look at the variables of the block when its angular momentum is constant, in other words when no external torque is acting on it.

1. Start up the PEARLS *Circular Motion* experiment, or if it is already running, restore the startup conditions by clicking the $\hookleftarrow$ button.

2. From *Display* choose *Angular Momentum Vector*.

3. From *Display* choose *Velocity Vector*.

4. Run the experiment.

5. While the experiment is running, change the radius to greater values with the slider. Observe the display. Observe the control window.

   (a) Does the *Angular Momentum Vector* change?

   (b) How does the *Tangential Velocity* change?

   (c) How does the *Angular Velocity* change?

6. From *Display* choose *Conserve Angular Momentum*.

7. Run the experiment.

8. While the experiment is running, change the radius to greater values with the slider. Observe the display. Observe the control window.

   (a) Does the *Angular Momentum* change?

   (b) How does the *Tangential Velocity* change?

   (c) How does the *Angular Velocity* change?

9. Restore the default values. Click $\hookleftarrow$.

   (a) Calculate the angular momentum for the default values.

10. Set *Radius* equal to 50 m.

    (a) To keep the angular momentum constant, what should the angular velocity ω be?

    (b) To what value will the tangential velocity change?

9.5 Review

Please circle the correct answers to the following multiple-choice questions. You may, if you like, use PEARLS to help you find the correct answer.

1. A particle travels in a circular path with constant speed v and radius r. When the speed is doubled the acceleration is

   (a) zero, because the speed is constant.

   (b) doubled also.

   (c) four times its original value.

2. A car can round a flat curve at a speed

   (a) unlimited

   (b) limited by its weight only.

   (c) limited by friction between the tires and the road.

3. You are whirling a ball on a string in a horizontal circle with radius 1 m. If the centripetal acceleration is equal to the acceleration due to gravity the ball is completing

   (a) 3.13 revolutions/second

   (b) 29.9 revolutions/second

   (c) 29.9 revolutions/minute

4. The tension in a string of 0.5 m whirling a ball of 1 kg around horizontally completing two circles in a second is:

   (a) 78.9 N

   (b) 19.7 N

   (c) 6.28 N

5. The angular speed of an hour hand of a watch is

   (a) 0.52 rad/h

   (b) 6.28 rad/h

   (c) 0.08 rad/h

6. Assume the car's speedometer reads proportional to the rotational speed of the wheels. The driver changes his tires to larger ones. The speedometer read will be

   (a) lower than its actual speed

   (b) the same as its actual speed

(c) higher than its actual speed.

7. A particle completes a circle of radius 4 cm in 8 seconds. Its linear velocity is:

   (a) 0.78 cm/s
   (b) 1.27 cm/s
   (c) 3.14 cm/s

8. A particle completes counter clockwise a circle of radius 4 cm in 8 seconds. At time $t = 0$ the x component is 4 cm. The x component of the position vector of the particle at $t = 1$ s will be

   (a) 3.14 cm
   (b) 2.83 cm
   (c) 6.28 cm

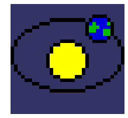
9. If a ball with a mass twice an other ball is whirled in a horizontal circle at the same speed its angular momentum will be:

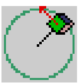
   (a) the same
   (b) twice the other ball's angular momentum
   (c) half the other ball's angular momentum

10. The angular momentum $\vec{L} = \vec{r} \times \vec{p}$ of a particle with mass m in uniform circular motion will continuously vary in direction because $\vec{r}$ and $\vec{v}$ change continuously in direction.

   (a) true
   (b) false

10 Circular Orbit

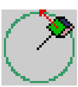




10.1 Prerequisite

A great help this assignment is the assignment *Circular Motion*.

10.2 Introduction



A body can only move in a circle when a centripetal force keeps it in that path. Centripetal means pointing towards the center. One example of a uniform circular motion is the revolution of the planets around the sun. The orbit, that is the path around the sun, only differs slightly from a circle. The force that keeps the planet in orbit is the gravitation. This force sets the centripetal acceleration for a certain planet and thus its velocity.

In the assignment *Circular Motion* we defined several new concepts. The angular velocity and angular momentum were defined in analogy with their linear counterparts. The angular quantities allow us to describe the rotational movement similar to the translational movement. In analogy with the uniform linear motion we can state that a body in a uniform circular motion has a constant angular velocity. The angular acceleration is zero.

10.3 Theory

The centripetal force, that keeps the planet in orbit around the sun (or a satelite around a planet) is the gravity. The universal gravitation law looks as follows:

$$F = G\frac{m_1 m_2}{r^2} \tag{54}$$

with F the magnitude of the force between two particles with mass m_1, m_2, respectively, separated by a distance r. G is the universal gravitation constant. (To find the gravitational force between the sun and the earth we fill in m_1 the mass of the sun, m_2 the mass of the earth and r the distance earth-sun.) The value of G is: $G = 6.6726 \times 10^{-11}$ m^3/kg.s^2.

The period of revolution T of a planet around the sun is the time the planet takes to complete a whole circle around the sun. The angular velocity ω for the uniform circular motion is given by:

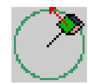
$$\omega = \frac{\Delta\theta}{\Delta t} \tag{55}$$

with $\Delta\theta$ the change in angular position occurred in the time Δt. For a whole revolution

$$\begin{aligned}\Delta\theta &= 2\pi \\ \Delta t &= T\end{aligned}$$

We obtain for T using equation (55):

$$T = \frac{2\pi}{\omega} \tag{56}$$



We recall also the relation between the linear velocity and the angular velocity (see assignment *Circular Motion*):

$$v = \omega r = \frac{2\pi r}{T} \tag{57}$$

The magnitude of the centripetal acceleration is:

$$a = \frac{v^2}{r} \tag{58}$$

10.4 Laboratory

This lab shows the planetary motion which is circular to a good approximation. The main purpose of this lab is to show that for any given orbital radius there is only one possible orbital velocity and acceleration.

10.4.1 Experiment A: Introduction and Setup

In this experiment, we will familiarize ourselves with the laboratory, especially with the display and the free body force diagram.

1. In the PEARLS window, select *Mechanics*.

2. Now launch the *Circular Orbit* item.

3. You will now see the experiment window, with a control window at the left and a laboratory display at the right.

4. Adjust the window size as desired.

5. Move the laboratory display to a convenient location on your screen by dragging the display with your mouse.

6. The default settings for this experiment are:

Radius	1.500×10^{11} m
Tangential Velocity	2.975×10^{4} m/s
Angular Velocity	1.983×10^{-7} rad/s
Mass of Sun	1.990×10^{30} kg
Mass of Planet	5.980×10^{24} kg
Gravity	6.670×10^{-11} Nm^2/kg^2
Zoom	6.870×10^{-10} pixels/meter
Time Zoom	6.000×10^{6}

7. You should see a yellow circle, the sun, and the earth on a background with stars. The $X - Y$ axis are displayed and the orbit (blue circle) of the planet.

8. Look at the control window.

   (a) What is the radius of the orbit?

   (b) What is the tangential velocity?

9. Set *Radius* equal to 1.08×10^{11} m (the distance Venus-Sun).

   (a) What other values changed in the *Control Window*?

10. From *Display*, choose *Velocity, Acceleration, Net Force*.

11. *Run* the experiment, observe until the planet has complete a revolution, *Stop* and *Reset*.

    (a) Is the velocity tangent to the circle at all times?

    (b) Is the acceleration centripetal at all times?

    (c) What causes the centripetal force $\vec{F}$?

10.4.2 Experiment B: The Period of Revolution

In this experiment we will become familiar with the concept period of revolution and study this for several planets.

1. Start up the PEARLS *Circular Orbit* experiment, or if it is already running, restore the startup conditions by clicking the $\hookleftarrow$ button.

2. Click *Display* and choose *Velocity*.

3. Click the graph button. Make a graph with *Angle* in the vertical axis and *Time* in the horizontal axis.

4. Run the experiment until the earth has completed two revolutions, stop, and reset.

   (a) The data from the *Control Window* and equation (57) are all we need to calculate the period of the earth around the sun. What is the period?
   (b) Look at the plot. When did the angle complete a whole circle (2π radians)? Does your calculated period agree with the value resulting from the graph?

5. Set *Radius* equal to 5.8×10^{10} m, the distance Mercury-Sun.
6. Run the experiment until the planet has completed two revolutions, stop and reset.
   (a) How does the tangential speed compare to the one of the earth in its orbit?
   (b) Calculate the period of the revolution of Mercury around the sun?
   (c) Does your calculated value agree with the graph value?
7. Set *Radius* equal to 2.28×10^{11} m, the distance Sun-Mars.
8. Run the experiment until the planet has completed two revolutions, stop and reset.
   (a) How does the tangential speed of Mars compare to the one of the earth in its orbit?
   (b) Calculate the period of the revolution of Mars around the sun?
   (c) Does your calculated value agree with the graph value?
9. Close all graphs.

10.4.3 Experiment C: Centripetal Acceleration

In this experiment we will study the centripetal acceleration for several planets, resulting from the gravitational force between them and the sun.

1. Start up the PEARLS *Circular Orbit* experiment, or if it is already running, restore the startup conditions by clicking the $\hookleftarrow$ button.
2. From *Display* choose *Velocity, Acceleration, Net Force.*
3. Recall the relation between centripetal acceleration a and tangential speed v in a uniform circular motion: $a = v^2/r$ with r the radius of the circle or orbit.
   (a) The data in the *Control Window* are by default the data for the motion of the earth around the sun. Calculate the centripetal acceleration of the earth.

4. Recall the equation (54) for the universal gravitation force and Newton's law: $\vec{F} = m\vec{a}$. The force is the gravitational force, m is the mass of the planet and a is its centripetal acceleration. With equation (54) we become:

$$a = Gm_{sun}/r^2. \tag{59}$$

with $G = 6.6726 \times 10^{-11}$ m^3/kg s^2 the universal gravitational constant.

   (a) Using your calculated a from step 3a what is the mass of the sun? Compare to the *Control Window* value.

5. Set *Radius* equal to 5.8×10^{10} m, the distance Mercury-Sun.

   (a) Looking at the display, how does the centripetal acceleration in this orbit compare to the one in step 3a?

   (b) With the data of the *Control Window* calculate the centripetal acceleration.

6. Set *Radius* equal to 2.28×10^{11} m, the distance Sun-Mars.

   (a) Looking at the display, how does the centripetal acceleration in this orbit compare to the one in step 3a?

   (b) With the data of the *Control Window* calculate the centripetal acceleration.

7. Change the mass of the planet to greater values with the slider.

   (a) Does the centripetal acceleration change?

   (b) Does the tangential speed change?

   (c) Does the angular speed change?

   (d) Does the gravitational force change?

8. Reset the simulation.

9. Change the mass of the sun to greater values with the slider.

   (a) If the planet stays in the same orbit, does the angular velocity change?

   (b) If the planet stays in the same orbit, does the tangential velocity change?

   (c) If the planet stays in the same orbit, does the centripetal acceleration change?

10. Reset the simulation.

11. Change the radius of the planet's orbit to greater values with the slider.

(a) Does the gravitational force change?
(b) Does the centripetal acceleration change?
(c) Does the tangential speed change?
(d) Does the angular speed change?

10.5 Review

Please circle the correct answers to the following multiple-choice questions. You may, if you like, use PEARLS to help you find the correct answer.

1. The acceleration of the planet Pluto, compared to the acceleration of Mercury is
   (a) much larger
   (b) much smaller
   (c) the same
2. The linear velocity of a planet farther from the sun, compared to the linear velocity of a planet closer to the sun is
   (a) greater
   (b) smaller
   (c) depending on the planet.
3. The acceleration of the planets around the sun is independent of their mass.
   (a) true
   (b) false
4. An electron is forced into a circular path by a magnetic field. It has a radial acceleration of 3.0×10^{14} m/s^2. Its path has a radius of 15 cm. The speed of this electron is:
   (a) 1.1×10^6 m/s
   (b) 6.7×10^6 m/s
   (c) 6.7×10^8 m/s
5. The electron in the previous question will have completed one circle in:
   (a) 1.4×10^{-7} s
   (b) 7.0×10^{-8} s
   (c) 8.6×10^{-5} s

6. A satellite moves at a constant speed in a circular orbit about the center of the earth, near the surface of the earth (take the orbit radius equal to the earth radius: 6370 km, the acceleration equal to 9.81 m/s^2.) Its speed is:

   (a) 250 m/s

   (b) 7.9 m/s

   (c) 7.91 km/s

7. The distance from the earth to the moon is approx. 3.82×10^8 m. The moon completes a whole orbit in 27.3 days. The radial acceleration of the moon is:

   (a) 2.7×10^{-3} m/s^2

   (b) 1.67 m/s^2

   (c) 9.8 m/s^2

8. The mass of Jupiter is expected to be 318 times the mass of the earth. Jupiter is 5 times farther away from the sun than the earth. Compare the centripetal force F_e excerted by the sun on the earth with the centripetal force F_J excerted by the sun on Jupiter:

   (a) $F_J = 0.08F_e$

   (b) $F_J = 12.7F_e$

   (c) $F_J = 63.6F_e$

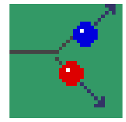
9. If the earth would have a smaller revolution period, it would have an orbit

   (a) closer to the sun

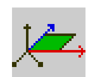
   (b) farther from the sun

   (c) the same as it has now.

11 Elastic Collisions



11.1 Prerequisite



A great help to this assignment are the *Vectors* simulations.

11.2 Introduction

Everything in the universe is composed of particles. If we can understand how any given pair of particles behaves in a collision, then we can understand a great deal about the universe.

In this assignment we will examine a relatively simple sort of collision: the elastic collision of two pool balls. In creating the laboratory for this assignment, we chose pool balls for the sake of concreteness, but don't take this too seriously. Real pool balls are more complicated than this. Also, we might just as well have chosen the collision of two inert gas atoms (Helium, for example), which behave similarly, though on a very different size scale.

The important thing is that the principles we will learn in this assignment apply to any collision, even in realms very far removed from the pool table.

11.3 Theory

11.3.1 Fundamentals

In the last assignment we introduced our first vector equation, Newton's Law:

$$\vec{F} = m\vec{a} \tag{60}$$

This states that the acceleration $\vec{a}$ (measured in meters/seconds^2) is proportional to the force $\vec{F}$ (measured in Newtons). The constant of proportionality is the mass m, usually measured in grams or kilograms.

We now proceed by defining two new physical quantities. The first of these is called the kinetic energy, which is a parameter associated with the motion of an object. It is defined as

$$K = \frac{1}{2}mv^2 \tag{61}$$

where m is the mass of the particle (usually in kilograms), and v is its speed (usually in meters per second). Note that equation (61) involves numbers rather than vectors, and thus the kinetic energy does not have any direction associated with it.

Another quantity associated with the motion of a particle is the momentum. Unlike the kinetic energy, momentum is a vector whose direction is the same as the particle's velocity. It is defined by the equation

$$\vec{P} = m\vec{v} \tag{62}$$

where again m is the mass, and $\vec{v}$ is the velocity vector of the particle. This vector equation is the equivalent of three scalar equations (equations involving ordinary numbers):

$$P_x = mv_x \tag{63}$$

$$P_y = mv_y \tag{64}$$

$$P_z = mv_z \tag{65}$$

In the above equations, we have used the components of the velocity and momentum vectors, which are their projections onto a cartesian coordinate system. Please refer to our earlier assignments on coordinate systems and vectors if you have questions about these equations.

11.3.2 Conservation Laws

In our analysis of the collision of two pool balls, we will make some simplifying assumptions. We will assume that the pools balls are not deformed, smashed, or stuck together as a result of the collision; this is called an elastic collision. We will also assume that no outside force of any kind (including friction) acts on the pool balls.

Under these conditions, it turns out that both quantities introduced above, momentum and kinetic energy, are conserved. This means that, no matter how we set up the pool balls, the total momentum and the total kinetic energy do not change as a result of the collision. It turns out that these two facts are related to important symmetries possessed by our universe.

