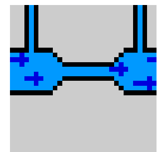
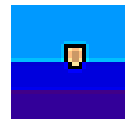
Part II

Fluid Mechanics



4 Buoyancy



4.1 Introduction

The happy Greek shout "Eureka" became associated with the laws of buoyancy thanks to Archimedes. While taking a bath, the story goes, he discovered the laws of bouancy. Ages later, the famous Archimedes' principle could be derived from Newton's laws.

In this assignment we will study the Archimedes' principle. We will examine how a body that is floating or submerged in a fluid is buoyed up by a force. This force is equal to the weight of the fluid, displaced by the body.

4.2 Theory

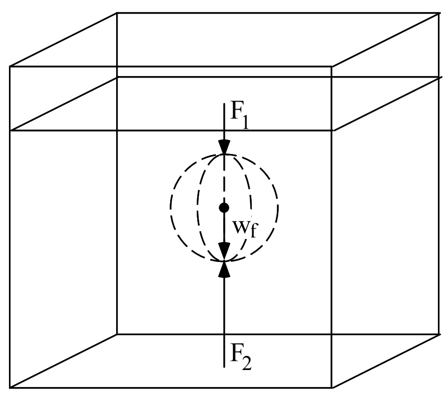
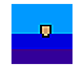


Figure 3: The Forces Acting on a Body Submerged in a Fluid

We consider a fluid at rest in a container. Close to the bottom of the container the pressure in the fluid is greater than close to the surface of the fluid because the weight of the fluid column. If we submerge a body into the fluid the pressure on the bottom of the body will be greater than on the top for this reason. When the pressures on a body differ, it means that the forces acting on the body differ. The sum of the forces by the fluid on the body will not be zero. See figure (3). There will be a net force. This force is called the *buoyant force*.



To derive Archimedes' principle we replace the body by fluid (think of a "fluid piece" in the same shape of the body). The forces that act on the "fluid piece" are:

- the buoyant force $\vec{B}$ (the resultant of the pressure differences in the fluid)
- gravity $\vec{w}_f$ (= weight of the fluid piece).

This "fluid piece" is at rest, this means that the sum of all forces on it is zero or

$$B = w_f \tag{9}$$

This expression is the same for different shapes of the "fluid piece". The weight of the piece is given by:

$$w_f = m_f g = \rho_f g V \tag{10}$$

with $m_f = \rho V$ the mass of the "piece", V the volume of the piece, ρ_f the density of the fluid, and g the gravity constant.

If we replace this "fluid piece" by the body with the same volume V but with a density ρ we see immediately when the body is going to sink or float.

- $\rho > \rho_f$ the body will sink because the weight is higher than the buoyant force.
- $\rho \leq \rho_f$ the body will float. The body will be totally or partly submerged. The buoyancy force is the force that holds up the body, in other words the buoyant force is equal to the weight of the body. Equation (9) tells us that the buoyant force is given by the weight of the fluid displaced by the submerged part of the body. We obtain:

  $$B = w \tag{11}$$

  or

  $$\rho_f g V' = \rho g V \tag{12}$$

  with V' the volume of the fluid displaced by the body (that is the volume of the submerged part of the body). This equation leads to an expression what fraction of the body is submerged:

  $$\frac{V'}{V} = \frac{\rho}{\rho_f} \tag{13}$$

4.3 Laboratory

In this laboratory we consider a ball in a container filled with fluid. The density of the fluid and the ball can be changed, so can the volume of the ball. (A limitation of the simulation: the simple harmonic oscillation of the ball in the liquid is a fixed oscillation, not depending on the density.)

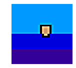
4.3.1 Experiment A: Introduction and Setup

In this experiment, we will familiarize ourselves with the laboratory, especially with the display.

1. In the PEARLS window, select *Fluid Mechanics*.
2. Now launch the *Buoyancy* item.
3. You will now see the experiment window, with a control window at the left and a laboratory display at the right.
4. Adjust the window size as desired.
5. Move the laboratory display to a convenient location on your screen by dragging the display with your mouse.
6. The default settings for this experiment are:

Sphere Radius	2.000 cm
Draft	1.000 cm
Sphere Density	8.000×10^{-1} g/cm^3
Fluid Density	1.000 g/cm^3
Zoom	2×10^1 pixels/cm

7. You should see a rectangular container, filled with a green fluid, with a ball dropped into it.
8. *Run* the experiment.
9. Press the *Rotation Control* in the control window, so you have a good view at the movement of the ball in the fluid.
   (a) After a while the ball is at rest. Is it floating?
   (b) What is the value of the *Draft*?
   (c) What is the value of the *Sphere Density*?
   (d) What is the value of the *Fluid Density*?
10. *Stop* the experiment and *Reset*.
11. Set *Sphere Density* equal to 1.1 g/cm^3.
12. *Run* the experiment, observe, *Stop* the simulation.
    (a) Is the ball floating?
    (b) Did the draft change?
13. Restore the default conditions by clicking the $\hookleftarrow$.



14. Set *Sphere Density* equal to 0.5 g/cm^3.
15. Push the ball to the bottom of the container: use the *Draft* slider to set the value to its maximum value.
16. *Run* the experiment until the ball is at rest, *Stop*, and *Reset*.
    (a) Did the ball stay at the bottom of the container?
    (b) Is the ball floating when it is at rest?

4.3.2 Experiment B: Density of the Ball and the Displaced Volume

In this experiment we will study how the fraction of the volume of the ball that will be submerged depends on its density.