Let us now state these facts in mathematical form. Conservation of energy states that

$$K_i = K_f \tag{66}$$

where K_i is the total kinetic energy initially (that is, before the balls collide) and K_f is the final kinetic energy (after the collision). K_i and K_f are obtained by adding together the kinetic energies of the two balls, like this:

$$K_i = \frac{1}{2}m_A {v_{Ai}}^2 + \frac{1}{2}m_B {v_{Bi}}^2 \tag{67}$$

$$K_f = \frac{1}{2}m_A {v_{Af}}^2 + \frac{1}{2}m_B {v_{Bf}}^2 \tag{68}$$

where the subscripts A and B refer to pool balls A and B respectively.

Conservation of momentum states that

$$\vec{P}_i = \vec{P}_f \tag{69}$$

where

$$\vec{P}_i = m_A \vec{v}_{Ai} + m_B \vec{v}_{Bi} \tag{70}$$

$$\vec{P}_f = m_A \vec{v}_{Af} + m_B \vec{v}_{Bf} \tag{71}$$

11.3.3 Reference Frames

The laws of physics should not depend on the velocity of our laboratory. If they did, we would need a new set of laws for every possible velocity that our lab could have, which is an uncountably infinite number. This idea is called the principle of relativity.

Thus, the equations given above should hold no matter what velocity our lab has. Experiment shows that they do in fact hold, so long as our pool balls move slowly compared with the speed of light.

In the case we will look at in the laboratory, it is possible for you (the observer) to move relative to the pool table with different velocities. Every possible value of the observer's velocity corresponds to a different reference frame. In every reference frame, the numbers that go into the equations will be different, but the equations themselves are always valid.

In the lab we will investigate how the appearance of a collision changes as we observe it from different reference frames.

11.4 Laboratory

11.4.1 Experiment A: Introduction

In this experiment, we will familiarize ourselves with the laboratory, especially the Controls and the Display Menu. In this lab, we will be creating collisions of pool balls, and observing the results. The pool table was chosen because it is something that most of us are familiar with. We could have chosen inert gas atoms instead of pool balls, and our results would still hold. Therefore, think in terms of a pool table if it helps you to understand what is happening; but bear in mind that the results are quite general and apply to systems very different from pool balls.

1. In the PEARLS window, select *Mechanics*.
2. Now launch the *Collisions* entry.
3. You will now see the laboratory window, with a Control Window at the left and a laboratory display at the right.

4. Adjust the window size as desired.

5. Move the laboratory display to a convenient location on your screen by dragging the display with your mouse.

6. The default settings for this experiment are:

Initial Velocity A	1.000 m/s
Impact Parameter	1.000×10^{-2} m
Mass A	5.000×10^{-1} kg
Mass B	5.000×10^{-1} kg
Observer Velocity X	0 m/s
Observer Velocity Y	0 m/s
Zoom	5.000×10^{2} pixels/meter
Time Zoom	1

7. You should now see two pool balls, and two black lines indicating the trajectories of the balls.

8. *Run* the experiment and observe the collision, then *Stop* and *Reset* to return the experiment to its initial state.

9. Set the *Initial Velocity of A* Control to 0.5 m/s and run the experiment again. Be sure to click the *Reset* Button at the end of each run.

   (a) Have the trajectories changed?

   (b) Has the speed of ball B after the collision increased or decreased?

10. Set the *Initial Velocity* of A Control back to 1 m/s.

11. Slowly increase the *Impact Parameter* Control to a value of 0.069 m.

    (a) In what way have the trajectories changed?

12. Using the *Display Menu*, turn off the *Trajectories* option and turn on the *Collision Point* option.

13. *Run* and observe the collision, then *Reset* to return the experiment to its initial state.

14. Set the *Observer Y Velocity* to -0.4 m/s and run the experiment again, then reset.

    (a) Did the collision point's position change?

11.4.2 Experiment B: Head-On Collisions

In this experiment, we will conduct a set of head-on and nearly head-on collisions, and observe how we can arrive at very different outcomes, depending on the exact conditions we specify.

1. Start up the PEARLS *Collision* experiment, or if it is already running, restore the default conditions by clicking the $\hookleftarrow$ button.

2. Set the *Impact Parameter* Control to zero. *Run* the experiment and then *Reset*.

   (a) What are the masses of the two balls?

   (b) What happens to ball A after the collision?

3. Set the *Mass of B* to 0.4 kg. *Run* the experiment and then *Reset*.

   (a) What happens to ball A after the collision?

4. Set the *Mass of A* to 0.3 kg. *Run* the experiment and then *Reset*.

   (a) What happens to ball A after the collision?

5. Set the Mass of both balls to 0.4 kg. Set the *Impact Parameter* to a small value: 6×10^{-4} m (you will need to type in this number as $6e-04$). *Run* the experiment and then *Reset*.

   (a) How does Ball A move after the collision?

6. Set the *Impact Parameter* back to zero. *Run* the experiment and *Reset*.

   (a) How does the motion of Ball A after the collision compare with step 5a?

11.4.3 Experiment C: Conservation of Energy

In this experiment we will check the equation for the conservation of kinetic energy.

1. Start up the PEARLS *Collision* experiment, or if it is already running, restore the default conditions by clicking the Defaults button $\hookleftarrow$.

2. Set the *Initial Velocity of A* control to 0.4 m/s.

3. Click the *Graph* Button and create a graph with *Kinetic Energy of A* on the vertical axis and *Time* on the horizontal axis. (If you are not sure how to create a graph, please consult the user's manual) Drag the new graph to the top of the window.

4. Click the Graph Button and create a graph with *Kinetic Energy of B* on the vertical axis and *Time* on the horizontal axis.

5. Arrange the two graphs so that they are both easily visible.

6. *Run* the experiment and then *Reset*. Examine the two graphs.

   (a) What happens to the kinetic energy of A as a result of the collision?

   (b) What happens to the kinetic energy of B as a result of the collision?

   (c) What happens to the total kinetic energy of A and B as a result of the collision?

7. Set the *Initial Velocity of A* to 0.3 m/s. *Run* the experiment and then *Reset*. Examine the two graphs.

   (a) How is the graph of the kinetic energy of A changed compared with step 6?

   (b) How is the graph of the kinetic energy of B changed compared with step 6?

   (c) Is the total kinetic energy greater or less than the kinetic energy in step 6?

8. Set the sl Impact Parameter to 0.056 m. *Run* the experiment and then *Reset*.

   (a) How is the graph of the kinetic energy of A changed compared with step 7?

   (b) How is the graph of the kinetic energy of B changed compared with step 7?

   (c) Is the total kinetic energy different from the total kinetic energy of step 7?

11.4.4 Experiment D: Conservation of X Momentum

In this experiment we will observe how conservation of momentum applies to collisions of different types.

1. Start up the PEARLS *Collision* experiment, or if it is already running, restore the default conditions by clicking the Defaults button ↩. Also, close any graphs that may be present.

2. Set the *Initial Velocity of A* control to 0.4 m/s.

3. Set the *Mass of A* and the *Mass of B* controls to 0.1 kg.

4. Click the Graph Button and create a graph with *X Momentum of A* on the vertical axis and *Time* on the horizontal axis. (If you are not sure how to create a graph, please consult the User Manual.) Drag the new graph to the top of the window.

5. Click the Graph Button and create a graph with *X Momentum of B* on the vertical axis and *Time* on the horizontal axis.

6. Arrange the two graphs so that they are both easily visible.

7. *Run* the experiment and then *Reset*. Examine the two graphs.

   (a) What happens to the *X momentum of A* as a result of the collision?

   (b) What happens to the *X momentum of B* as a result of the collision?

   (c) What happens to the total x momentum of A and B as a result of the collision?

8. Set the *Initial Velocity of A* to 0.3 m/s. *Run* the experiment and then *Reset*. Examine the two graphs.

   (a) How is the graph of the *X momentum of A* changed compared with step 7?

   (b) How is the graph of the *X momentum of B* changed compared with step 7?

   (c) Is the *total X momentum* greater or less than the *total X momentum* in step 7?

9. Set the *Impact Parameter* to 0.05 m. *Run* the experiment and then *Reset*.

   (a) How is the graph of the *X momentum of A* changed compared with step 8?

   (b) How is the graph of the*X momentum of B* changed compared with step 8?

   (c) Is the *total X momentum* different from the *total X momentum* of step 8?

11.4.5 Experiment E: Conservation of Y Momentum

In this experiment we will observe how conservation of momentum applies to collisions of different types. This time we will examine the vertical (y) momentum.

1. Start up the PEARLS *Collision* experiment, or if it is already running, restore the default conditions by clicking the Defaults button $\hookleftarrow$. Also, close any graphs that may be present.

2. Set the *Mass of A* and the *Mass of B* controls to 0.1 kg.

3. Click the Graph Button and create a graph with *Y Momentum of A* on the vertical axis and *Time* on the horizontal axis. (If you are not sure how to create a graph, please consult the User Manual.) Drag the new graph to the top of the window.

4. Click the Graph Button and create a graph with *Y Momentum of B* on the vertical axis and *Time* on the horizontal axis.

5. Arrange the two graphs so that they are both easily visible.

6. *Run* the experiment and then *Reset*. Examine the two graphs.

   (a) What happens to the *Y momentum of A* as a result of the collision?

   (b) What happens to the *Y momentum of B* as a result of the collision?

   (c) What happens to the *total Y momentum* of A and B as a result of the collision?

7. Set the *Mass of A* and the *Mass of B* controls to 0.2 kg. *Run* the experiment and then *Reset*. Examine the two graphs.

   (a) How is the graph of the *Y momentum of A* changed compared with step 6?

   (b) How is the graph of the *Y momentum of B* changed compared with step 6?

   (c) Is the *total Y momentum* greater or less than the *total X momentum* in step 6?

8. Set the *Impact Parameter* to 0.015 m. *Run* the experiment and then *Reset*.

   (a) How is the graph of the *Y momentum of A* changed compared with step 7?

   (b) How is the graph of the *Y momentum of B* changed compared with step 7?

   (c) Is the *total Y momentum* different from the *total Y momentum* of step 7?

11.4.6 Experiment F: Center of Mass Motion

In this experiment we will observe how conservation of momentum applies to collisions of different types.

1. Start up the PEARLS *Collision* experiment, or if it is already running, restore the default conditions by clicking the Defaults button ↩. Also, close any graphs that may be present.

2. With the *Display Menu*, turn on the *Center of Mass* display.
3. Using the *Display Menu*, turn on the *Labels* option.
4. Set the *Initial Velocity of A* control to 0.35 m/s.
5. *Run* the experiment and then *Reset*. Observe the motion of the center of mass of the two pool balls.
   (a) Is the center of mass motion affected by the collision?
6. Set the *Impact Parameter* to zero. *Run* the experiment and then *Reset*.
   (a) Is the center of mass motion affected by the collision?
7. Decrease the *Mass of A* to 0.15 kg. *Run* the experiment and then *Reset*.
   (a) How does the mass change affect the position of the center of mass?
   (b) Is the center of mass motion affected by the collision?

11.4.7 Experiment G: Reference Frames

In this experiment we will observe collisions while moving relative to the pool table. We will see how this affects our view of collisions.

1. Start up the PEARLS *Collision* experiment, or if it is already running, restore the default conditions by clicking the Defaults button ↩. Also, close any graphs that may be present.
2. Using the *Display Menu*, turn on the *Labels* option.
3. With the *Display Menu*, turn on the *Pool Table* display.
4. Turn on the *Reference Frame* display.
5. Turn on the *Center of Mass* display.
6. Set the *Impact Parameter* to zero.
7. Set the *Initial Velocity of A* control to 0.4 m/s. Run the experiment and then Reset.
   (a) Does the center of mass move?
   (b) Does the pool table move?
   (c) Is one of the balls stationary initially?
   (d) What is the name of this reference frame?
8. Set the *Observer X Velocity* to 0.5 m/s. Run the experiment and then Reset.

   (a) Does the center of mass move?
   (b) Does the pool table move?
   (c) Is either ball stationary initially?

9. Set the *Observer X Velocity* to 0.4 m/s. Run the experiment and then Reset.
   (a) Does the center of mass move?
   (b) Does the pool table move?
   (c) Is either ball stationary initially? Why?

10. Set the *Observer X Velocity* to 0.2 m/s. Run the experiment and then Reset.
   (a) Does the center of mass move?
   (b) Does the pool table move?
   (c) Is either ball stationary initially?
   (d) What is the name of this reference frame? Why?

11.5 Review

Please circle the correct answers to the following multiple-choice questions. You may, if you like, use PEARLS to help you find the correct answer.

1. Consider a head on collision (zero impact parameter) of two pool balls, A and B of equal mass. Ball B is initially stationary. What is the velocity of ball A after the collision?
   (a) 0.0 meters/sec
   (b) 5.0 meters/sec
   (c) 10.0 meters/sec
   (d) 100.0 meters/sec

2. Consider again the previous question. As the impact parameter is varied, is it possible for the recoil angle of ball A to be greater than 90 degrees? (The recoil angle is the angle between its initial velocity and its final velocity.)
   (a) yes
   (b) no

3. In the collision of two particles with no external forces, does the combined center of mass move at constant velocity?

(a) yes
(b) no

4. If the collision of two particles is viewed from the center of mass frame, are the momentum vectors of the two particles always equal and opposite?

   (a) yes
   (b) no

5. Consider a head on collision (zero impact parameter) of two pool balls, A and B. Ball B is initially stationary, and is heavier than A. In what direction does ball A move after the collision?

   (a) in its original direction
   (b) opposite its initial direction
   (c) 90 degrees from its original direction

6. Pool balls A and B are made to collide. B is more massive than A. In the center of mass frame, which ball moves more rapidly?

   (a) A
   (b) B

7. Is conservation of kinetic energy satisfied in all reference frames?

   (a) yes
   (b) no

8. Is conservation of momentum satisfied in all reference frames?

   (a) yes
   (b) no

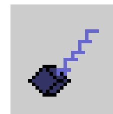
9. Does the total kinetic energy of two particles change if the observer's velocity changes?

   (a) yes
   (b) no

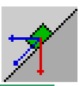
10. Does the total momentum of two particles change if the observer's velocity changes?

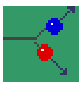
    (a) yes
    (b) no

12 Simple Harmonic Oscillator



12.1 Prerequisite





A great help to understand some of the concepts in this assignment are the simulations *Block on Incline*, *Collision*, and *Oscillating Functions*.

12.2 Introduction

If a particle is moving back and forth, following the same path, it is oscillating. If this oscillation repeats itself after the same time interval the oscillation is called harmonic or periodic. This oscillation normally undergoes a certain damping, e.g. friction, and will stop after a while.

The harmonic oscillator is an important system in physics. The oscillations of the pendulum or the mass on the spring bouncing up and down can be described by the same mathematical equations as the oscillations of the charges in an inductor-capacitor circuit, of a tuning fork, of the atoms in a crystal, the pistons in the car engine, etc.

In this assignment we will study the harmonic oscillation of a mass on a spring. We will study the kinematics and the laws of conservation for the undamped and the more realistic damped oscillator.

12.3 Theory

12.3.1 Harmonic Oscillator without Damping

The Motion Equations Let's attach a block to a spring and put it on a frictionless table. We stretch the spring by pulling on the block. When we release the block, it is not just going to return to its original position. The spring will pull it back, the block will pass its equilibrium and compress the spring. Then the spring will push the block out. The block will pass its equilibrium, stretch the spring, and so on. The block is oscillating. By equilibrium we mean the position in which the spring is neither stretched nor compressed.

Experiments have shown that the spring will pull the block back with a force that is proportional to the distance over which we stretched the spring.This result is valid as long as we do not stretch the spring to far.