1. Start up the PEARLS *Buoyancy* experiment, or if it is already running, restore the startup conditions by clicking the $\hookleftarrow$ button.
2. Click the graph button and make a graph with *Displaced Volume* in the vertical axis and *Time* in the horizontal axis.
3. Set *Sphere Density* equal to 0.5 g/cm^3.
4. *Run* the experiment until the ball is at rest. *Stop* the simulation.
    (a) What is the *Fluid Density*? Compare to the *Sphere Density*.
    (b) Which fraction of the volume is submerged? Can you tell it from the display? Use the rotation control to have a better view.
    (c) Read the value of the submerged volume from the plot when the ball is at rest.
5. *Reset*.
6. Set *Sphere Density* equal to 0.25 g/cm^3.
7. *Run* the experiment until the ball is at rest, *Stop*.
    (a) Compare the *Fluid Density* to the *Sphere Density*.
    (b) Which fraction of the volume is submerged? Can you tell it from the display?
    (c) Read the value of the submerged volume from the plot. Compare to the value of step 4c.
8. *Reset*.
9. Set *Sphere Density* equal to 0.75 g/cm^3.
10. *Run* the experiment until the ball is at rest, *Stop*.

(a) Compare the *Fluid Density* to the *Sphere Density*.
(b) Which fraction of the volume is submerged? Can you tell it from the display?
(c) Read the value of the submerged volume from the plot. Compare to the value of step 4c and 7c.

11. Set *Sphere Density* equal to 1 g/ cm^3.

12. *Run* the experiment until the ball is at rest, *Stop*.

    (a) Compare the *Fluid Density* to the *Sphere Density*.
    (b) Which fraction of the volume is submerged? Can you tell it from the display?
    (c) Read the value of the submerged volume from the plot. Compare to the value of step 4c, 7c, and 10c.
    (d) Is the ball floating or sinking?

13. *Reset*.

14. Close the graph.

4.3.3 Experiment C: The Same Ball in Different Fluids

In this experiment we will study how the fraction of the volume of the ball that will be submerged depends on the density of the fluid.

1. Start up the PEARLS *Buoyancy* experiment, or if it is already running, restore the startup conditions by clicking the $\hookleftarrow$ button.

2. Click the graph button and make a graph with *Displaced Volume* in the vertical axis and *Time* in the horizontal axis. Change the *Maximum* in the vertical axis to 5.

3. Set *Radius* of the ball equal to 1 cm.

4. Set *Fluid Density* equal to 1 g/cm^3

5. Set *Sphere Density* equal to 1 g/cm^3

6. *Run* the experiment until the ball is at rest, *Stop*.

    (a) Compare the *Fluid Density* to the *Sphere Density*.
    (b) Did the ball sink or float in this fluid?
    (c) Which fraction of the volume is submerged? Can you tell it from the display?
    (d) Read the value of the submerged volume from the plot.

7. *Reset*.

8. Set *Fluid Density* equal to 0.5 g/cm^3.

9. *Run* the experiment until the ball is at rest, *Stop*.

   (a) Compare the *Fluid Density* to the *Sphere Density*.

   (b) Did the ball sink or float in this fluid?

   (c) Which fraction of the volume is submerged? Can you tell it from the display?

   (d) Read the value of the submerged volume from the plot. Compare to the value of step 6d.

10. *Reset*.

11. Set *Fluid Density* equal to 2 g/cm^3.

12. *Run* the experiment until the ball is at rest, *Stop*.

    (a) Compare the *Fluid Density* to the *Sphere Density*.

    (b) Did the ball sink or float in this fluid?

    (c) Which fraction of the volume is submerged? Can you tell it from the display?

    (d) Read the value of the submerged volume from the plot. Compare to the value of step 6d.

13. *Reset*.

14. Close the graph.

4.3.4 Experiment D: The Buoyancy Force

In this experiment we will throw different size balls, but with the same density, into the same liquid.

1. Start up the PEARLS *Buoyancy* experiment, or if it is already running, restore the startup conditions by clicking the $\hookleftarrow$ button.

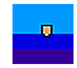
2. Click the graph button. Make a graph with *Buoyancy Force* in the vertical axis and *Time* in the horizontal axis.

3. Calculate the volume of the sphere. (Remember volume of a sphere $4\pi r^3/3$ with r the radius).

   (a) What is the value of the volume of the sphere?

4. Set *Sphere Density* equal to 0.5 g/cm^3.

5. *Run* the experiment until the ball is at rest, *Stop*.
   (a) Calculate the weight of the ball.
   (b) What fraction of the ball is submerged in the liquid? What is its volume?
   (c) What is the value of the buoyancy force? Read the value of the graph. Compare to the weight of the ball.
   (d) What is the weight of the displaced liquid? Compare to the value of the buoyancy force.
6. *Reset*.
7. Set *Sphere Density* equal to 0.75 g/cm^3.
8. *Run* the experiment until the ball is at rest, *Stop*.
   (a) Calculate the weight of the ball.
   (b) What fraction of the ball is submerged in the liquid? What is its volume?
   (c) What is the value of the buoyancy force? Compare to the weight of the ball.
   (d) What is the weight of the displaced liquid? Compare to the value of the buoyancy force.
9. *Reset*.
10. Set *Sphere Density* equal to 1 g/cm^3.
11. *Run* the experiment until the ball is at rest, *Stop*.
    (a) Calculate the weight of the ball.
    (b) What fraction of the ball is submerged in the liquid? What is its volume?
    (c) What is the value of the buoyancy force? Compare to the weight of the ball.
    (d) What is the weight of the displaced liquid? Compare to the value of the buoyancy force.
12. *Reset*.
13. Set *Sphere Density* equal to 2 g/cm^3.
14. *Run* the experiment until the ball is at rest, *Stop*.
    (a) Calculate the weight of the ball.