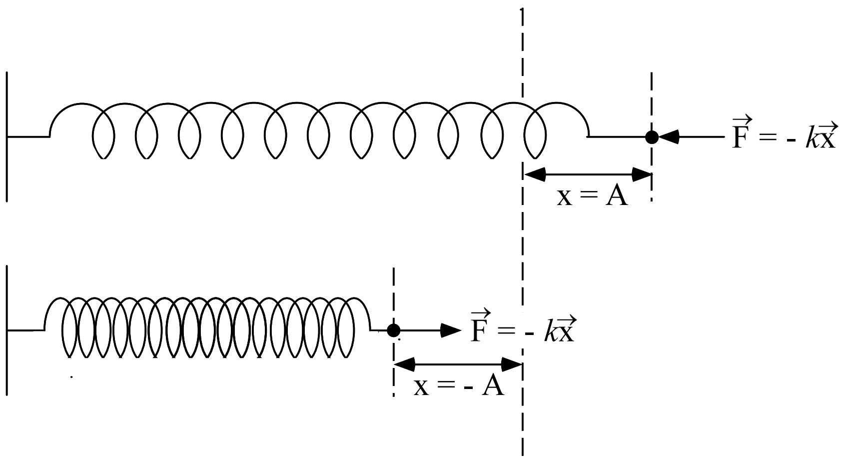


Figure 10: A Block on a Spring

Here we suppose that the spring is ideal: no friction, no overstretching, etc. The magnitude of the force by the spring on the block is thus:

$$F = -kx \tag{72}$$

with k the spring constant specific of the spring and x the distance over which the block is stretched. The units of the spring constant k are Newton/meter. The minus sign tells us that the force is a restoring force, it means that it will try to bring the block back into its unstretched, uncompressed position, i.e. its equilibrium. When the block is going in the positive x direction the force is going in the negative direction and so on (see figure (10)). This law is called *Hooke's law* and is as said, an empirical law.

We suppose that the block is sliding over a surface without any friction. The only force acting on the block is the force by the spring. We can write Newton's law $F = ma$ as:

$$F = ma = -kx \tag{73}$$

The motion is totally linear (let's say along the x-axis) so the acceleration can also be written as:

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} \tag{74}$$

We put this into the equation (73) and become:

$$m\frac{d^2x}{dt^2} = -kx \tag{75}$$

This equation is called a differential equation and is typical for the simple harmonic motion. This is the famous equation that turns up in so many physical problems.

The solution of this equation describes the motion. The block is oscillating, so the expression for $x(t)$ will have to show that type of motion.

You can verify that the following function is a solution for the famous differential equation:

$$x = A\cos(\omega t + \phi) \tag{76}$$

with A the amplitude of the oscillation, ω the angular frequency and ϕ the phase constant.

- The amplitude A is the maximum distance the block moves away from its equilibrium point during the oscillation. It is determined by the initial conditions. What this means will become clear during the laboratory.

- The angular frequency ω is given by:

$$\omega = \sqrt{\frac{k}{m}} \tag{77}$$

  This frequency is often called the natural frequency. In this assignment we are studying natural oscillations, they are not forced as opposite to the ones we will study in the assignment *Resonance*.

- The period T of the oscillation is the duration of a oscillation or the time after which the motion repeats itself. The function (76) repeats itself after $t = 2\pi/\omega$ or

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \tag{78}$$

- The frequency f of the oscillation is per definitions the number of oscillations in a second, thus:

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{79}$$

- The constant ϕ is the phase constant. (Also in assignment *Wavemix* you can become familiar with phase, phase constant, phase difference). It is determined by the initial conditions of the oscillation just as the amplitude is.

Doing some algebra you find the expressions for the speed and the acceleration of the block on the spring.

$$\begin{aligned}
x &= A\cos(\omega t + \phi) && (80)\\
v &= \frac{dx}{dt} = -\omega A\sin(\omega t + \phi) && (81)\\
a &= \frac{dv}{dt} = -\omega^2 A\cos(\omega t + \phi) && (82)
\end{aligned}$$

The Energy Equations For all energy considerations of a system we need a reference level where we put the potential energy zero. (In the assignment *Block on Incline* we studied this already.) For the spring-block system the equilibrium position of the block (the spring is not stretched, not compressed) is chosen as reference level. In this section we consider an oscillation without friction, in other words only a conservative force is acting on the block. The potential energy in function of the position of the oscillating block can be shown to be:

$$E_{pot} = -\int_0^x (-kx)dx = \frac{1}{2}kx^2 \tag{83}$$

The kinetic energy is given by:

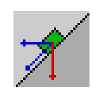
$$E_k = \frac{1}{2}mv^2 \tag{84}$$

Only a conservative force is acting so the total energy should be constant at all times and positions. Using the expressions (81) and (82) we find for E:

$$E = E_k + E_{pot} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \tag{85}$$

12.3.2 The Harmonic Motion with Damping

In the previous section we described an oscillation that would go on for ever. In reality friction (in the spring, with the surface...) would damp the oscillation and finally stop it. In the assignment *Block on Incline* we studied the friction between the block and the surface on which it slides. Experiments have shown that for regular speeds this friction was as good as independ of the speed of the block. In most cases (springs, viscous fluids...) friction is proportional to the velocity, but of course opposite to it (because it will slow down the movement). The friction has the form:



$$F_f = -bv = -b\frac{dx}{dt} \tag{86}$$

with b a positive constant that gives the magnitude of the damping. The units of this damping constant are kg/s. The equation (75) becomes then:

$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt} \tag{87}$$

or the differential equation for the damped oscillation becomes:

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0 \tag{88}$$

If the damping is small a solution for this equation is:

$$x = Ae^{-bt/2m}\cos(\omega' t + \phi) \tag{89}$$

with

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} \tag{90}$$

This means that for small damping we become again a harmonic oscillation. The amplitude is not constant anymore but decreases exponentially in time. The frequency of that oscillation is ω' which is for small damping: $\omega' \simeq \omega$.

You can damp the motion so much that actually no oscillation occurs anymore. That means that the ω' is zero. Putting this in equation (90 gives:

$$b_c = 2\sqrt{km} = 2m\omega \tag{91}$$

with m the mass of the oscillating object and ω the natural frequency. All damping that is greater than the critical damping is called overdamping. The block will have a hard time to return to its equilibrium position.

Energy Friction is a non-conservative force. Some of the energy is "lost" to heat. The total energy of the block will not be constant. As the total energy is proportional to the square of the amplitude it can be shown that the total energy decreases exponentially due to the negative work of the damping force on the system:

$$E = E_0 e^{-(b/m)t} \tag{92}$$

with E_0 the total energy at $t = 0$.

12.4 Laboratory

In this laboratory we will perform experiments with a harmonic oscillator without damping and with damping. One purpose is to show the kinematic relations between displacement, velocity and acceleration. Another purpose is to examine the law of energy conservation and energy loss. We will also have a closer look at the difference between light, heavy and critical damping.

12.4.1 Experiment A: Introduction and Setup

In this experiment, we will familiarize ourselves with the laboratory.

1. In the PEARLS window, select *Mechanics*.

2. Now launch the *Harmonic Oscillator* item.

3. You will now see the experiment window, with a control window at the left, and a laboratory display at the right.

4. The default settings for this experiment are:

Mass	1.000 kg
Spring Constant	1.500×10^{1} N/m
Damping	4.000×10^{-1} Ns/m
Period	1.622 s
Natural Angular Frequency	3.873 rad/s
Natural Frequency	6.164×10^{-1} Hz
Initial Displacement	5.000×10^{-2} m
Initial Velocity	0.000 m/s
Space Zoom	1×10^{3} pixels/m

5. Adjust the window size as desired.
6. Move the laboratory display to a convenient location on your screen by dragging the display with your mouse.
7. You should see a piston in a cylinder (the spring, the block, and the damper are visible).
8. Run the simulation. Stop and reset.
9. Have a look at the *Control Window*.
   (a) What is the value of the *Initial Displacement*?
   (b) What is the value of the *Initial Velocity*?
10. Set *Damping* equal to zero.
11. *Run* and observe. *Stop* and *Reset*.
    (a) When does the displacement have its greatest value?
    (b) What is the value of the displacement of the block for an unstretched, uncompressed spring position or the equilibrium position?
12. Click the *Display* button. Choose *Acceleration Vector* and *Total Force Vector*. The vectors are displayed on the Y-axis. You might have to move the display to make them visible.
13. *Run*, observe, *Stop*, and *Reset*.
    (a) When is the force zero during the motion?
    (b) When is the acceleration maximum?
14. Restore the default values by clicking the$\hookleftarrow$.
15. From *Display* choose *Grid* and *Distances*.
16. *Run* the simulation and observe.
    (a) Does the maximum displacement (fully stretched or fully compressed) change during the motion?

12.4.2 Experiment B: The Kinematic Relations

In this experiment we will study the undamped simple harmonic oscillator. We will see how the displacement, velocity and acceleration change in time and relate to each other.

1. Start up the PEARLS *Harmonic Oscillator* experiment, or if it is already running, restore the startup conditions by clicking the $\hookleftarrow$ button.
2. From *Display* choose *Acceleration Vector*, *Velocity Vector*.
3. *Run* for at least a complete oscillation.
4. Compare the vectors in the display.
   (a) At which position was the acceleration maximum?
   (b) Did the acceleration become zero? If so, when?
   (c) At which position was the velocity maximum?
   (d) Did the velocity become zero? If so, when?
   (e) When are the acceleration vector and velocity vector pointing in the same direction?
   (f) When are the acceleration vector and velocity vector pointing in the opposite direction?
   (g) At which position does the acceleration vector change direction?
   (h) At which position does the velocity change direction?
5. *Stop* and *Reset* the experiment.
6. *Run* the simulation. Observe the clock: *Time* =.
   (a) How long does a whole oscillation take? Compare to the value of the *Period* in the control window.
7. Click the graph button. Make a graph with *Displacement* in the vertical axis and *Time* in the horizontal axis.
8. Click the graph button again. Make a graph with *Velocity* in the vertical axis and *Time* in the horizontal axis.
9. Click the graph button again. Make a graph with *Acceleration* in the vertical axis and *Time* in the horizontal axis.
10. Drag the graphs away from each other, so you can see all three.
11. *Run* the experiment for at least two whole oscillations, *Stop* and *Reset*.
    (a) How does the displacement vary in time?

(b) How does the velocity vary in time?

(c) How does the acceleration vary in time?

(d) Is the period of all these functions the same?

12. Find the times when the displacement is zero (or the spring is neither stretched nor compressed).

    (a) Is the absolute value of the velocity maximum or zero?

    (b) Is the absolute value of the acceleration maximum or zero?

13. Find the times when the displacement is maximum. By maximum we mean maximum compression or maximum stretching.

    (a) Is the absolute value of the velocity maximum or zero?

    (b) Is the absolute value of the acceleration maximum or zero?

12.4.3 Experiment C: Period, Frequency and Amplitude of the Oscillation

In this experiment we will become familiar with the concept amplitude, period and frequency of the undamped simple harmonic oscillator. We will also study how the initial conditions (the position and the velocity of the block at $t = 0$) influence the motion.

1. Start up the PEARLS *Harmonic Oscillator* experiment, or if it is already running, restore the startup conditions by clicking the $\hookleftarrow$ button.

2. Set *Damping* equal to zero.

3. From *Display* choose *Acceleration Vector* and *Velocity Vector*.

4. Click the graph button. Make a graph with *Displacement* in the vertical axis and *Time* in the horizontal axis.

5. Click the graph button again. Make a graph with *Velocity* in the vertical axis and *Time* in the horizontal axis.

6. Click the graph button again. Make a graph with *Acceleration* in the vertical axis and *Time* in the horizontal axis.

7. Drag the graphs away from each other, so you can see all three.

8. Run the experiment for at least two whole oscillations, stop, and reset.

    (a) What is the value of the *Spring Constant*?

    (b) Recall the formula for the natural frequency of the oscillator $\omega = \sqrt{k/m}$. Calculate the natural frequency and compare to the value in the *Natural Frequency* control.

(c) Calculate the period of the oscillation and compare to the graph and the value in the control window.

(d) Do all three curves have the same period?

(e) What is the value of the phase constant (see equation (81) and (82))?

(f) What is the value of the amplitude of the oscillation? Is it different from the initial displacement?

(g) What is the value of the amplitude of the velocity? Recall equation (82).

(h) What is the amplitude of the acceleration? Recall equation (82).

9. Set *Mass* equal to 0.50 kg.

10. Run the experiment for at least two whole oscillations, stop and reset.

    (a) What is the value of the *Spring Constant*?

    (b) Calculate the natural frequency and compare to the value in the control window.

    (c) Calculate the period of the oscillation and compare to the graph an the period of step 8c.

    (d) Do all new curves have that same period? Compare to the plots from step 8d.

    (e) What is the value of the amplitude of the oscillation? Compare to the graph from step 8f.

    (f) What is the amplitude of the velocity versus time graph? Compare to the graph from step 8g.

    (g) What is the value of the amplitude of the acceleration versus time graph? Compare to the graph from step 8h.

11. Set *Mass* equal to 4 kg.

12. Run the experiment for at least a whole oscillation, stop and reset.

    (a) Compare the *Natural Frequency* with its values in the previous steps.

    (b) Compare the period of the oscillation with its value in the previous steps.

    (c) Did the maximum displacement change?

    (d) Compare the maximum velocity to its value in step 8g.

    (e) Compare the maximum acceleration to its value in from step 8h.

13. Click the $\hookleftarrow$ button.

14. Set *Damping* equal to zero.

15. Run the experiment until the block has completed at least a whole oscillation.
    (a) What is the value of the amplitude of the oscillation?
    (b) What is the value of the period of the oscillation?
16. Set *Initial Displacement* equal to -0.05 m.
17. Run the experiment until the block has completed a whole oscillation.
    (a) Did the amplitude of the oscillation change compared to step 15?
    (b) Did the period of the oscillation change compared to step 15?
18. Set *Initial Displacement* to 0.10 m.
    (a) Did the period change compared to the previous step?
    (b) Did the velocity change compared to the previous step?
    (c) Did the acceleration change compared to the previous step?

    .
19. Set *Initial Velocity* equal to 0.1 m/s.
20. Run the experiment until the block has completed a whole oscillation.
21. Observe the motion of the block.
    (a) Was the initial displacement of the block the maximum displacement?
22. Run the experiment again until the block has completed a whole oscillation.
23. Observe the plots.
    (a) Compare the amplitude of the oscillation to the one in step 15. Is the value still the same as the initial displacement?
    (b) Compare the period with step 15.
    (c) What is the difference with the plot in step 15?
24. Recall that the motion of the block is described by the equations (81) and (82). Initial conditions mean $t = 0$.
    (a) Filling in the values for the initial conditions in both equations, what is the value of the phase constant?
    (b) Using this calculated phase constant, what is the amplitude of this oscillation? Compare to the maximum value given by the plot.

12.4.4 Experiment D: Force, Spring Constant and Period of the Oscillation

In this experiment we will examine the elastic force, its relation with the spring constant and the period of the oscillation.

1. Start up the PEARLS *Harmonic Oscillator* experiment, or if it is already running, restore the startup conditions by clicking the $\hookleftarrow$ button.

2. From *Display* choose *Damping Force Vector, Spring Force Vector*, and *Total Force Vector*.

3. Click the graph button. Make a graph with *Force*(that is the total force) in the vertical axis and *Displacement* in the horizontal axis.

4. Click the graph button again. Make a graph with *Displacement* in the vertical axis and *Time* in the horizontal axis.

5. Drag the graphs away from each other so you can see them both.

6. Set *Damping* equal to zero.

7. Run the experiment until the block has completed at least a whole oscillation and reset.

   (a) Is the total force different from the spring force?

   (b) What is the value of the period of the oscillation

8. Have a look at the plots.

   (a) How does the force relate to the displacement?

   (b) What does the downwards slope mean?

   (c) What is the maximum force?

   (d) For which displacement is the force zero?

   (e) What is the value of the period of the oscillation?

9. Set the *Spring Constant* equal to 7.5 N/m, half the original value.

10. Run the experiment for a whole oscillation, stop and reset.

    (a) What is the value of the period? Compare to step 8e.

11. Have a look at the plots.

    (a) Compare the slope with step 8.

    (b) What is the maximum force?

    (c) For which displacement is the force zero?

12. Set the *Spring Constant* equal to zero.
13. Run the experiment, observe and reset.
    (a) Is there any elastic force?
    (b) Does the block's position change?
14. Set *Initial Velocity* equal to 10 m/s.
15. Run the experiment, observe and reset.
    (a) Is there any elastic force?
    (b) Does the block's position change?
    (c) Is this still an oscillation? How would you call this motion? What can you predict about the velocity of the block? Check your answer with the plot *Displacement vs. Time*.

12.4.5 Experiment E: Conservation of Energy

In this experiment we will examine the kinetic, potential and total energy. We will study how they vary in function of time and displacement.