(b) What fraction of the ball is submerged in the liquid? What is its volume?

(c) What is the value of the buoyancy force? Compare to the weight of the ball.

(d) What is the weight of the displaced liquid? Compare to the value of the buoyancy force.

15. *Reset*.

16. Close the graph.

4.4 Review

Please circle the correct answers to the following multiple-choice questions. You may, if you like, use PEARLS to help you find the correct answer. Some approximate densities are: (given at $t = 0°\text{C}$ and $\text{P} = 1$ atm)

Density	$\times 10^3$ kg/m^3
ice	0.92
seawater	1.03
lead	11.3
ethanol	0.81
gold	19.3

1. A slab of ice floats on a fresh water lake. The minimum volume the slab must have for a 24 kg penguin to stand on it without getting its feet wet is

   (a) 0.30 m^3

   (b) 0.92 m^3

   (c) 1.09 m^3

2. The ice of an iceberg is frozen fresh water. An iceberg floats in seawater. The fraction of its volume that is submerged

   (a) depends on its shape

   (b) 89%

   (c) 92%

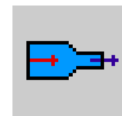
3. Because of the buoyancy force, an object will appear to weigh less in seawater than in fresh water.

   (a) True

   (b) False

4. A hoisting cable pulls a sunken treasure out of the water. The tension in the cable will be less when the treasure is still totally immersed in the water.
   (a) True
   (b) False
5. A hydrometer (a floating instrument that is used to measure the density of liquids) will be floating higher in denser fluids than in less dense ones.
   (a) true
   (b) false
6. When floating in water 1/5th of a cork's volume is submerged. When floating in ethanol, the part of its volume that will be submerged is:
   (a) impossible to calculate for lack of data.
   (b) 1/4th
   (c) 100%
7. A jewel, of supposedly, pure gold weighs 1 Newton in the air suspended from a spring scale. The jewel is submerged in water hanging from the same spring scale. In water it appears to weigh 0.91 N. The jewel must be made of
   (a) pure gold
   (b) gold and less dense metals than gold
   (c) gold and denser metals than gold.
8. The buoyancy force on an object depends on the shape of the object.
   (a) True
   (b) False

5 Continuous Flow



5.1 Introduction

In this assignment we consider a non-viscous fluid that flows in a steady state (non-turbulent) through a tube with a changing cross section. (We will study the viscous fluid in the assignment *Viscous Flow*.) We also assume that the fluid is incompressible, a good assumption for most fluids.

5.2 Theory

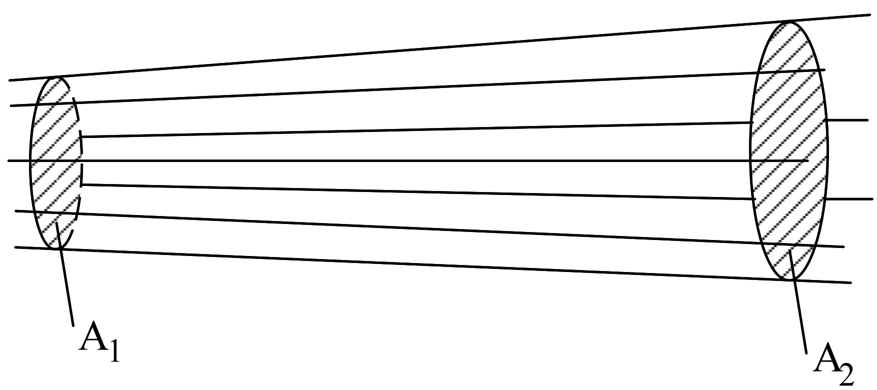


Figure 4: Incompressible fluid flowing in a tube of varying cross section

Figure (4) shows a liquid that flows through a tube with varying cross section. In the time interval Δt a volume ΔV of fluid passes through the cross section A_1. If we assume the fluid is having a speed v_1 this volume ΔV is:

$$\Delta V = A_1 v_1 \Delta t \tag{14}$$

Because the fluid is incompressible and flowing steadily, this same volume of fluid has to leave the tube in the time interval Δt. The cross section at the end is A_2. If we assume the speed of the fluid is there v_2 the volume is:

$$\Delta V = A_2 v_2 \Delta t \tag{15}$$

Since the volumes are equal, we have:

$$\begin{aligned} A_1 v_1 \Delta t &= A_2 v_2 \Delta t \\ A_1 v_1 &= A_2 v_2 \end{aligned} \tag{16}$$

The quantity Av is called the *volume flow rate* or volume of fluid that passed in a second. It has the dimensions of meter^3/second. The equation (16) is also called the *continuity equation*: the volume flow rate in a steady flow of an incompressible fluid is constant:

$$Av = \text{constant.}$$

5.3 Laboratory

5.3.1 Experiment A: Introduction and Setup

In this experiment, we will familiarize ourselves with the laboratory, especially with the display.

1. In the PEARLS window, select *Fluid Mechanics*.
2. Now launch the *Continuous Flow* item.
3. You will now see the experiment window, with a control window at the left, and a laboratory display at the right.
4. Adjust the window size as desired.
5. Move the laboratory display to a convenient location on your screen by dragging the display with your mouse.
6. You should see a tube with changing cross section, filled with a flowing liquid. The velocity vectors of the fluid $\vec{v}_1$ and $\vec{v}_2$ are displayed.
7. The default settings for this experiment are:

Radius 1	3.000×10^{-2} m
Radius 2	1.000×10^{-1} m
Velocity 1	6.000×10^{-2} m/s
Velocity 2	5.400×10^{-3} m/s
Zoom	1.000×10^{2} pixels/m
Time Zoom	1.000

8. *Run* the experiment, observe the flowing of the liquid, *Stop* and *Reset*.
9. Set *Radius 1* equal to 13×10^{-2} m.

   (a) Did the cross section of the first part of the tube change?

  (b) Did the velocity $\vec{v}_1$ change?
  (c) Did the velocity $\vec{v}_2$ change?

10. Set *Velocity 2* equal to 3×10^{-1} m/s.
  (a) Did the velocity $\vec{v}_1$ change?

5.3.2 Experiment B: Continuity Equation and the Cross Section

In this experiment we will study what happens when we change the cross section of one part of the tube in which a fluid flows steadily.

1. Start up the PEARLS *Continuous Flow* experiment, or if it is already running, restore the startup conditions by clicking the $\hookleftarrow$ button.

2. Run the experiment, stop and reset.
   (a) What is the value of the cross section A_1? Remember that the surface of a circle is πr^2.
   (b) What is the value of the cross section A_2?
   (c) What is the volume flow rate through 1?
   (d) What is the volume flow rate through 2? Compare to step 2c.

3. Set *Radius 1* equal to 6×10^{-2} m.
   (a) Does the fluid flow faster or slower through the tube (look at the velocity vector) than in step 2?
   (b) What is the value of the speed v_1?
   (c) What is the flow rate through the first part of the tube?
   (d) What is the flow rate through the second part of the tube? Compare to 3c.

4. Set *Radius 1* equal to 10×10^{-2} m.
   (a) Does the fluid flow faster or slower through the first part of the tube (look at the velocity vector) than in step 3a?
   (b) What is the value of the speed v_1? Compare to v_2.
   (c) What is the flow rate through the first part of the tube?
   (d) What is the flow rate through the second part of the tube? Compare to 4c.

5. Set *Radius 2* equal to 5×10^{-2} m.
   (a) Does the fluid flow faster or slower through the second part of the tube (look at the velocity vector) than in step 4a?

(b) What is the value of the speed v_1? Compare to v_2.

(c) What is the flow rate through the first part of the tube?

(d) What is the flow rate through the second part of the tube? Compare to 5c.

5.4 Review

Please circle the correct answers to the following multiple-choice questions. You may, if you like, use PEARLS to help you find the correct answer. In all these exercises we assume that the fluids are not viscous.

1. Water is flowing in a pipe with varying cross sections. The pipe is totally filled with water. At the points where the diameter of the pipe is halved, the speed of the water is

   (a) divided by 4

   (b) halved

   (c) doubled

   (d) multiplied by 4.

2. Blood (disregard its viscosity), flows in an aorta of radius 9 mm at 30 cm/s. The volume flow rate is:

   (a) 0.076 liter/second

   (b) 0.019 liter/second

   (c) 7634 mm^3

3. Blood (disregard its viscosity), flows from the aorta to capillaries which have a much smaller radius. But there are many capillaries and their total cross section is larger than the aorta's. If blood flows at 30 cm/s through the aorta with radius 9 mm, and through the capillaries at 1 mm/s, the total cross section of the capillaries is:

   (a) 7.63×10^{-2} m^2.

   (b) 7634 mm^2

   (c) 1179 m^2

4. Water flows through garden hose of 3 cm diameter with a speed of 0.65 m/s. The nozzle of the hose is 0.30 cm. The water speed through the nozzle is:

   (a) 0.065 m/s

   (b) 6.5 m/s

(c) 65 m/s

5. Water flows through a horizontal garden hose of 2 cm diameter with a speed of 0.5 m/s. To fill a bucket with 10 liter water, it will take:

   (a) 1 minute and 4 seconds
   (b) 6 seconds
   (c) 15 minutes

6. Water is flowing in a pipe with varying cross sections. The pipe is completely filled with water. At the points where the cross section of the pipe doubles, the speed of the water flowing through it will be

   (a) divided by 4
   (b) halved
   (c) doubled
   (d) multiplied by 4

6 Viscous Flow

6.1 Introduction

In the assignment *Continuous Flow* we studied the flow of a non-viscous fluid. In this assignment we will look at the more realistic case of a viscous fluid. When you let the honey flow from your spoon, some portion sticks to the spoon. The fluids are not flowing steadily anymore, some of the fluid lacks behind. This is a quite complicated problem that, in many cases, can only be solved empirically.