1. Start up the PEARLS *Harmonic Oscillator* experiment, or if it is already running, restore the startup conditions by clicking the ↩ button.
2. Click the graph button. Make a graph with *Potential Energy* in the vertical axis and *Time* in the horizontal axis.
3. Click the graph button again. Make a graph with *Kinetic Energy* in the vertical axis and *Time* in the horizontal axis.
4. Click the graph button again. Make a graph with *Total Energy* in the vertical axis and *Time* in the horizontal axis.
5. Drag the graphs away from each other so you can see them both.
6. Set *Damping* equal to zero.
7. Run the experiment until the block has completed at least a whole oscillation and reset.
    (a) How does the total energy vary in time? What does this mean?
    (b) Recall the expression for the total energy: equation (85). Calculate its value and compare to the graph.
    (c) What is the maximum potential energy? When does its first maximum occur?

(d) What is the maximum kinetic energy? When does its first maximum occur?

(e) At $t = 0$ the kinetic energy is zero. When is the next time it is zero again? Compare to the period of oscillation given by: $T = 2\pi\sqrt{m/k}$.

8. Close all graphs.

9. Click the graph button. Make a graph with *Potential Energy* in the vertical axis and *Displacement* in the horizontal axis.

10. Click the graph button again. Make a graph with *Kinetic Energy* in the vertical axis and *Displacement* in the horizontal axis.

11. Click the graph button again. Make a graph with *Total Energy* in the vertical axis and *Displacement* in the horizontal axis.

12. Run the experiment until the block has completed at least a whole oscillation and reset.

    (a) What is the value of the total energy at all positions?

    (b) At which positions is the kinetic energy zero?

    (c) At which positions is the kinetic energy maximum?

    (d) At which positions is the potential energy zero? Is this the equilibrium position?

    (e) At which positions is the potential energy maximum?

13. Set the *Spring Constant* equal to 7.5 N/m.

14. Run the experiment until the block has completed at least a whole oscillation and reset.

15. Compare to step 12.

    (a) Did the total energy change?

    (b) Did the potential energy change?

    (c) Did the kinetic energy change?

    (d) Did the positions change at which the potential energy is maximum?

16. Set the *Spring Constant* back to 15 N/m.

17. Set the *Initial Displacement* equal to 2.5×10^{-2} m.

18. Run the experiment until the block has completed at least a whole oscillation and reset.

19. Compare to step 12.

(a) Did the total energy change? What is its value?
(b) Did the potential energy change?
(c) Did the kinetic energy change?
(d) Did the positions change at which the potential energy is maximum?

20. Set the *Initial Displacement* back to 0.05 m.
21. Set the *Mass* of the block equal to 4 kg.
22. Run the experiment until the block has completed at least a whole oscillation and reset.
23. Compare to step 12.
    (a) Did the total energy change?
    (b) Did the potential energy change?
    (c) Did the kinetic energy change?

12.4.6 Experiment F: Forces in the Damped Oscillator

In this experiment we will examine the acting forces on the damped oscillator.

1. Start up the PEARLS *Harmonic Oscillator* experiment, or if it is already running, restore the startup conditions by clicking the $\hookleftarrow$ button.
2. From *Display* choose *Spring Force Vector*.
3. Run the experiment and observe.
    (a) Compare the motion of the displacement vector and the motion of the spring force vector.
    (b) When is the spring force maximum?
4. Stop and reset the experiment.
5. From *Display* choose *Damping Force Vector* and *Velocity Vector*.
6. Run the experiment and observe.
    (a) Compare the motion of the velocity vector and the motion of the damping force vector.
    (b) When is the damping force maximum? Is the damping force maximum at the same time the spring force is maximum?
    (c) When is the damping force zero? Is the damping force zero at the same time the spring force is zero?

7. Stop and reset the experiment.
8. From *Display* choose *Acceleration Vector* and *Total Force*.
9. Run the experiment.
   (a) Compare the motion of the acceleration vector and the total force vector.

12.4.7 Experiment G: Oscillation with Damping

In this experiment we will examine the motion of the oscillator when the motion is damped. We will study the difference between light, critical, and overdamping.

1. Start up the PEARLS *Harmonic Oscillator* experiment, or if it is already running, restore the startup conditions by clicking the ↩ button.
2. Click the graph button. Make a graph with *Displacement* in the vertical axis and *Time* in the horizontal axis.
3. Set *Damping* equal to zero.
4. Run the experiment for 10 seconds, stop, and reset.
5. Set *Damping* equal to 0.4 Ns/m.
6. Run the experiment for 10 seconds, stop and reset.
   (a) Compare the amplitude of the motion with step 4.
   (b) Compare the period of the motion with step 4.
7. Set *Damping* equal to 0.8 Ns/m.
8. Run the experiment for 10 seconds, stop and reset.
   (a) Is the block still oscillating?
9. Set *Damping* equal to 1.6 Ns/m.
10. Run the experiment for 10 seconds, stop and reset.
    (a) Is the block still oscillating?
    (b) How is the displacement varying in time?
11. From *Display* choose *Damping* = *critical*.
12. Run the experiment for 10 seconds, stop and reset.
    (a) What is the value of the damping?

(b) Is the block oscillating?

(c) Is the block returning fast to its equilibrium position? How long does it take the block to return to its equilibrium?

(d) Look at the plot. How does the displacement vary in time?

13. Set *Damping* equal to 15 Ns/m.

    (a) How long does it take the block to reach its equilibrium position? Compare to step 12c.

14. Set *Mass* of the block equal to 4 kg.

    (a) Calculate the critical damping. Recall: $b_c = 2m\omega$.

15. Set *Damping* equal to the calculated value.

16. Run the experiment for 10 seconds, stop, and reset.

    (a) Does the displacement exponentially decrease in time?

12.4.8 Experiment H: Energy Considerations for the Damped Harmonic Oscillator

In this experiment we will examine the total, kinetic and potential energy of the oscillator when the motion is lightly, critically, and heavily damped.

1. Start up the PEARLS *Harmonic Oscillator* experiment, or if it is already running, restore the startup conditions by clicking the $\hookleftarrow$ button.

2. Click the graph button. Make a graph with *Total Energy* in the vertical axis and *Time* in the horizontal axis.

3. Click the graph button again. Make a graph with *Potential Energy* in the vertical axis and *Time* in the horizontal axis.

4. Click the graph button. Make a graph with *Kinetic Energy* in the vertical axis and *Time* in the horizontal axis.

5. Drag the graphs so you can see all three at the same time.

6. Drag the display so you can observe the motion of the block too.

7. Set *Damping* equal to zero.

8. Run the experiment for a whole oscillation period, stop, and reset.

    (a) Is the total energy constant in time?

9. Set damping equal to 4×10^{-1} Ns/m.

10. Run the experiment for a whole oscillation period, stop, and reset.
    (a) Is the total energy constant in time?
    (b) At some times the total energy loss is minor. At what points in the period? What is the speed of the block at those times?
11. Set damping equal to 0.8 kg/s.
12. Run the experiment for a whole oscillation period, stop, and reset.
13. Compare the three plots in each graph window.
    (a) Is the loss or dissipation of energy greater than in step 10a?
14. Set damping equal to critical damping. (Choose from *Display*: *damping = critical*.
15. Run the experiment for a whole oscillation period, stop, and reset.
16. Compare the new plot with the previous ones in each graph window.
    (a) How does the total energy behave in time?
    (b) How does the kinetic energy of the block compare to the one for less damping?
    (c) Did the block oscillate?
17. Set damping equal to heavy damping 15 Ns/m..
18. Run the experiment for a whole oscillation period, stop, and reset.
19. Compare the new plot with the previous ones in each graph window.
    (a) How does the total energy behave in time? Compare to step 16a.

12.5 Review

Please circle the correct answers to the following multiple-choice questions. You may, if you like, use PEARLS to help you find the correct answer.

1. If the amplitude of a simple harmonic oscillator is doubled, its total energy is changed by a factor:
    (a) 1.4
    (b) 2
    (c) 4

2. Consider two identical blocks attached to two identical ideal springs on the same surface. Block one is pulled out over 10 cm and block two is pulled out over 5 cm. Both blocks are released at the same time. Block one will pass by the equilibrium position:

   (a) before block two.

   (b) at the same time as block two.

   (c) after block two.

3. When in a spring block system the block is replaced by a block with a mass 4 times as big the period of the oscillation is:

   (a) the same

   (b) half the original period

   (c) twice the original period

   (d) four times the original period

4. When in a spring block system the spring is replaced by a 4 times stiffer spring the maximum velocity of the block will be

   (a) one fourth the original velocity

   (b) half the original velocity

   (c) twice the original velocity

   (d) four times the original velocity

5. The motion of the piston in a car engine is approximately simple harmonic. If the stroke of an engine (a stroke = 2× amplitude) is 10 cm and the engine runs at 2500 rev/min, the acceleration of the piston at the end point of a stroke is:

   (a) zero

   (b) 13.09 m/s^2

   (c) 86.8 m/s^2

   (d) 3.43×10^3 m/s^2

6. A 0.4 kg mass on the end of a spring with force constant $k = 300$ N/m is damped by a force $-bv$. The motion will be critically damped for

   (a) $b = 21.9$ kg/s

   (b) $b = 0.03$ kg/s

   (c) $b = 10.95$ kg/s

7. A 2 kg object oscillates with a initial amplitude of 3 cm on a spring with spring constant $k = 400$ N/m. The energy decreases by 1 percent per period. The damping constant is:

   (a) $b = 0.045$ kg/s

   (b) $b = 0.29$ kg/s

   (c) $b = 56.6$ kg/s

8. An oscillator has a period of 3 s. Its amplitude decreases by 5 percent during each cycle. The total energy decreases during each cycle by:

   (a) 5 percent

   (b) 10 percent

   (c) 25 percent

9. A damped oscillator initially oscillates with an amplitude of 25 cm. The block has a mass of 2 kg. The spring constant is 10 N/m. The damping force is $F = -bv$. The amplitude falls to 3/4 of its original value in four complete cycles. The value of the damping is:

   (a) 0.020 kg/s

   (b) 0.025 kg/s

   (c) 0.102 kg/s

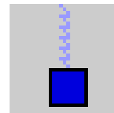
10. The block with mass 5 kg in the spring-block system is given an initial velocity of 10 m/s towards the equilibrium. The initial displacement is 50 cm. The spring constant is 1000 N/m. The total energy of the system is:

    (a) 125 Joule

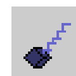
    (b) 262.5 Joule

    (c) 375 Joule

13 Resonance



13.1 Prerequisite



A great and indispensable help to this assignment is the assignment *Simple Harmonic Oscillator*.

13.2 Introduction

In the assignment *Simple Harmonic Oscillator* we have seen that in damped oscillations energy is lost and the oscillation finally will stop. These oscillations are called natural oscillations. If we want the oscillation to continue we will have to put energy into the system. Just as you have to pump if you want to keep on swinging on a swing. In other words we will have to force the oscillations. In this assignment we will study how the driving force does affect the forced oscillation. We wil examine the conditions under which we even can become oscillations or vibrations with an amplitude that is much bigger than the magnitude of the driving force. This phenomenon is called *resonance*.

13.3 Theory

The natural frequency of an oscillator (see assignment *Simple Harmonic Oscillation*) is the frequency of the oscillator when it is oscillating all by itself without friction. In *Simple Harmonic Oscillator* the oscillator is a block attached to a spring, which other end is held in a fixed place. If we move this point of support in a harmonic motion, at first we get a complicated motion, but after a while a *steady state* is reached. The block will oscillate with the same frequency as the support and the amplitude will be constant.

The driving force is putting energy into the oscillating system, which loses energy because of damping. When as much energy is put in as is dissipated the system reaches the steady state. The energy of the oscillating system is directly related to the amplitude of the oscillation (see *Simple Harmonic Oscillator*). This amplitude is not only going to depend on the amplitude, i.e. the magnitude, of the driving force but is also going to depend on its frequency. If the frequency of the driving is approximately equal to the natural frequency the system will oscillate with an amplitude much larger than the amplitude of the driving force. This phenomenon is called *resonance*.

We saw in the assignment *Simple Harmonic Oscillator* that the forces working on the oscillator were

- the elastic force $-kx$ with k the spring or force constant.
- the damping force $-bv$ with b the damping factor and v the speed of the oscillating block.

Now we have added the driving force which is an oscillating force:

$$F_{ext} = F_0 \cos \omega t \tag{93}$$

Newton's law becomes then:

$$-kx - bv + F_0 \cos \omega t = ma \tag{94}$$

This leads to the differential equation:

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0 \cos \omega t \tag{95}$$

Let's not worry about how to solve this. The solution has two parts: the part that looks exactly like the damped oscillator and is dissapearing after some time (remember the exponentially decreasing amplitude) and another part that is the steady state solution, which does not depend on the initial conditions. This steady state solution looks like:

$$x = A\cos(\omega t + \phi) \tag{96}$$

with ω the frequency of the driving force, A the amplitude of the oscillation in steady state and ϕ a phase constant. The amplitude and the phase constant are not arbitrary but depend on the driving force amplitude F_0 and frequency ω. The steady state amplitude is given by:

$$A = \frac{F_0}{\sqrt{m^2({\omega_0}^2 - \omega^2)^2 + b^2\omega^2}} \tag{97}$$

Resonance means amplification for small damping. For frequencies close to the resonance frequency the amplitude becomes:

$$A = \frac{F_0}{b\omega_0} = \frac{F_0}{b\sqrt{k/m}} \tag{98}$$

A force F_0 on a spring would make it elongate over a distance F_0/k (see assignment *Simple Harmonic Oscillator*. Equation (98) gives that at the resonance frequency the amplitude will be $\sqrt{km}/b$ times larger. The phase constant is given by:

$$\tan \phi = -\frac{b\omega}{m({\omega_0}^2 - \omega^2)}. \tag{99}$$

The meaning of this complicated expressions will become a lot clearer in the laboratory. The phase constant has to be negative: the motion of the oscillating block lags behind the driving force. For $\omega = \omega_0$, $\tan\phi = 0$ or $\phi = -\pi/2$. Just one more word about the velocity of the block.

$$v = \frac{dx}{dt} = -A\omega\sin(\omega t + \phi) \tag{100}$$

In case of resonance $\phi = \pi/2$ or

$$v = A\omega\cos\omega t \tag{101}$$

If we compare this equation for the velocity with the driving force expression (93) we see that at resonance the block is moving in the direction of the driving force.

The reason why we have to drive the oscillation is the energyloss due to the damping. The quality factor Q of a system is a measure for the energyloss in the system. The quality factor is defined as:

$$Q = \frac{\omega_0 m}{b} \tag{102}$$

The quality factor Q is small for highly damped systems. In the laboratory we will see that systems with low damping (and high Q factors) show a sharply peaked resonance curve. The ratio of the resonance frequency to the width of the resonance curve is the quality factor:

$$Q = \frac{\omega_0}{\Delta\omega} \tag{103}$$

with $\Delta\omega$ the width of the curve.

13.4 Laboratory

In this laboratory we will perform experiments with a piston in a cylinder. The spring, connected at one end to the piston, is connected to an oscillating rod at the other end. We will see how the motion of the piston depends on the amplitude and the frequency of the oscillation of the rod. We will also study the influence of the stiffness of the spring and of the damping.

13.4.1 Experiment A: Introduction and Setup

In this experiment, we will familiarize ourselves with the laboratory.

1. In the PEARLS window, select *Mechanics*.

2. Now launch the *Resonance* item.

3. You will now see the experiment window, with a control window at the left and a laboratory display at the right.

4. Adjust the window size as desired.

5. Move the laboratory display to a convenient location on your screen by dragging the display with your mouse.

6. The default settings for this experiment are:

Forcing Frequency	4.000×10^{-1} Hz
Force Amplitude	2.500×10^{-1} N
Damping	1.000 kg/s
Block Mass	1.000 kg
Spring Constant	9.870 N/m
Zoom	7.500×10^{2} pixels/meter
Time Zoom	1.000

7. You should see a piston in a cylinder attached to a spring, which is attached to a rod. The length of the steady state amplitude A_s is marked on the display. Two graphs are displayed.