6.2 Theory

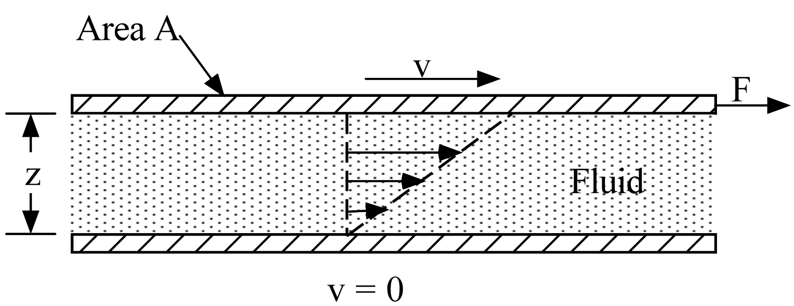


Figure 5: Viscous drag between two parallel plates

A viscous fluid does stick to the wall of the pipe through which it is flowing. The pressure in the pipe will not stay constant as in *Continuous Flow* but it will drop. If we don't push the fluid through the pipe, the fluid will finally stop flowing. This is so because one layer of fluid will drag on the next layer of fluid. (Think of the fluid as divided in layers, flowing over each other: we call this laminar flow).

The volume flow rate, the volume of fluid that passes through the pipe per unit of time, depends on that pressure drop. If the pressure drop is too high, no fluid will flow anymore. The more resistance the fluid offers to the motion, the more the pressure will drop, or

$$\Delta p = I_v R \tag{17}$$

with $\Delta p = p_1 - p_2$ with p_1, p_2 being the pressure at the beginning and the end of the pipe, respectively. I_v is the volume flow rate and R is the resistance to flow.

To determine the drag force on the fluid we think of it as between two parallel plates with an area A and separated by a distance z. The force needed to move the top layer at a speed v is proportional to that speed v, because the drag force (the friction from one layer of fluid on the next) is proportional to the speed. The force is also proportional to the area A. The force F is given by:

$$F = \eta \frac{vA}{z} \tag{18}$$

The proportionality constant η is called the *viscosity*. Its units are Pascal$\times$second (Pa.s). Fluids that behave this way are called Newtonian fluids. Not all fluids show this proportionality between applied force and velocity.

The flow speed v is maximum in the center of the pipe. At a distance r from the axis of a cylindrical pipe with radius R the speed v is smaller than in the center:

$$v = \frac{\Delta p}{4\eta L}(R^2 - r^2) \tag{19}$$

with $\Delta p = p_1 - p_2$. $\Delta p/L$ is called the pressure gradient: the pressure change over the length of the pipe.

The volume flow rate can be calculated from this speed distribution and is:

$$\frac{dV}{dt} = \frac{\pi R^4}{8\eta}\frac{\Delta p}{L} \tag{20}$$

Equation (20) is called *Poiseuille's Law*. This law is valid for laminar flow of a fluid with constant viscosity (that is a viscosity independent of the velocity). By laminar we mean a viscous flow that is not turbulent: the velocity diminishes gradually from the center of the tube towards the sides.

If the velocity of the fluid becomes sufficiently great the flow will not be laminar anymore. The velocity above which the flow will become turbulent depend on the density ρ and the viscosity η of the fluid and of the radius R of the tube. The flow of a fluid can be characterized by a dimensionless number, *Reynolds number*, N_R given by:

$$N_R = \frac{2R\rho v}{\eta} \tag{21}$$

Experiments have shown that the flow is laminar if the Reynold's number is less than 2000.

6.3 Laboratory

This laboratory examines the case of a laminar flow of a fluid through a pipe of uniform circular cross section. A limitation of the lab is that the flow wil remain laminar even for high Reynold's numbers. The display will tell you when the flow might not be laminar and the laws described in the theory section are not valid.

6.3.1 Experiment A: Introduction and Setup

In this experiment, we will familiarize ourselves with the laboratory.

1. In the PEARLS window, select *Fluid Mechanics*.
2. Now launch the *Viscous Flow* item.
3. You will now see the experiment window, with a control window at the left and a laboratory display at the right.
4. Adjust the window size as desired.
5. Move the laboratory display to a convenient location on your screen by dragging the display with your mouse.
6. You should see a pipe in which a viscous fluid flows. The velocity vectors of different layers of the fluid are displayed, as are the values of Reynolds number and the cebnter velocity.
7. The default settings for this experiment are:

Pressure Gradient	0.500 N/m^3
Density	9.980×10^2 kg/m^3
Viscosity	1.000×10^{-3} Ns/m^2
Radius	1.000×10^{-2} m
Zoom	8.000×10^2 pixels/meter
Time Zoom	1.000

8. From *Display* choose: *Velocity Distribution.*
9. Run the experiment.
   (a) Is the fluid flowing faster in the middle of the tube than at the sides? Concentrate on the velocity distribution.
   (b) Do the particles in the fluid seem to follow flow lines?
10. Set *Pressure Gradient* equal to zero.
    (a) Is the fluid flowing?
    (b) What is the value of the *Center Velocity*?
    (c) What is the value of the *Reynold's Number*?
11. Set *Pressure Gradient* equal to a negative value.

(a) Which way is the fluid flowing now?

12. Restore the default values: click $\hookleftarrow$.

13. Run the simulation.

14. Slide *Viscosity* to higher values.

    (a) Does the fluid flow faster or slower at higher values of viscosity?

    (b) Did the *Center Velocity* change?

    (c) Did the *Reynolds number* change?

15. Stop and reset the simulation.

16. Set *Viscosity* equal to 0.5×10^{-3} Ns/m^2.

17. Slide the *Pressure Gradient* to higher values.

    (a) Does the fluid flow faster or slower at higher values of pressure gradient?

6.3.2 Experiment B: Volume Flow Rate and Pipe Radius

In this experiment we will keep the pressure gradient constant and see how the pipe radius affect the flow rate.

1. Start up the PEARLS *Viscous Flow* experiment, or if it is already running, restore the default conditions.

2. Set *Radius* equal to 0.5×10^{-2} m.

3. *Run* the simulation and observe. *Reset*.

    (a) What is the value of the *Center Velocity*?

    (b) Calculate the volume flow rate. Recall equation (20).

4. Set *Radius* equal to 1×10^{-2} m.

5. *Run* the simulation and observe. *Reset*.

    (a) Does the fluid flow faster or slower than in step 3a?

    (b) Compare the value of the *Center Velocity* with the value of step 3a.

    (c) Calculate the volume flow rate and compare with the value of step 3b.

6. Set *Radius* equal to 2×10^{-2} m.

    (a) Does the fluid flow faster or slower than in step 5a?

   (b) Compare the value of the *Center Velocity* with the value of step 3a and 5b.
   (c) Calculate the volume flow rate and compare with the value of step 3b and 5c.

6.3.3 Experiment C: Volume Flow Rate and Pressure Gradient

In this experiment we will keep the radius of the tube constant. We will apply different pressure gradients and study how they affect the volume flow rate.

1. Start up the PEARLS *Viscous Flow* experiment, or if it is already running, restore the startup conditions by clicking the $\hookleftarrow$ button.
   (a) What is the value of the *Pressure Gradient*?
   (b) What is the value of the *Center Velocity*?
   (c) Calculate the volume flow rate.
2. Set *Pressure Gradient* equal to 1 Newton/m^3.
3. *Run* the simulation and observe. *Reset*.
   (a) Compare the *Center Velocity* with step 1b?
   (b) Calculate the volume flow rate and compare to the value of step 1c.
4. Set *Pressure Gradient* equal to 2 Newton/m^3.
5. *Run* the simulation and observe. *Reset*.
   (a) Compare the *Center Velocity* with step 1b and 3a?
   (b) Calculate the volume flow rate and compare to the value of step 1c and 3b.

6.3.4 Experiment D: How to Keep the Volume Flow Rate constant in Clogged Pipes

In this experiment we will change the radius of the tube and study how we can keep the flow constant.

1. Start up the PEARLS *Viscous Flow* experiment, or if it is already running, restore the startup conditions by clicking the $\hookleftarrow$ button.
2. Set *Radius* equal to 2×10^{-2} m.
   (a) What is the value of the center velocity?
   (b) Calculate the volume flow rate.
3. Set *Radius* equal to 1×10^{-2} m/s.

(a) Compare the value of the *Center Velocity* with step 2a.

(b) Calculate the volume flow rate and compare to the value of step 2b.

(c) How do you have to change the pressure gradient to obtain the same value of *Center Velocity* as in step 2a? (Try adjusting the pressure gradient with the slider).

(d) Does the formula for the center velocity give you the same result?

4. Restore the default values.

5. Set *Radius* equal to 0.5×10^{-2} m.

   (a) What is the value of the *Center Velocity*?

   (b) How do you have to change the pressure gradient to obtain the same value of *Center Velocity* as in step 2a? (Try adjusting the pressure gradient with the slider. If the slider does not let you go far enough type in the number).

   (c) Does the formula for the center velocity give you the same result?

6.3.5 Experiment E: Viscosity

In this experiment we will change the viscosity of the fluid and study the influence on the center velocity of the fluid.

1. Start up the PEARLS *Viscous Flow* experiment, or if it is already running, restore the startup conditions by clicking the $\hookleftarrow$ button.

   (a) What is the value of the *Viscosity*?

   (b) What is the value of *Center Velocity*?

2. Set *Viscosity* equal to 2×10^{-3} Ns/m^2.

   (a) Compare the value of *Center Velocity* with step 1b.

3. Set *Viscosity* equal to 4×10^{-3} Ns/m^2.

   (a) Compare the value of *Center Velocity* with step 1b and step 2a.

4. Knowing the radius of the pipe, measuring the pressure drop and the center velocity, we can determine the viscosity of the fluid. Recall equation (19). Calculate the viscosity and compare with the value in the control window.

6.3.6 Experiment F: Reynolds Number

In this experiment we will change the presuure gradient of a certain fluid and study how it affects the Reynolds number.

1. Start up the PEARLS *Viscous Flow* experiment, or if it is already running, restore the startup conditions by clicking the ↩ button.
2. Set *Pressure Gradient* equal to 0.5 N/m^3.
   (a) What is the value of the *Center Velocity*?
   (b) What is the value of the Reynolds Number?
3. Set *Pressure Gradient* equal to 4 dyne/cm^3.
   (a) Compare the *Center Velocity* with step 2a?
   (b) What is the value of the Reynolds Number? Compare to the value of step 2b.
4. Set *Pressure Gradient* equal to 8 dyne/cm^3.
   (a) Compare the *Center Velocity* with step 2a and 3a?
   (b) What is the value of the Reynolds Number? Compare to the value of step 2b and 3b.
5. Set the viscosity equal to 0.5×10^{-3} Ns/m^2.
   (a) What is the value of Reynolds number? Is the flow still laminar?.

6.4 Review

Please circle the correct answers to the following multiple-choice questions. You may, if you like, use PEARLS to help you find the correct answer.