8. Drag the graphs so you can see them both and the display.

9. One graph is the *Steady State Amplitude versus (Forcing) Frequency* or $A_s(\omega)$. The other graph gives the *Steady State Phase versus (Forcing) Frequency* or $\phi(\omega)$.

10. Observe the position of the piston before the driving force is started.

11. *Run* the experiment.
    (a) Did you observe the complicated motion in the beginning?
    (b) Was the amplitude in the first oscillation the shown steady state amplitude?

12. *Stop* and *Reset*.

13. Look at the plots.
    (a) Read the steady state amplitude from the plot.
    (b) Is a greater amplitude possible for this system? At which frequency?

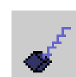
14. Change the *Force Amplitude* to greater values using the slider.
    (a) Does the steady state amplitude change?
    (b) Does the maximum amplitude value in the plot $A(\omega)$ change?

15. Change *Damping* to a lower value.
    (a) Did the steady state amplitude change?
    (b) Which change did you see in the plot $A(\omega)$?
    (c) Which change did you see in the plot $\phi(\omega)$?

13.4.2 Experiment B: The Steady State Amplitude and the Frequency of the Forcing Force

In this experiment we will vary the frequency of the driving force and see how the oscillation amplitude of the piston changes. We will examine for which frequency the motion becomes amplified.

1. Start up the PEARLS *Resonance* experiment, or if it is already running, restore the startup conditions by clicking the $\hookleftarrow$ button.
2. Close the plot *Steady State Phase vs. Forcing Frequency* if displayed.
3. With the slider change the *Forcing Frequency* slowly from its minimum to its maximum value. Observe the plot and the display.
    (a) Does the steady state amplitude stay constant when you change the frequency of the driving force?
    (b) For which frequency is the amplitude maximum?

    (a) What is the value of the *Spring Constant* k?
    (b) What is the value of the *Block Mass* m?
    
    (c) Recall the expression (see assignment *Harmonic Oscillator*) to calculate the natural angular frequency ω_0 of the oscillating block: $\omega_0 = \sqrt{k/m}$. Calculate the angular natural frequency.
    (d) What is the natural frequency f_0? Recall $f_0 = \omega_0/2\pi$. Is your calculated value equal to the *Resonance Frequency* displayed?
4. Run and observe the experiment.
    (a) Is the steady state reached immediately?
    (b) What is the value of the *Forcing Frequency*? How does this value compare to the *Resonance Frequency*?
5. *Stop* and *Reset* the experiment.
6. Recall the equation (97).
    (a) What is the value of ω_0?
    (b) What is the value of ω?

(c) What is the value of the *Damping* b?

(d) What is the value of the *Block Mass*?

(e) Calculate the steady state amplitude. Compare your calculated value with the value displayed.

7. Look at the plots.

   (a) Look at the plot $A(\omega)$. How does this steady state amplitude compare to the maximum steady state amplitude possible for this system?

8. Set *Forcing Frequency* equal to 0.1 Hz.

9. Run,observe, stop and reset the experiment

   (a) Compare the *Forcing Frequency* with the *Resonance Frequency*.

   (b) What is the value of the *Steady State Amplitude*? Compare to the maximum possible value.

   (c) Compare the steady state amplitude to F/k, the elongation when a force F would pull on a spring with a spring constant k. Is the motion amplified?

10. Set *Forcing Frequency* equal to 2 Hz.

11. Run, observe, stop and reset the experiment

    (a) Compare the *Forcing Frequency* with the *Resonance Frequency*.

    (b) What is the value of the *Steady State Amplitude*? Compare to the maximum value possible for this system.

    (c) Compare the steady state amplitude to F/k. Is the motion amplified?

12. Set *Forcing Frequency* equal to the natural or resonance frequency.

13. Calculate the steady state amplitude. The equation (97) becomes simple for $\omega = \omega_0$. $A = F_0/b\omega$.

    (a) What is the value of the *Steady State Amplitude*? Compare to the maximum value possible for this system.

    (b) Compare the steady state amplitude to F/k. Is the motion amplified?

13.4.3 Experiment C: The Steady State Phase and the Frequency of the Driving Force

In this experiment we will vary the frequency of the driving force and see how the oscillation of the piston changes.

1. Start up the PEARLS *Resonance* experiment, or if it is already running, restore the startup conditions by clicking the $\hookleftarrow$ button.

2. Drag the plots so you can see them both and the display.
   (a) What is the value of the resonance frequency?
3. Set *Forcing Frequency* equal to 0.1 Hz.
4. *Run* the experiment.
   (a) Once the piston has reached the steady state, is it moving in the same direction the forcing rod is moving?
   (b) Compare the forcing frequency to the resonance frequency.
   (c) Look also at the plot *Steady State Amplitude vs. Forcing Frequency*. Does this forcing frequency fall in the frequency interval where the amplitude is amplified?
   (d) In the plot *Steady State Phase vs. Forcing Frequency* find the phase constant ϕ for this forcing frequency.
   (e) Look back at the motion. Do you see this phase difference between the forcing rod and the piston in the motion?
5. *Stop* the experiment and *Reset*.
6. Set *Forcing Frequency* equal to 0.7 Hz.
7. *Run* the experiment.
   (a) Once the piston has reached the steady state, is it moving the same way the forcing rod is moving?
   (b) Compare the forcing frequency to the resonance frequency.
   (c) Look also at the plot *Steady State Amplitude vs. Forcing Frequency*. Does this forcing frequency fall in the frequency interval where the amplitude is amplified?
   (d) In the plot *Steady State Phase vs. Forcing Frequency* find the phase constant ϕ for this forcing frequency. Compare to your answer of step 7a.
8. *Stop* the experiment and *Reset*.
9. Set *Forcing Frequency* equal to 0.5 Hz.
10. *Run* the experiment for 10 seconds, stop and reset.
    (a) Once the piston has reached the steady state, is it moving the same way the forcing rod is moving?
    (b) Compare the forcing frequency to the resonance frequency.

(c) Look also at the plot *Steady State Amplitude vs. Forcing Frequency*. Does this forcing frequency fall in the frequency interval where the amplitude becomes greater?

(d) In the plot *Steady State Phase vs. Forcing Frequency* find the phase constant ϕ for this forcing frequency.

13.4.4 Experiment D: The Steady State Amplitude, Phase, and Damping

In this experiment we will vary the damping of the system and see how the oscillation of the piston changes. We will study the quality factors of different systems.

1. Start up the PEARLS *Resonance* experiment, or if it is already running, restore the startup conditions by clicking the $\hookleftarrow$ button.

2. Drag the plots so you can see them both and the display.

3. Set forcing frequency equal to resonance frequency 0.5 Hz.

4. Set *Force Magnitude* equal to 0.5 N.

5. Set *Damping* equal to 1 kg/s.

   (a) Is the plot *Steady State Amplitude vs Forcing Frequency* symmetric?

   (b) What is the value of the steady state amplitude at resonance frequency?

   (c) Is the maximum steady state amplitude the amplitude at resonance frequency?

   (d) What is the value of the quality factor for this system? Recall equation (102).

6. Run the simulation. Observe, stop, and reset the simulation.

7. Set *Damping* equal to 0.1 kg/s.

   (a) Is the plot *Steady State Amplitude vs Forcing Frequency* symmetric?

   (b) Compare the width of the resonance peak to the one in step 3.

   (c) What is the value of the steady state amplitude at resonance frequency? Compare to the one in step 3.

   (d) Is the maximum steady state amplitude the amplitude at resonance frequency?

   (e) Compare the plot *Steady State Phase vs Forcing Frequency* with the one of step 3.

   (f) What is the value of the quality factor for this system? Recall equation (102). Compare to the quality factor of step 5d.

8. Run and observe. Compare to step 6. Stop and reset the simulation.

9. Set *Damping* equal to 10 kg/s.

   (a) Is the plot *Steady State Amplitude vs Forcing Frequency* symmetric?
   (b) Do you still observe a resonance peak as in step 3 and step 7?
   (c) What is the value of the steady state amplitude at resonance frequency? Compare to the one in step 3 and 7.
   (d) Is the maximum steady state amplitude the amplitude at resonance frequency?
   (e) Compare the plot *Steady State Phase vs Forcing Frequency* with the one of step 3 and 7.
   (f) What is the value of the quality factor for this system? Recall equation (102).

10. Run the simulation and observe. Compare the motion of the block to step 6 and step 9.

11. Decrease the *Damping* with the slider and observe how the shape of the resonance curve varies.

13.4.5 Experiment E: Resonance without Damping

In this experiment we will examine what happens if the system is not damped.

1. Start up the PEARLS *Resonance* experiment, or if it is already running, restore the startup conditions by clicking the ↩ button.

2. Click the graph button and make a graph with *Position* in the vertical axis and *Time* in the horizontal axis.

3. Click the graph button again and make a graph with *Total Energy* in the vertical axis and *Time* in the horizontal axis.

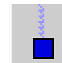
4. Run the experiment and look at the plot.

   (a) How does the total energy vary in time?
   (b) How does the *Position* vary in time?

5. Set *Damping* equal to 0 kg/s.

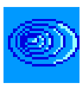
6. Set *Forcing Frequency* equal to the resonance frequency 0.5 Hz.

7. Run the experiment for 20 seconds, stop and reset.



(a) What can you tell about the motion in the display?
(b) Does the system reach a steady stage?
(c) Do you see the same pattern in the graph *Position vs. Time*?
(d) Is the total energy reaching a steady state as in step 4a?

8. Set *Forcing Frequency* equal to 0.45 Hz.
9. Clear the graph *Position vs. Time*.



10. Run the experiment for 40 seconds, stop and reset.
    (a) Recall the beat phenomenom in the assignment *Waves*. Do you see the same beat pattern in the motion of the block and in the graph *Position vs. Time*?
    (b) How may beats do you see in 40 seconds? What is the beat frequency?.
    (c) Compare the beat frequency to the difference between forcing frequency and natural frequency.
11. Set *Forcing Frequency* equal to 0.3 Hz.
12. Run the experiment.
    (a) What is the difference between the *Forcing Frequency* and the resonance frequency?
    (b) Do you see a beat pattern?
    (c) Is the motion of the piston simple harmonic?

13.4.6 Experiment F: The Steady State Amplitude at Low Frequencies

In this experiment we will see by which parameters the steady state amplitude is influenced for a driving force with low frequency in comparison to the resonance frequency.

1. Start up the PEARLS *Resonance* experiment, or if it is already running, restore the startup conditions by clicking the ↩ button.
2. Set *Force Amplitude* equal to 0.5 N.
3. Set *Forcing Frequency* equal to 0.01 Hz.
4. Set *Spring Constant* equal to 20 N/m.
    (a) What is the resonance frequency?
    (b) How does the forcing frequency compare to the resonance frequency?

   (c) What is the steady state amplitude?

5. Set *Spring Constant* equal to 40 N/m.
   (a) What is the resonance frequency?
   (b) How does the forcing frequency compare to the resonance frequency?
   (c) What is the steady state amplitude? Compare to step 4.

6. Set *Spring Constant* equal to 80 N/m.
   (a) What is the resonance frequency?
   (b) How does the forcing frequency compare to the resonance frequency?
   (c) What is the steady state amplitude?Compare to step 4 and 5.

7. Set *Spring Constant* equal to 40 N/m.

8. Set *Block Mass* equal to 2 kg.
   (a) What is the resonance frequency?
   (b) How does the forcing frequency compare to the resonance frequency?
   (c) What is the steady state amplitude?

9. Run the simulation. Observe, stop and reset.

10. Set *Block Mass* equal to 4 kg.
    (a) What is the resonance frequency?
    (b) How does the forcing frequency compare to the resonance frequency?
    (c) What is the steady state amplitude? Compare to step 8.

11. Run the simulation. Observe, stop and reset.

12. Set *Block Mass* equal to 8 kg.
    (a) What is the resonance frequency?
    (b) How does the forcing frequency compare to the resonance frequency?
    (c) What is the steady state amplitude? Compare to step 8 and 10.

13. Run the simulation. Observe, stop and reset.

14. Recall equation (97). For frequencies low in comparison to the resonance frequency the steady state amplitude becomes: $A = F_0/m{\omega_0}^2$ or $A = F_0/k$ with k the spring constant.

(a) The mass doesn't appear in this equation for low frequencies. Did you become the same result? Check your answer by sliding the *Block Mass* slider from its minimum to its maximum value, while you read the steady state amplitude in the plot.

(b) The spring constant is important in this equation. Slide the spring constant from its minimum to its maximum value and observe the influence on the steady state amplitude.

13.4.7 Experiment G: The Maximum Steady State Amplitude, the Spring and the Block Mass

In this experiment we will see how the maximum of the resonance curve changes when you change the stiffness of the spring or the mass of the piston.

1. Start up the PEARLS *Resonance* experiment, or if it is already running, restore the startup conditions by clicking the $\hookleftarrow$ button.

2. Set *Force Amplitude* equal to 0.5 N.

3. Set *Damping* equal to 0.5 kg/s.

4. Have a look at the plot *Steady State Amplitude vs. Forcing Frequency*.

5. Set *Spring Constant* equal to 20 N/m.

   (a) What is the value of the resonance frequency?

   (b) What is the value of the steady state amplitude at resonance frequency?

   (c) Compare this maximum with F_0/k ($F =$ the force magnitude, k the spring constant) or the amplitude for $f = 0$. What is the amplification? Is this value the same as: $\sqrt{km}/b$?

6. Set *Spring Constant* equal to 40 N/m.

   (a) What is the value of the resonance frequency? Did the resonance curve shift to lower or higher frequencies? Compare to step 5.

   (b) What is the value of the steady state amplitude at resonance frequency? Compare to step 5.

   (c) Compare this maximum with F_0/k or the amplitude for $f = 0$. What is the amplification? Is this value the same as: $\sqrt{km}/b$?

7. Set *Spring Constant* equal to 80 N/m.

   (a) What is the value of the resonance frequency? Did the resonance curve shift to lower or higher frequencies? Compare to step 5 and 6.

(b) What is the value of the steady state amplitude at resonance frequency? Compare to step 5 and 6.

(c) Compare this maximum with F_0/k. What is the amplification? Is this value the same as: $\sqrt{km}/b$?

8. Set *Spring Constant* equal to 40 N/m.

9. Set *Block Mass* equal to 2 kg.

   (a) What is the value of the resonance frequency?

   (b) What is the value of the steady state amplitude at resonance frequency?

   (c) Compare this maximum with F_0/k. What is the amplification? Is this value the same as: $\sqrt{km}/b$?

10. Run the simulation. Observe, stop and reset.

11. Set *Block Mass* equal to 4 kg.

    (a) What is the value of the resonance frequency? Did the resonance curve shift to lower or higher frequencies? Compare to step 9.

    (b) What is the value of the steady state amplitude at resonance frequency? Compare to step 9. item Compare this maximum with F_0/k. What is the amplification? Is this value the same as: $\sqrt{km}/b$?

12. Run the simulation. Observe, stop, and reset.

13. Set *Block Mass* equal to 8 kg.

    (a) What is the resonance frequency? Did the resonance curve shift to lower or higher frequencies? Compare to step 9and 11.

    (b) What is the value of the steady state amplitude at resonance frequency? Compare to step 9 and 11.

    (c) Compare this maximum with F_0/k. What is the amplification? Is this value the same as: $\sqrt{km}/b$?

13.5 Review

Please circle the correct answers to the following multiple-choice questions. You may, if you like, use PEARLS to help you find the correct answer.

1. Consider a driven oscillating (block-spring) system that is rather heavily damped. The block will move the least when the frequency of the driving force is:

   (a) much larger than the resonance frequency

(b) smaller than the resonance frequency
(c) equal to the resonance frequency
(d) for all frequencies

2. At the frequencies low compared to the resonance frequency the piston in a driven oscillation will be moving
   (a) in the direction of the driving force
   (b) in the opposite direction of the driving force
   (c) in a direction independent of the direction of the driving force.

3. If we double the mass of the block in the driven spring-block system, the amplitude at resonance frequency for the same driving force will be approximately:
   (a) half the original resonance amplitude.
   (b) the same as the original resonance amplitude.
   (c) the original resonance amplitude $\times\sqrt{2}$.
   (d) four times the original resonance amplitude.