1. The nurse changes a hypodermic needle for one with a radius half the first one. To inject at the same volume rate the nurse will have to press
   (a) half as hard
   (b) twice as hard
   (c) four times as hard
   (d) eight times as hard
   (e) sixteen times as hard
2. The water pipes are clogged to half their normal radius. The velocity of the water in the center of the pipe is
   (a) 1/16 of the original velocity

(b) 1/8 of the original velocity

(c) 1/4 of the original velocity

(d) 1/2 of the original velocity

3. Water of 60°C has a viscosity of 0.469 centipoise. Water of 20°C has a viscosity of 1.005 centipoise. (1 centipoise $= 10^{-2}$ poise $= 10^{-3}$ N.s/m^2.) In a similar pipe under the same pressure the volume flow rate for warm water is

   (a) 0.22 times the flow rate of cold water

   (b) 0.47 times the flow rate of cold water

   (c) 2.14 times the flow rate of cold water

   (d) 4.59 times the flow rate of cold water

4. Water of 20°C (viscosity $= 1.005 \times 10^{-3}$ N.s/m^2) is flowing in a pipe with radius 20 cm. If the water velocity at the center is 2.6 m/s, the water speed at 10 cm from the center of the pipe is:

   (a) 0.65 m/s

   (b) 1.30 m/s

   (c) 1.95 m/s

5. In viscous flow, the pressure drop along a pipe is proportional to the flow rate.

   (a) True

   (b) False

6. A liquid with viscosity 4×10^{-3} N.s/m^2, is flowing at 30 cm/s through a pipe of radius 1.0 cm. Its density is 1060 kg/m^3. The Reynolds number for this fluid is:

   (a) 1590

   (b) 1.59×10^6

   (c) 1.59×10^7

7. To pass through 1 mm long capillary with a diameter of 7×10^{-6} m, with a pressure drop of 2.60×10^3 Pa, blood takes 1 second. The viscosity of blood calculated from this data is:

   (a) 3.98×10^{-3} Pa.s

   (b) 7.96×10^{-3} Pa.s

   (c) 15.92×10^{-3} Pa.s

8. To have the same volume of glycerin as water (glycerin's viscosity at 20° C is 1000 times the viscosity of water) at the end of an equivalent pipe in the same amount of time, you will have to apply a pressure that

   (a) is 1/1000 times the pressure on the water pipe.

   (b) is 1000 times the pressure on the water pipe.

   (c) can be more than 1000 times the pressure on the water pipe depending on the length of the pipe.

9. Laminar flow implies that the velocity of the fluid gradually diminishes from the middle of the pipe towards the sides.

   (a) True

   (b) False

10. The velocity at which the flow of a viscous fluid through a pipe changes from laminar to turbulent does depend on:

    (a) the viscosity of the fluid only

    (b) the radius of the pipe only

    (c) the viscosity and the radius only

    (d) neither of the above

Part IX
Answers

Answers: Buoyancy

4.3.1 9a The ball is floating.

9b 2.85 cm

9c 0.8 g/cm^3

9d 1 g/cm^3

12a No, the ball sank.

12b The draft became its maximum value. The ball is at the bottom of the container. The draft is 6.34 cm.

16a No.

16b The ball is floating, half submerged.

4.3.2 4a $\rho_f = 1$ g/cm^3, $\rho = 1/2\rho_f$

4b Half the sphere is submerged.

4c $V' = 16.8$ cm^3

7a $\rho = 1/4\rho_f$

7b 1/4 of the volume is submerged.

7c $V' = 8.4$ cm^3.

10a $\rho = 3/4\rho_f$

10b 3/4 of the volume is submerged.

10c $V' = 25$ cm^3.

12a $\rho = \rho_f$

12b The sphere is totally immerged.

12c $V' = 33.5$ cm^3.

12d The ball is floating.

4.3.3 6a $\rho_f = \rho$

6b The ball floats.

6c All of it.

6d $V' = 4.2$ cm^3.

9a $\rho_f = 1/2\rho$

9b The ball is sinking

9c All of it.

9d $V' = 4.2$ cm^3.

12a $\rho_f = 2\rho$

12b The ball floats

12c Half the volume is submerged.

12d $V' = 2.1$ cm^3.

4.3.4 3a $V = 33.51 \times 10^{-6}$ m^3.

5a $W_{ball} = \rho g V$, $W_{ball} = 33.51 \times 0.5 \times 9.81 \times 10^{-3}$ N or $W_{ball} = 1.64 \times 10^{-1}$ N.

5b Half the ball is submerged. $V_{sub} = 1/2V = 16.76 \times 10^{-6}$ m^3.

5c $B = 1.64 \times 10^{-1}$ N $= W_{ball}$. The buoyant force is equal to the weight of the ball: the ball is floating.

5d $W_{fluid} = 1/2V \times 1 \times 9.81 \times 10^{-3}$ N $= 1.64 \times 10^{-1}$ N = B. The weight of the displaced fluid is equal to the buoyant force.

8a $W_{ball} = 2.47 \times 10^{-1}$ N.

8b 3/4 of the ball is submerged. $V_{sub} = 3/4V = 25.13 \times 10^{-6}$ m^3.

8c $B = W_{ball}$. The buoyant force is equal to the weight of the ball: the ball is floating.

8d $W_{fluid} = 3/4V \times 1 \times 9.81 \times 10^{-3}$ N $= B$. The weight of the displaced fluid is equal to the buoyant force.

11a $W_{ball} = 3.29 \times 10^{-1}$ N.

11b The ball is totally submerged. $V_{fluid} = V_{ball}$

11c $B = W_{ball}$. The buoyant force is equal to the weight of the ball: the ball is floating.

11d $Wfluid = B = W_{ball}$. The weight of the displaced fluid is equal to the buoyant force.

14a $W_{ball} = 6.57 \times 10^{-1}$ N.

14b The ball sank totally. $V_{fluid} = V_{ball}$.

14c $B = 3.29 \times 10^{-1}$ N. $B < W_{ball}$. The buoyant force is smaller than the weight of the ball: the ball sank.

14d $W_{fluid} = 3.29 \times 10^{-1}$ N. $W_{fluid} = B$. The weight of the displaced fluid is equal to the buoyant force.

Answers: Continuous Flow

5.3.1 9a Yes, it became larger.

9b It became smaller: $v_1 = 3.195 \times 10^{-3}$ m/s

9c No. $v_2 = 5.400 \times 10^{-3}$ m/s

10a Yes, v_1 became larger.

5.3.2 2a $A_1 = 28.3 \times 10^{-4}$ m^2.

2b $A_2 = 314.1 \times 10^{-4}$ m^2.

2c $A_1 v_1 = 1.696 \times 10^{-4}$ m^3/s.

2d $A_2 v_2 = 1.696 \times 10^{-4}$ m^3/s, the volume flowrate is constant.

3a Slower, the velocity vector is shorter.

3b $v_1 = 1.5 \times 10^{-2}$ m/s.

3c $A_1 v_1 = 1.131 \times 10^{-2} \times 1.5 \times 10^{-2}$ m^3/s $= 1.696 \times 10^{-4}$ m^3/s.

3d $A_2 v_2 = 1.696 \times 10^{-4}$ m^3/s, the volume flowrate is constant.

4a Slower.

4b $v_1 = 5.4 \times 10^{-3}$ m/s, $v_1 = v_2$

4c $A_1 v_1 = 1.696 \times 10^{-4}$ m^3/s.

4d $A_2 v_2 = 1.696 \times 10^{-4}$ m^3/s, the volume flowrate is constant.

5a Faster.

5b $v_1 = 2.16 \times 10^{-2}$ m/s. $v_1 < v_2$.

5c $A_1 v_1 = 1.696 \times 10^{-4}$ m^3/s

5d $A_2 v_2 = 1.696 \times 10^{-4}$ m^3/s, the volume flowrate is constant.

Answers: Viscous Flow

6.3.1

9a Yes.

9b Yes, this is a laminar flow.

10a No, without pressure a viscous fluid cannot flow.

10b The center velocity is of course zero, because the fluid is not flowing.

10c Reynold's Number is zero.

11a The fluid is flowing in the other direction than before.

14a The fluid flows slower at higher values of viscosity. The velocity vectors are shorter.

14b Yes, the meter shows a smaller value.

14c Yes, the Reynolds number became smaller.

17a The fluid flows faster at higher values of the pressure gradient.

6.3.2

3a 3.13×10^{-3} m/s.

3b The volume flow rate is 2.45×10^{-7} m^3/s.

5a The fluid flows a lot faster.

5b 1.25×10^{-2} m/s. The fluid is flowing four times faster.

5c The volume flow rate is 3.927×10^{-6} m^3/s. Per second sixteen times more volume flows through this pipe than through the one of step 3b.

6a The fluid flows a lot faster.

6b 5×10^{-2} m/s. The fluid is flowing four times faster.

6c The volume flow rate is 6.28×10^{-5} m^3/s. Per second sixteen times more volume flows through this pipe than through the one of step 5c.

6.3.3 1a $\Delta p/L = 5 \times 10^{-1}$ N/m^3.

1b $v = 1.25 \times 10^{-2}$ m/s.

1c The volume flow rate is 1.96×10^{-6} m^3/s.

3a $v = 2.50 \times 10^{-2}$ m/s, twice the velocity in step 1b.

3b The volume flow rate is 3.93×10^{-6} m^3/s, twice the volume flow rate in step 1c.

5a The center velocity is 5×10^{-2} m/s.

5b The volume flow rate is 7.86×10^{-6} m^3/s, twice the volume flow rate of step 3b and four times the one of step 1c.

6.3.4 2a $v = 5 \times 10^{-2}$ m/s.

2b The volume flow rate is 3.14×10^{-5} m^3/s.

3a $v = 1.25 \times 10^{-2}$ m/s, four times smaller than in step 2a.

3b The volume flow rate is 1.96×10^{-6} m^3/s, 16 times smaller than the rate in step 2b.

3c The pressure gradient has to be 4 times as high, or $\Delta p/L = 2$ N/m^3.

3d Yes.

5a $v = 3.13 \times 10^{-3}$ m/s.

5b The pressure gradient has to be 4 times as high as in step 3c and 16 times as high as in step 2a, or $\Delta p/L = 8$ N/m^3.

5c Yes.

6.3.5 1a $\eta = 1 \times 10^{-3}$ Ns/m^2.

1b $v = 1.25 \times 10^{-2}$ m/s.

2a $v = 6.25 \times 10^{-3}$ m/s, half the velocity in step 1b.

3a $v = 3.13 \times 10^{-3}$ m/s, half the velocity in step 2a.

4 $\eta = 4 \times 10^{-3}$ Ns/m^2.

6.3.6 2a $v = 1.25 \times 10^{-2}$ m/s.

2b $N_R = 250$

3a $v = 2.50 \times 10^{-2}$ m/s.

3b $N_R = 499$

4a $v = 5. \times 10^{-2}$ m/s.

4b $N_R = 998$

5a $N_R = 3990$, the flow could be turbulent.