4. If we drive a harmonic oscillator without damping with a forcing frequency equal to the resonance frequency of the oscillator no steady state will be reached.
   (a) true
   (b) false

5. The sum of two undamped harmonic oscillations with frequencies very close to each other results in a periodic motion with
   (a) an amplitude that is the sum of both amplitudes
   (b) an amplitude that varies harmonically
   (c) a continuously increasing amplitude

6. When the frequency of the driving force with amplitude F_0 is very low compared to the resonance frequency of the driven oscillation of a block-spring system the amplitude of the oscillation will be
   (a) depending on the mass of the block
   (b) $= F_0/k$ with k the spring constant
   (c) smaller than F_0/k because of damping

7. When the driving force frequency is equal to the resonance frequency of a damped driven system, the steady state amplitude is

(a) maximum

(b) close to maximum

(c) zero

8. System 1 has the same resonance frequency as system 2, but the quality factor of system 1 is twice the one of system 2. The width of the resonance curve of system 1 is:

    (a) twice the width of system 2's resonance curve

    (b) half the width of system 2's resonance curve

    (c) equal the width of system 2's resonance curve

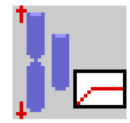
9. When you tap a glass made of fine crystal it will ring longer than one made of ordinary glass. A sound wave of the right frequency can break a glass. Using this method, it will be easier to break

    (a) the fine crystal glass

    (b) the ordinary glass

    (c) neither one, nor the other.

14 Stress and Strain



14.1 Introduction

Most solids will, to some extent, return to their original shape after you pushed or pulled on them with an equal force on two opposite sides. We call them elastic. Some solids are more elastic than others. We also know that if we push or pull too hard the solid will break.

In this assignment we will study the law of elasticity. We will see how solid rods made out of different materials or with different lengths or with different cross sections will stretch differently when pulled uniformly using a very simple model. We will also study what their breaking point is.

14.2 Theory

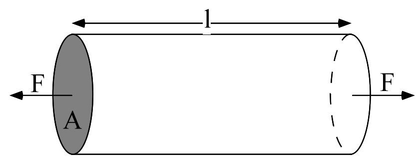


Figure 11: A force $\vec{F}$ stretches a solid rod.

A cylindrical rod with length l and crossection A is stretched by pulling each end equally in opposite directions (see figure (11)). The rod will stretch and the length will increase with an amount Δl. We suppose that this length increase is a small fraction of the total lenght of the rod. For a large number of materials experiments have shown that the force is proportional to the length increase. This is Hooke's law:

$$F \propto \Delta l. \tag{104}$$

We have met this law before in the assignment *Simple Harmonic Oscillator* when we discussed the restoring force excerted by the spring. A rod with a length twice as long will stretch twice as much. This means that the force is invertionally proportional to the length of the rod, or

$$F \propto \frac{\Delta l}{l}. \tag{105}$$

The fractional change in length of the rod $\frac{\Delta l}{l}$ is called *strain*. For a rod with a cross section twice the cross section of a rod of the same length and the same material we will have to pull twice as hard to increase the length over the same amount. Putting this into the previous expression, we become:

$$F \propto A\frac{\Delta l}{l}. \tag{106}$$

Finally we can write Hooke's law as:

$$F = YA\frac{\Delta l}{l}. \tag{107}$$

The force per unit area is called the *stress*. The units are Newton/meter^2. The equation (107) can be rewritten as:

$$\frac{F}{A} = Y\frac{\Delta l}{l} \tag{108}$$

The constant factor Y is called Young's modulus and depend only on the nature of the material. The units for Y are Newton/meter^2.

$$\text{Stress} = \text{Youngs Modulus} \times \text{Strain}.$$

The maximum stress the rod can sustain without breaking is called the *tensile strength*.

Up to now we discussed what happens to a rod when you pull on it. For many materials Hooke's Law and the Young's modulus will be the same when you compress the material. The length will then of course decrease, we call the stress *compressive stress* and the maximum compressive stress you can apply before breakage occurs the *compressive strength*.

14.3 Laboratory

The purpose of this lab is to present a very simple model of the behavior of materials under stress. In particular we wish to demonstrate the concepts of stress and strain. We will also become familiar with the material constants Young's modulus and Breaking Stress, also called Tensile Strength.

14.3.1 Experiment A: Introduction and Setup

In this experiment, we will familiarize ourselves with the laboratory, especially with the display.

1. In the PEARLS window, select *Mechanics*.
2. Now launch the *Stress and Strain* item.

3. You will now see the experiment window, with a control window at the left, and a laboratory display at the right.

4. Adjust the window size as desired.

5. Move the laboratory display to a convenient location on your screen by dragging the display with your mouse.

6. The default settings for this experiment are:

Force	4.000×10^{-1} N
Young's Modulus	1.000×10^{6} Pa
Breaking Stress	6.366×10^{5} Pa
Stress	1.273×10^{5} Pa
Strain	1.273×10^{5} Pa
Radius	1.000×10^{-3} m
Unstressed Length	1.000×10^{-1} m
Change in Length	1.273×10^{-2} m
Zoom	1.000×10^{3} pixels/meter

7. You should see a blue wire, stretched by the force $\vec{F}$, a red wire which is the same wire as the blue one, before stretching. By default a graph *Stress vs. Strain* is displayed as is the change in length $\Delta l = 1.27 \times 10^{-2}$ m.

8. Look at the control window.

   (a) What is the value of the applied force?

   (b) What is the value of the stress?

   (c) What is the radius of the wire?

9. Change the *Radius* slowly to its maximum value with the slider.

   (a) Did the value *Stress* change?

   (b) Did the aspect of the wires change?

   (c) Did the length difference Δl change?

10. Restore the default conditions. (Click $\hookleftarrow$).

11. Set the force to zero.

    (a) Is the blue wire shorter or longer than the red wire?

    (b) What is the value of the stress?

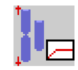
12. Restore the defaults by clicking $\hookleftarrow$.

13. Run the simulation.
14. Augment the *Force* using the slider until the wire breaks.
    (a) What was the magnitude of the *Force* for which the wire breaks?
    (b) Compare the stress with the breaking stress the moment the rod breaks.
15. Stop the simulation.
16. Restore the default values.
17. Augment the *Force* using the slider. Observe the graph.
    (a) At which point in the graph does the wire break?

14.3.2 Experiment B: Elongation and Initial Length

In this experiment we will examine if the elongation of a certain wire indeed is proportional to its original length.

1. Start up the PEARLS *Stress and Strain* experiment, or if it is already running, restore the startup conditions by clicking the ↩ button.
2. Set *Unstressed Length* equal to 0.1 m.
    (a) What is the elongation?
    (b) Did you see a change in the plot?
3. Set *Unstressed Length* equal to 0.2 m.
    (a) What is the elongation? Compare to the elongation of step 2a.
4. Set *Unstressed Length* equal to 0.4 m.
    (a) What is the elongation? Compare to the elongation of step 2a and step 3a.
5. Set *Force* to 0.8 Newton.
6. Set *Unstressed Length* equal to 0.1 m.
    (a) What is the elongation?
7. Set *Unstressed Length* equal to 0.2 m.
    (a) What is the elongation? Compare to the elongation of step 6a.
8. Set *Unstressed Length* equal to 0.4 m.
    (a) What is the change in length? Compare to the change in length of step 6a and step 7a.



14.3.3 Experiment C: The Change in Length and the Cross section

. In this experiment we will examine if the change in length of a certain wire changes when the cross section changes.

1. Start up the PEARLS *Stress and Strain* experiment, or if it is already running, restore the startup conditions by clicking the ↩ button.
   (a) What is the change in length?
   (b) Calculate the cross section.
   (c) Calculate the stress, compare to the displayed one.
2. Set *Radius* equal to $2. \times 10^{-3}$ m.
   (a) What is the change in length?
   (b) Calculate the cross section.
   (c) Calculate the stress, compare to the displayed one and the stress in step 1c.
   (d) Did something change in the plot? What does this change mean?
3. Set *Radius* equal to 0.5×10^{-3} m.
   (a) What is the change in length?
   (b) Calculate the cross section.
   (c) Calculate the stress, compare to the displayed one and the stress in step 1c and step 2c.
   (d) Did something change in the plot? What does this change mean?
4. Set *Radius* equal to 0.4×10^{-3} m.
   (a) What happened to the wire?
   (b) Calculate the stress. Compare to *Breaking Stress*. Why did the wire break?
   (c) Could you see the same result in the plot?

14.3.4 Experiment D: Young's constant, the Elongation and the Breaking Point.

In this experiment we will examine the elongation and breakingspoint for for different materials, that is materials with different Young's Constants.

1. Start up the PEARLS *Stress and Strain* experiment, or if it is already running, restore the startup conditions by clicking the ↩ button.
   (a) What is the value of Young's Modulus?

   (b) What is the change in length?
   (c) What is the strain?

2. Set *Force* equal to 0.8 Newton.
   (a) What is the change in length? Compare to the value of step 1b.
   (b) What is the value of the strain?

3. Set *Force* equal to 2 Newton.
   (a) What is the maximum stress allowed without breaking the wire?
   (b) Is the applied force the maximum force that can be applied to the wire without breaking it?
   (c) What is the change in length?
   (d) What is the strain for the maximum stress?

4. Restore the default values.

5. Set *Young's Constant* equal to 2×10^7 Pa.
   (a) Did you see a change in the plot?
   (b) What is the change in length? Compare to the one in 1b.
   (c) What is the value of the strain?

6. Set *Force* equal to 0.8 N.
   (a) What is the change in length? Compare to the value of step 2a.
   (b) What is the strain?

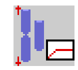
7. Set *Force* equal to 2 N.
   (a) Is this the maximum force that can be applied to the rod without breaking it? Compare to step 3b.
   (b) What is the change in length for the maximum stress for this material?
   (c) What is the strain for this maximum stress? Compare to step 3d.

14.3.5 Experiment E: Breaking Stress

In this experiment we will examine the breaking stress of different materials.

1. Start up the PEARLS *Stress and Strain* experiment, or if it is already running, restore the startup conditions by clicking the ↩ button.

2. Observe the plot while you change the *Breaking Stress* with the slider to higher and then to lower values.



3. Set *Breaking Stress* equal 2×10^5 Pa.

   (a) Use the slider to find how thin you can make the wire before it breaks.

   (b) Calculate the minimum radius of the wire for which the wire will just not break. Compare to you measured value.

4. Set *Breaking Stress* equal 4×10^5 N/m^2.

   (a) Use the slider to find how thin you can make the wire before it breaks. Compare with 3a.

   (b) Calculate the minimum radius of the wire for which the wire will just not break. Compare to you measured value.

5. Set *Breaking Stress* equal 8×10^5 N/m^2.

   (a) Use the slider to find how thin you can make the wire before it breaks. Compare with 3a.

   (b) Calculate the minimum radius of the wire for which the wire will just not break. Compare to you measured value.

14.4 Review

Please circle the correct answers to the following multiple-choice questions. You may, if you like, use PEARLS to help you find the correct answer.

1. Human muscles can approximately exert the same maximum stress. Muscles with larger cross sections are able to exert

   (a) only the same force.

   (b) a larger force.

   (c) only a smaller force.

2. A ball of 50 kg is suspended from a steel wire of length 5 m and radius 2 mm. Young's modulus for steel is: 200×10^9 N/m^2. The wire will stretch by:

   (a) 6.125×10^{-6} m

   (b) 0.19 mm

   (c) 0.976 mm

3. Young's modulus for bone is 1.0×10^{10} N/m^2. Bone can only take 1.0% change in length. The maximum force a bone with cross section 3 cm^2 can undergo is:

   (a) 3×10^4 N

(b) 3×10^6 N
(c) 3×10^8 N

4. Nickel has a Young's modulus that is approx. twice the Young' modulus of copper. We apply the same stress to wires with equal length made of the two materials. The copper wire will elongate

   (a) the same as the nickel wire
   (b) twice as much as the nickel wire
   (c) half as much the nickel wire

5. A wire is stretched 1 mm by a force of 1 kN. A wire of the same material, same length but four times that diameter will be stretched:

   (a) 1/4 mm
   (b) 1/8 mm
   (c) 1/16 mm
   (d) depends on the length

6. A vertical steel post 15 cm in diameter and 3 m long has to support a load of 6000 kg. The stress in the post is:

   (a) 3.33×10^2 N
   (b) 3.33×10^2 N/m^2
   (c) 3.33×10^6 N/m^2
   (d) 3.33×10^6 N

7. Copper has a breaking stress of 3×10^8 N/m^2. The maximum load that can be hung from a copper wire with diameter 0.5 mm is:

   (a) 58.9 N
   (b) 235.6 N
   (c) 300 N

8. A steel wire with diameter 1 mm can maximum support a load of 408 N. To support a load of 816 N a steel wire is needed with a diameter minimum:

   (a) 2 mm
   (b) 1.41 mm
   (c) 0.7 mm

9. A wire 1 m long has a cross section of 2 mm^2. It is hung vertically and stretches 0.2 mm when a 10 kg block is attached to it. The Young's modulus of this wire is:

   (a) 9.8×10^3 $\mathrm{N/m^2}$

   (b) 2.4×10^8 $\mathrm{N/m^2}$

   (c) 2.5×10^{10} $\mathrm{N/m^2}$

   (d) 2.4×10^{11} $\mathrm{N/m^2}$

10. A wire of length l under a stress S is stretched 1 mm. A wire of the same material under the same stress but with length $2l$, will be stretched

    (a) 0.5 mm

    (b) 1 mm

    (c) 2 mm

Part IX

Answers

Answers: Block on Incline

7.4.1 7a 35 degrees.

7b $\mu_s = 0.5$.

7c $\mu_k = 0.3$.

7d The weight $\vec{W}$, the normal force from the inclined plane on the block $\vec{N}$, and the $\vec{f}$, the friction.

8a Yes.

9a Two components: $W_{\|}$, parallel to the plane, $W_{\perp}$ perpendicalar to the plane.

9b The sum of the forces is not zero. ($W_{\|} > F_s$ and $W_{\perp} = N$

11a f became larger.

11b f is the kinetic friction.

11c Yes.

11d No, $F_{\Sigma} \neq 0$.

14a The recent f is greater than the previous f.

15a No: $f = W_{\|}$

15b No, the sum of the forces is now equal to zero.

16a The block is at rest.

17a Each component became twice as long.

17b The block will not move because the sum of the forces is still zero.

18a No.

20a f did not change.

20b f is the static friction.

20c $F_{\Sigma} = 0$.

7.4.2 2a 35 degrees.

2b $\mu_s = 0.5$

2c $\theta_s = 26.56$ degrees ($\tan 26.56° = 0.5$).

4a No.

4b Yes, the static friction is maximum at this point.

6a Yes.

6b For angles $\theta > \theta_s$ the block will slide down.

8a No.

8b For angles $\theta < \theta_s$ the block will not slide down.

9a $\mu_s = \tan 30° = 0.58$

9b No.

9c Yes. $\theta > \theta_s$.

7.4.3 5a $W_{||} = 10 \times 9.8 \times \sin 30°$ kg m/s^2 or $W_{||} = 49$ kgm/s$^2 = 49$ N

5b $F_k = 0.3 \times 10 \times 9.8 \cos 30°$ kgm/s^2 or $F_k = 25$ N

5c $F_\Sigma = (49 - 25)$ N= 24 N

7a The frictional force is constant in time.

7b The value in the graph is 25 N.

9a $a = F_\Sigma/m = 24/10\text{m/s}^2 = 2.4$ m/s^2

11a The acceleration is constant in time. This agrees with Newton's law: if a constant force is acting on a body the acceleration will be constant.

11b The value is 2.4 m/s^2.

15a The velocity changes linear in time. The slope of the linear curve is the acceleration, hence constant.

15b The displacement changes with t^2.

17a $W_{||} = 69$ N.

17b The weight component is now larger.

17c $F_k = 21$ N

17d The friction force is less.

17e $F_\Sigma = 48$ N.

17f The total force is approx. twice the previous total force.

17g $a = 48/10$ m/s$^2 = 4.8$ m/s^2.

7.4.4 6a $\mu_k = 0.3$

6b Yes. A downward force.

7a The velocity changes linear with the time.

7b The acceleration is constant of course.

8a 16.70 degrees.

8b 29.56 degrees.

8c $\theta_k < \theta_s$.

9a Yes. A velocity vector appeared.

10a The total force is zero.

11a The velocity is constant in time.

11b The acceleration is of course zero.

7.4.5 8a The potential energy is proportional to t^2.

8b The kinetic energy is proportional to t^2.

8c The total energy is constant.

8d No, the total energy stayed constant. All potential energy is transformed into kinetic energy.

8e Yes.

11a The total energy changes now proportional to t^2.

11b Yes, the total energy is not constant.

11c No, friction is a non-conservative force.

14a The energy is decreasing in time.

14b More energy is lost: friction is higher.

7.4.6 7a The kinetic energy changes linearly with the displacement (see equation (38)).

7b The potential energy is given by mgy with $y = s \sin\theta$ so their relation is linear if θ is constant.

7c The total energy is constant. Its value is 0.

7d No energy is lost.

7e All forces are conservative.

10a The potential energy is the same.

10b The kinetic energy is less, the speed is less because the acceleration is not $g \sin\theta$ but smaller.

10c The total energy is not constant anymore. Energy is lost to heat.

10d Not all forces are conservative.

13a The friction is higher.

13b The energy loss is higher.

13c The potential energy is the same.

13d The kinetic energy is smaller at all spots.

16a The friction is higher.

16b The energy loss is higher.

16c The potential energy is the same.

16d The kinetic energy is smaller at all spots.

17a $-mgs \sin\theta = -6936$ Joule.

17b $mas = 10 \times 0.69 \times 100$ Joule $= 690$ Joule.

17c Total energy: $(-6936 + 690)$ Joules $= -6246$ Joule Energy loss 6246 Joule.

7.4.7 3a All angles above 26.56°.

4a No, the static friction is maximum for this angle.

5a Yes. The sum of the forces was not zero anymore.

7a The forces becames all twice as large.

7b The sum of the forces is still zero.

8a No.

9a Yes.

12a No. All forces grew proportionally, the sum stayed zero.

13a Yes.

7.4.8 10a Yes, the friction and the acceleration (and thus the total force) are twice the original value.

18a The potential energy drops twice as fast.

18b The kinetic energy climbs twice as fast.

18c The total energy drops twice as fast.

Answers: Center of Mass

8.3.1 8a The system center of mass moved closer to the alligator.

9a The system center of mass moved towards the canoe's center of mass.

9b The system center of mass is midway between the alligator and the canoe.

10a The system center of mass moved towards the canoe's center of mass.

11a The system center of mass moved towards the alligator's center of mass.

11b The system center of mass is again midway between the alligator and the canoe.

11c The center of mass of a system of two equal masses is always halfway between the two.

12a The system center of mass moved towards the alligator.

12b The system center of mass is at the alligator's center of mass.

8.3.2 5a No. The system center of mass remained stationary.

5b The alligator's position relative to the shore is -1 meter.

5c The canoe's position relative to the shore is $+1$ meter.

6a No. The system center of mass remained stationary.

6b The alligator's position relative to the shore is $+1$ meter.

6c The canoe's position relative to the shore is -1 meter.

8.3.3 7a No. The system center of mass remained stationary.

7b The alligator's position relative to the shore is -0.0198 meter.

7c The canoe's position relative to the shore is $+1.98$ meter.

7d The canoe moved much further than the alligator (relative to the shore).

8a No. The system center of mass remained stationary.

8b The alligator's position relative to the shore is $+0.0198$ meter.

8c The canoe's position relative to the shore is -1.98 meter.

8d The canoe moved much further than the alligator (relative to the shore).

13a In between $+0.0198$ m and -0.0198 m.

13b In between -1.98 m and 1.98 m.

13c The position of the center of mass didn't change relative to the shore.

13d When the gator is in the middle of the canoe, all positions relative to the shore are zero.

13e When the gator moves to the right the canoe moves to the left, so that the center of mass of the system doesn't move.

8.3.4 7a No. The system center of mass remained stationary.

7b The alligator's position relative to the shore is -1.98 meter.

7c The canoe's position relative to the shore is $+0.0198$ meter.

7d The alligator moved much further than the canoe (relative to the shore).

8a No. The system center of mass remained stationary.

8b The alligator's position relative to the shore is $+1.98$ meter.

8c The canoe's position relative to the shore is -0.0198 meter.

8d The alligator moved much further than the canoe (relative to the shore).

13a In between -1.98 m and 1.98 m.

13b In between $+0.0198$ m and -0.0198 m.

13c The position of the center of mass didn't change relative to the shore.

13d When the gator is in the middle of the canoe, all positions relative to the shore are zero.

13e When the gator moves to the right the canoe moves to the left, so that the center of mass of the system doesn't move.

Answers: Circular Motion

9.4.1
- 8a Counterclockwise.
- 8b θ is positive, as the convention says.
- 8c ω is positive.
- 10a Clockwise.
- 10b θ is negative.
- 12a In the $X-Y$ plane.
- 13a $\theta = 0$
- 13b The block is on the X axis.

9.4.2
- 5a The $X-Y$-plane.
- 5b The $X-Y$-plane.
- 7a $r = 75$ m
- 7b $\omega = 1.333$ rad/s.
- 7c $v = 100$ m/s.
- 7d $v = 1.33 \times 75$ m/s $= 100$ m/s.
- 9a ω did not change.$\omega = 1.333$ rad/s.
- 9b $v = 200$ m/s, double the original value.
- 9c The magnitude of the velocity vector doubled, so did the radius.
- 11a ω did not change.
- 11b $v = 50$ m/s half its original value.
- 11c The magnitude of the velocity vector and the radius are half their original value.
- 13a $v = 200$ m/s, double the original value.
- 16a $v = -200$ m/s.
- 16b Clockwise.
- 16c The velocity vector is pointing in the direction opposite to the one in step 5.

9.4.3
- 4a A whole circle means 2π radians $= 6.28$ radians. The motion is uniform , so it wil take the block 4 seconds to complete the circle.
- 10a $\theta_{max} = 6.28$ radians.
- 10b θ_{max} is reached after 4 seconds.
- 10c θ varies linear in time, the motion is uniform.
- 12a 4 seconds.

15a θ is negative because the block is turning clockwise.

15b -6.28 rad/s.

15c 4 seconds

15d Linear.

17a ω stayed the same.

17b The plot is the same, because the angular velocity is the same.

18a 2 seconds.

19a θ reaches its maximum after 2 seconds.

9.4.4 5a For $\theta = \pi/2$ and $\theta = 3\pi/2$ rad. (The block passes the Y axis.)

5b For $\theta = 0$ and π rad. (The block passes the X axis).

5c When $Y = 0$ or for $\theta = 0$ and π rad. (The block passes the X axis).

5d When $X = 0$ or for $\theta = \pi/2$ and $\theta = 3\pi/2$ rad. (The block passes the Y axis.)

5e Z is zero at all times because the block is turning in the $X - Y$ plane.

8a As a cosine.

8b As a sine.

8c After 5 seconds Y is zero for the second time.

13a For $\theta = \pi/2$ and $\theta = 3\pi/2$ rad. (The block passes the Y axis.)

13b For $\theta = 0$ and π rad. (The block passes the X axis).

13c When $Y = 0$ or for $\theta = 0$ and π rad. (The block passes the X axis).

13d When $X = 0$ or for $\theta = \pi/2$ and $\theta = 3\pi/2$ rad. (The block passes the Y axis.)

13e For $\theta = \pi/4$ rad.

14a The maxima are higher but occur at the same times as in 8a and 8b.

14b The maxima are higher but occur at the same θ.

17a The period of the sine and cosine is half the period of step 8a and 8b. The block completes a circle in half the original time.

17b The plots stay the same.

9.4.5 5a $X - Y$ plane.

5b $X - Y$ plane.

5c Z axis.

7a The angular momentum points towards the negative Z axis.

7b The definition is a cross product $\vec{L} = \vec{r} \times m\vec{v}$. The velocity changed direction so will the angular momentum.

8a The force became larger since the acceleration is larger.

8b The angular momentum changed direction and became larger.

9.4.6 5a Yes. The angular momentum becomes greater.

5b The tangential velocity increases.

5c No, the angular velocity stays the same.

8a No, the angular momentum is kept constant.

8b The tangential velocity decreased.

8c The angular velocity decreases.

9a $L = m\omega r^2 = 1.4 \times 75^2$ kg.m^2/s = 7875 kg.m^2/s.

10a $\omega = 7878/50^2$ rad/s = 3.15 rad/s.

10b $v = \omega r = 3.15 \times 50$ m/s = 157.5 m/s.

Answers: Circular Orbit

10.4.1 8a 1.5×10^{11} m.

8b 2.975×10^4 m/s.

9a The *Tangential Velocity* became 3.506×10^4 m/s. The *Angular Velocity* changed too.

11a Yes.

11b The centripetal force is the gravitational force between the sun and the planet earth.

10.4.2 4a 3.15×10^7 s = 365 days.

4b After 3.15×10^7 s.

6a The linear speed is higher than the one for the earth orbit.

6b $T = 2\pi r/v$ or $T = 2\pi 5.8 \times 10^{10}/(4.8 \times 10^4)$ s. $T = 7.61 \times 10^6$ s or $T = 0.241$ year.

6c Yes.

8a The linear speed for the Mars orbit is smaller than for the earth orbit.

8b $T = 2\pi 2.28 \times 10^{11}/2.41 \times 10^4$ s = 1.88 year.

8c Yes. After the calculated T the angle has reached 6.28 rad.

10.4.3 3a $a = v^2/r = (2.98 \times 10^4)^2/1.5 \times 10^{11}$ m/s^2 = 5.9×10^{-3} m/s^2

4a $m_{sun} = 1.99 \times 10^{30}$ kg.

5a The centripetal acceleration is a lot higher, the distance to the sun is so much smaller.

5b $a = v^2/r = 39 \times 10^{-3}$ m/s^2.

6a The acceleration is smaller.

6b $a = 2.5 \times 10^{-3}$ m/s^2.

7a No, the acceleration does not change.

7b No.

7c No.

7d Yes, because the mass of the planet changed.

9a The angular velocity becomes greater.

9b The value of the tangential velocity becomes greater.

9c The centripetal acceleration is greater.

11a Yes, it becomes smaller.

11b Yes, it becomes smaller.

11c Yes, it becomes smaller.

11d Yes, it becomes smaller.

Answers: Elastic Collisions

11.4.1 9a No. The trajectories are the same.

9b The speed of ball B after the collision is decreased.

11a Ball A's trajectory after the collision is only slightly different from its initial trajectory. Ball B goes off with a final trajectory nearly perpendicular to the x axis.

11.4.2 2a The mass of both balls is 500 grams.

2b Ball A is stationary after the collision.

3a Ball A continues to move forward after the collision, but at a slower speed.

4a Ball A recoils backwards after the collision.

5a Ball A has a very small velocity after the collision, nearly perpendicular to its initial velocity.

6a Now ball A has zero velocity after the collision.

11.4.3 6a The kinetic energy of ball A decreases.

6b The kinetic energy of B increases.

6c The total kinetic energy is unchanged; the amount lost by A is the amount gained by B.

7a The initial kinetic energy of A is less than before. The final kinetic energy of A is about the same as in step 6.

7b The final kinetic energy of B is less than in step 6; its initial kinetic energy is zero, as it was in step 6.

7c The total kinetic energy is less.

8a The initial kinetic energy of A is the same as step 7. The final kinetic energy of A is greater than in step 7.

8b The final kinetic energy of B is less than in step 6; its initial kinetic energy is zero, as it was in step 7.

8c The total kinetic energy is the same as in step 7. The difference is that less kinetic energy was transferred from A to B in this case, because of the glancing nature of the collision.

11.4.4 7a The x momentum of ball A decreases.

7b The x momentum of B increases.

7c The total x momentum is unchanged, because the amount lost by A is the amount gained by B.

8a The initial x momentum of A is less than before. The final x momentum of A is about the same as in step 7.

8b The final x momentum of B is less than in step 7; its initial x momentum is zero, as it was in step 7.

8c The total x momentum is less than before, because of A's lower inital velocity.

9a The initial x momentum of A is the same as step 8. The final x momentum of A is greater than in step 8.

9b The final x momentum of B is less than in step 8; its initial x momentum is zero, as it was before.

9c The total x momentum is the same as in step 8. The difference is that less x momentum was transferred from A to B in this case, because of the glancing nature of the collision.

11.4.5 6a The y momentum of ball A increases from zero to a positive number.

6b The y momentum of ball B decreases from zero to a negative number.

6c The total y momentum is zero, both before and after the collision. The increase in A's y momentum is offset by the decrease in B's.

7a The initial y momentum of A is zero, as before. The final y momentum of A is greater than in step 6.

7b The initial y momentum of B is zero, as before. The final y momentum of B is more negative than in step 6.

7c The total y momentum is greater than before.

8a The initial y momentum of A is the same as step 7. The final y momentum of A is less than in step 7.

8b The final y momentum of B is less negative than in step 7; its initial y momentum is zero, as it was before.

8c The total y momentum is the same as in step 7. The difference is that less y momentum was transferred from A to B in this case, because of the glancing nature of the collision.

11.4.6 5a No. Neither the direction nor the magnitude of the center of mass velocity is affected by the collision.

6a No. Neither the direction nor the magnitude of the center of mass velocity is affected by the collision.

7a The center of mass is closer to B, at all times.

7b No. Neither the direction nor the magnitude of the center of mass velocity is affected by the collision.

11.4.7 7a Yes, the center of mass moves to the right.

7b No, the pool table does not move.

7c Ball B is initially stationary.

7d This is the “lab” frame.

8a Yes, the center of mass moves to the left.

8b Yes, the pool table moves to the left.

8c No. Neither ball is initially stationary.

9a Yes, the center of mass moves to the left.

9b Yes, the pool table moves to the left.

9c Yes. Ball A is initially stationary.

10a No.

10b Yes, the pool table moves to the left.

10c No.

10d This is the center of mass frame, so named because the observer is moving at the same speed as the center of mass.

Answers: Simple Harmonic Oscillator

12.4.1 9a $x_0 = 0.05m$

9b $v_0 = 0$ m/s

11a The displacement is the greatest when the spring is totally compressed or stretched.

11b The displacement is zero.

13a The force is zero when the acceleration is zero, or when the block is going to the equilibrium.

13b The acceleration is maximum when the spring is fully stretched or fully compressed.

16a Yes, the amplitude of the motion becomes smaller.

12.4.2 4a When the displacement is maximum, in other words when the spring is most compressed or most stretched.

4b Yes. At the equilibrium point $x = 0$.

4c At the equilibrium point $x = 0$.

4d When the displacement is maximum.

4e When the block moves from maximum displacement to the equilibrium position.(By maximum we mean the positive as well the negative displacement maximum.)

4f When the block moves from the equilibrium position out, in other words when the spring is stretched or compressed again.

4g At the equilibrium point.

4h At maximum displacement.

6a The period of an oscillation is the time of an entire oscillation takes. $T = 1.622$ s.

11a As a cosine.

11b As a sine.

11c As a cosine.

11d Yes.

12a Maximum.

12b Zero.

13a Zero.

13b Maximum.

12.4.3 8a 15 N/m

8b $\omega = 3.873$ rad/s.

8c $T = 2\pi/\omega = 1.622$ s.

8d Yes. Displacement, velocity and acceleration repeat the same behavior after the period of the oscillation.

8e The phase constant is zero (For $t = 0$ the displacement is the amplitude).

8f $A = 0.05$ m. The amplitude = the initial displacement.

8g $-\omega A = -0.19$ m/s.

8h $-\omega^2 A = -0.75$ m/s^2.

10a 15 N/m.

10b $\omega = 5.477$ rad/s.

10c $T = 2\pi/5.477 = 1.147$ s.The period is $\sqrt{2}$ times smaller.

10d Yes. The new plots show a shorter period than the ones of step 8d.

10e $A = 0.05$ m. The same as in step 8f.

10f $-\omega A = -0.27$ m/s. The maximum velocity is $\sqrt{2}$ times the value of step 8g.

10g $-\omega^2 A = -1.50$ m/s^2. The maximum acceleration is 2 times the value in step 8h.

12a $\omega = 1.94$ rad/s, half the value of step 8c.

12b $T = 3.245$ s.The period is twice its value of step 8f.

12c $A = 0.05$ m. The amplitude stayed the same.

12d $-\omega A = -1.94 \times 0.05$ m/s $= -0.092$ m/s. The maximum velocity is half the original value.

12e $-\omega^2 A = -(1.94)^2 \times 0.05 = 0.18$ m/s^2. The maximum acceleration is one fourth of the original one.

15a $A = 0.05$ m

15b $T = 1.622$ s

17a The absolute value of the amplitude did not change. The spring starts from fully compressed.

17b No, the period is not affected by the initial displacement.

18a The period is not affected by the initial displacement.

18b The velocity is not affected by the initial displacement.

18c The acceleration is not affected by the initial displacement.

21a No, the block moves farther out.

23a The amplitude is greater than the initial displacement.

23b The period is not affected by the initial velocity.

23c The new plot differs by a phase constant from the old plot.

24a Initial conditions $t = 0$:$x_0 = 0.10m = A\cos\phi$ and $v_0 = 0.1m/s = -A\omega\sin\phi$. Dividing both equations gives: $\tan\phi = -0.26$ or $\phi = 14.5$ degrees.

24b $A = 0.1m/\cos 14.5 = 0.097$ m.

12.4.4 7a The total force is equal to the spring force because there is no damping.

7b $T = 1.622$ s.

8a The force varies linear with the displacement.

8b The slope is $-k$, and means that the force is a restoring force (opposite to the displacement).

8c $Fmax = kA = 15 \times 0.05$ N $= 0.75$ N.

8d For $x = 0$ or the equilibrium point.

8e The period is 1.622 s.

10a $T = 2.294$ s, $\sqrt{2}$ larger than the one in step 8e.

11a The slope is less steep than in step 8 because the spring constant is half the first value.

11b $F_{max} = kA = 7.5 \times 0.05$ N.

11c For $x = 0$ or the equilibrium point.

13a No.

13b No, no forces are acting on the block.

15a No.

15b The position of the block changes linear in time.

15c No. This is a linear motion with constant velocity. No force is acting on the block.

12.4.5 7a The total energy is constant.The energy is conserved.

7b $E_{tot} = 1.88 \times 10^{-2}$ Joule.

7c The maximum potential energy is equal to the total energy: 1.88×10^{-2} Joule. Its first maximum occurs at $t = 0$.

7d The maximum kinetic energy is equal to the total energy: 1.88×10^{-2} Joule. Its first maximum occurs at $t = 0.4$ s.

7e The period of oscillation is: $T = 1.622$s. The kinetic energy becomes zero at $\frac{1}{2}T = 0.8$ s.

12a The total energy is constant and equal 1.88×10^{-2} Joule.

12b For $x = A$ and $x = -A$ with the amplitude $A = 0.05$ m.

12c For $x = 0$ the equilibrium point.

12d For $x = 0$, the equilibrium position.

12e For $x = A$ and $x = -A$ with the amplitude $A = 0.05$ m.

15a Yes. The total energy $E = 1/2kA^2 = 9.4 \times 10^{-4}$ Joule, half the value in step 12

15b Yes. The curve is less steep. The maximums are half the ones in step 12.

15c Yes. The curve is less steep. The maximum is half the ones in step 12.

15d No.

19a Yes. A is half its original value, so the energy is one quart of the value from step 12

19b The curve is shorter..

19c Yes.

19d $x = A$ and $x = -A$ with $A = 100$m.

23a No. The total energy of the block does not depend on the mass of the block.

23b No.

23c No.

12.4.6 3a The spring force vector points in the opposite direction to the displacement vector.

3b The spring force is maximum when the displacement is maximum.

6a The damping force points in the opposite direction as the velocity vector.

6b The damping force is maximum when the velocity is maximum, in other words, when the block passes through its equilibrium, or when the spring force is zero.

6c The damping force is zero when the velocity is zero, in other words when the block is at its maximum displacement. The spring force is maximum then.

9a The total force is proportional to the acceleration, they move in the same direction.

12.4.7 6a The amplitude is diminishing during the motion.

6b The period of the damped motion is approx. the same as for the undamped motion.

8a Very small oscillation.

10a Yes.

10b The amplitude of the displacement is decreasing fast.

12a The critial damping is 7.746 Ns/m.

12b No.

12c It takes the block approx. 1.6 seconds to return to its equilibrium.

12d The displacement varies exponentially in time.

13a It takes approximately 3.9 seconds to reach its equilibrium, which is a lot longer than at critical damping.

14a $b = 15.49$ Ns/m.

16a Yes.

12.4.8 8a Yes.

10a No, the total energy is diminishing in time.

10b At $t = T/2$ and $t = T$. The speed is small and zero at those times.

13a Yes.

16a Drops fast to zero.

16b The block almost doesn't have kinetic energy.

16c No, it returned to its equilibrium position.

19a Drops slower to zero in time than in step 16a.

Answers: Resonance

13.4.1 11a Yes. For a short time the motion is not steady.

11b No. The amplitude is smaller at the beginning.

13a The steady state amplitude $A_s = 5.74 \times 10^{-2}$ m.

13b Yes. The maximum possible amplitude is $A_{max} = 0.806 \times 10^{-2}$ m. The frequency for which the amplitude is maximum is approximately 0.5 Hz.

14a Yes, it becomes greater.

14b The maximum of the plot has greater values, too.

15a Yes, it became larger.

15b The width of the resonance curve decreases. The height increases.

15c The phase drops faster around the resonance frequency.

13.4.2 3a No, the steady state amplitude varies with the frequency of the driving force.

3b The amplitude is maximum for a forcing frequency equal to 0.5 Hz.

3a $k = 9.87$ N/m.

3b $m = 1$ kg.

3c $\omega_0 = 3.14$ rad/s.

3d $f_0 = 0.5$ Hz, the resonance frequency.

4a It takes a short time before the block is oscillating with an amplitude equal to the steady state amplitude.

4b $f = 0.4$ Hz. The forcing frequency is close to the resonance frequency.

6a $\omega_0 = 2\pi \times 0.5$ rad/s= 3.14 rad/s

6b $\omega = 2\pi \times 0.4$ rad/s $= 2.51$ rad/s.

6c $b = 1$ kg/s.

6d $m = 1$ kg.

6e $A = 5.74 \times 10^{-2}$ m.

7a The amplitude is very close to the maximum steady state amplitude for this system.

9a The forcing frequency is much lower than the resonance frequency.

9b $A = 2.63 \times 10^{-2}$ m. The steady state amplitude is much lower than the maximum possible value.

9c $F/k = 0.25/9.87$ m $= 2.5 \times 10^{-2}$ m. The motion is only slightly amplified.

11a The forcing frequency is much lower than the resonance frequency.

11b $A = 1.68 \times 10^{-3}$ m is much smaller than the maximum possible value.

11c The motion is not amplified quite the opposite.

13a $A = 7.96 \times 10^{-2}$ m. This is very close to the maximum value.

13b The steady state amplitude is much larger than $F/k = 2.5 \times 10^{-2}$ m. The motion is amplified (approx. 3 times).

13.4.3 2a $f_0 = 0.5$ Hz.

4a Yes.

4b The forcing frequency is quite smaller than the resonance frequency.

4c The forcing frequency is smaller than the frequency in the interval.

4d $\phi \approx 0$.

4e The piston and the rod move in the same direction, there is almost no phase difference between their motion.

7a Piston and rod are moving opposite ways.

7b The forcing frequency is higher than the resonance frequency.

7c The frequency is higher than the frequencies in the interval.

7d $\phi \approx -162$ degrees. This phase difference shows indeed that the rod should move almost in the opposite direction the block is moving.

10a The piston lags behind the rod.

10b The forcing frequency is equal to the resonance frequency.

10c At this frequency the amplitude is very close to maximum.

10d $\phi = 90$ degrees.

13.4.4 5a The plot is symmetric in the region around the resonance frequency but assymmetric en total.

5b $A = 1.59 \times 10^{-1}$ m

5c No, it is slightly different.

5d The quality factor $Q = \omega_0 m/b = 1.6$.

7a Yes.

7b The width of the peak is a lot smaller than in step 3.

7c $A = 1.59$ m. The amplitude is ten times higher.

7d Yes.

7e The Phase drop is steeper around the resonance frequency.

7f $Q = 15.7$ The quality factor is ten times the quality factor of step 5d. Less energy is lost because the damping is smaller.

9a No, very asymmetric.

9b No, the damping is too high.

9c $A = 1.59 \times 10^{-2}$ m. The amplitude is half the value of the one in step 3 and 100 times smaller than the one in step 7.

9d No, the difference between both is bigger than in the previous cases.

9e The phase drops a lot slower than in the case with less damping.

9f The quality factor $Q = 0.16$ is ten times smaller than in step 5d, because the damping is ten times higher.

13.4.5 4a The total energy is varying harmonically in time, once the steady state is reached.

4b The position of the block varies harmonically in time once the steady state is reached.

7a The piston is wildly oscillating.

7b No. The amplitude becomes larger and larger.

7c Yes. The amplitude becomes larger and larger as time goes by.

7d No, the energy becomes larger and larger.

10a Yes. The block's amplitude increases to a maximum, to decrease again.

10b In 40 seconds we see 2 beats. Or the beat frequency is $f_{beat} = 2/40$ Hz$= 0.05$ Hz.

10c $f_{diff} = f_0 - f = 0.05$ Hz $= f_{beat}$.

12a $f_{diff} = 0.5 - 0.3$ Hz $= 0.2$ Hz.

12b No, the difference in frequency of the two harmonic motions is too large.

12c No. The position versus time is not a simple cosine or sine function.

13.4.6 4a $f_0 = 0.712$ Hz.

4b $f \ll f_0$.

4c $A = 0.0255$ m

5a $f_0 = 1.01$ Hz.

5b $f \ll f_0$.

5c $A = 0.01262$ The amplitude is half the amplitude in step 4.

6a $f_0 = 1.42$ Hz.

6b $f \ll f_0$.

6c The amplitude 1/4 the amplitude of step 4 and 1/2 the amplitude of step 5

8a $f_0 = 0.712$ Hz

8b $f \ll f_0$.

8c $A = 1.28 \times 10^{-2}$ m.

10a $f_0 = 0.503$ Hz.

10b $f \ll f_0$.

10c $A = 1.30 \times 10^{-2}$ m. The amplitude almost didn't change.

12a $f_0 = 0.356$ Hz.

12b $f \ll f_0$.

12c $A = 1.36 \times 10^{-2}$ m. The change in amplitude is very small.

14a Yes, changing the mass doesn't affect the amplitude at low frequencies while the amplitude is invertionally proportional to the spring constant.

13.4.7 5a $f_0 = 0.712$ Hz.

5b $A_1 = 0.2227$ m.

5c $F_0/k = 0.025$ m. The amplification is: $0.2227/0.025 = 9$ $\sqrt{km}/b = \sqrt{20}/0.5 = 9$.

6a $f_0 = 1.01$ Hz.The curve shifted to higher frequencies.

6b $A_2 = 0.156$ m. $A_1/A_2 = 1.41 = \sqrt{2}$

6c $F_0/k = 0.0125$ m. The amplification is $0.156/0.0125 = 12.5$.

7a $f_0 = 1.42$ Hz. The resonance curve shifted to higher frequencies.

7b $A_3 = 0.110$ m.$A_1/A_3 = 2 = \sqrt{4}$, $A_2/A_3 = 1.41 = \sqrt{2}$

7c $F_0/k = 0.00625$ m. The amplification is $0.110/0.00625 = 17.6$.

9a $f_0 = 0.712$ Hz.

9b $A_4 = 0.219$ m

9c $F/k = 0.0125$ m. The amplification is $0.219/0.0125 = 17.5$.

11a $f_0 = 0.503$ Hz. The curve shifted to lower frequencies.

11b $A_5 = 0.305$ m $A_4/A_5 = 0.7 = 1/\sqrt{2}$

11b The amplification is $0.305/0.0125 = 24$.

13a $f_0 = 0.356$ Hz. The curve shifted to lower frequencies.

13b $A_6 = 0.419$ m $A_4/A_6 = 0.5 = 1/2$, $A_5/A_6 = 0.71 = 1/\sqrt{2}$

13c The amplification is $0.419/0.0125 = 34$.

Answers: Stress and Strain

14.3.1

8a $F = 0.4$ N.

8b The stress is equal to 1.273×10^5 Pa.

8c The radius of the wire is 1×10^{-3} m.

9a Yes. The stress becomes smaller as the radius becomes larger.

9b The radius of the wire change so it is thicker.

9c Yes. The length difference becomes smaller as the radius becomes larger.

11a Both wires are the same length.

11b The stress is of course zero. No force is applied.

14a $F = 2$ Newton.

14b The stress becomes greater than the tensile strength.

17a The breaking point of the wire is the point where the plot becomes horizontal.

14.3.2

2a $\Delta l = 1.27 \times 10^{-2}$ m

2b No change in the plot.

3a $\Delta l = 2.55 \times 10^{-2}$ m. This is twice the elongation of step 2a.

4a $\Delta l = 5.09 \times 10^{-2}$ m This is twice the elongation of step 3a and four times the one of step 2a.

6a $\Delta l = 2.55 \times 10^{-2}$ m

7a $\Delta l = 5.09 \times 10^{-2}$ m. This is twice the elongation of step 6a.

8a $\Delta l = 1.02 \times 10^{-1}$ m. This is twice the elongation of step 7a and four times the one of step 6a.

14.3.3 1a $\Delta l = 1.27 \times 10^{-2}$ m.

1b $\pi r^2 = 3.14 \times 10^{-6}$ m^2.

1c $F/A = 1.273 \times 10^5$ Pa.

2a $\Delta l = 3.18 \times 10^{-3}$ m.

2b $\pi r^2 = 1.25 \times 10^{-6}$ m^2.

2c $F/A = 3.183 \times 10^4$ Pa.

2d The cross hairs moves down the curve. We are farther from the breaking point.

3a $\Delta l = 5.09 \times 10^{-2}$ m

3b $\pi r^2 = 7.85 \times 10^{-7}$ m^2

3c $F/A = 5.093 \times 10^5$ Pa.

3d We are close to the breaking point.

4a The wire broke.

4b $F/A = 7.958 \times 10^{-5}$ Newton/m^2. The stress is greater than the tensile strength or breaking stress.

4c We are past the breaking point.

14.3.4 1a $Y = 1 \times 10^6$ Pa.

1b $\Delta l = 1.27 \times 10^{-2}$ m.

1c Strain $= 1.273 \times 10^{-1}$

2a $\Delta l = 2.55 \times 10^{-2}$ m, twice the first elongation.

2b Strain $= 2.546 \times 10^{-1}$

3a $F/A = 6.366 \times 10^5$ Pa.

3b $F_{max} = 2$ N. Yes, this is the maximum force, because for a greater force the stress will be greater than the tensile strength or the breaking stress.

3c The change is length is 6.37×10^{-2} m.

3d Strain $= 6.366 \times 10^{-1}$.

5a Yes. The slope became steeper for this higher value of Young's constant.

5b $\Delta l = 6.37 \times 10^{-4}$ m. The elongation is 20 times smaller, the material is 20 times stiffer.

5c $\Delta l/l = 6.366 \times 10^{-3}$.

6a $\Delta l = 1.27 \times 10^{-3}$ m.

6b $\Delta l/l = 1.273 \times 10^{-2}$.

7a Yes, it is the same maximum force as in step 3a.

7b $\Delta l = 3.18 \times 10^{-3}$ m, 20 times smaller than the elongation of 3c.

7c $\Delta l/l = 3.183 \times 10^{-2}$. The strain is 20 times smaller than for the less stiff material.

14.3.5 3a Around 7.6×10^{-4} m.

3b 7.98×10^{-4} m

4a 5.5×10^{-4} m. Because the tensile strength is higher the wire can be made thinner before it breaks.

4b 5.64×10^{-4} m.

5a 3.98×10^{-4} m. The minimum radius is half the minimum radius of step 3a.

5b 3.99×10^{-4} m.