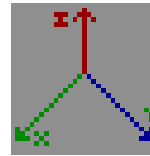
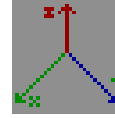


Part I
Coordinate Systems



1 Cartesian Coordinates



1.1 Introduction

Fundamental to the study of physics is the ability to locate a point in space. The Cartesian coordinate system is one way of doing this. It is mathematically the simplest coordinate system (later we will see more complicated systems), and it is frequently the most convenient system to use.

Many calculations in physics require us to choose in advance a coordinate system. The exact choice we make is arbitrary: there is an infinite number of ways to set up a coordinate system, all of which are correct so long as they are used consistently throughout the calculation. We mustn't change coordinate systems halfway through a problem if we wish to arrive at a correct answer.

Thus, the choice of coordinate system is dictated largely by convenience; certain choices can make a calculation much easier. An important factor here is the symmetry of the physical system.

1.2 Theory

1.2.1 Fundamentals

The space we live in is three-dimensional. This means that three numbers are required to specify the location of a point in space without ambiguity. Consider, for example, a cardboard box. If we want to specify the location of a point inside the box, we could do this as follows:

- Pick out one corner of the box. We will call this corner the origin.
- With a ruler, measure the distance from the point to each of the three sides of the box that meet at the origin. Be sure, when doing this, that the ruler meets the side of the box at a right angle (an angle of 90 degrees).
- Write down the three numbers on a sheet of paper, something like this:

(12.3, 5.4, 22.5)

If your ruler is marked with a centimeter (abbreviated cm) scale, then the point inside the box is 12.3 cm, 5.4 cm, and 22.5 cm from the three sides, respectively. These three numbers are called the Cartesian coordinates of the point.



The three sides that you used in your measurements above meet to form three lines, which intersect at the origin (the corner). These three lines are all perpendicular to one another. These lines are called the X , Y and Z axes of the coordinate system.

The three numbers you wrote down above may be represented by variables. So in general we refer to the three numbers as the x , y and z coordinates of the point. So we may say that the coordinates of the point are

$$(x, y, z)$$

where

$$x = 12.3 \text{ cm}$$

$$y = 5.4 \text{ cm}$$

$$z = 22.5 \text{ cm.}$$

Thus, when we think of a Cartesian coordinate system, our mind's eye should picture a point inside a box, with a ruler to measure distances.

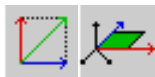
1.2.2 The Position Vector

The three coordinates of a point may be thought of as the components of the position vector of the point. We will call the position vector \vec{R} . Thus we may write

$$\vec{R} = (x, y, z) \quad (1)$$

The length of \vec{R} is denoted by R and is given by the Pythagorean Theorem:

$$R = \sqrt{(x^2 + y^2 + z^2)}. \quad (2)$$



We will discuss vectors in more detail in assignments *Vector Addition* and *Vector Cross Product*.

1.3 Laboratory

1.3.1 Experiment A: Customizing the Display

The purpose of this experiment is to learn how the lab display can be modified for different purposes.

1. In the PEARLS window, select *Coordinate Systems*.
2. Now launch the *Cartesian Coordinates* entry.
3. You will now see the laboratory window, with a Control Window at the left and a laboratory display at the right.
4. Adjust the window size as desired.

5. Move the laboratory display to a convenient location on your screen by dragging the display with your mouse.
6. The default settings for this experiment are:

<i>X Component</i>	$7.000 \times 10^1 \text{ m}$
<i>Y Component</i>	$1.100 \times 10^2 \text{ m}$
<i>Z Component</i>	$1.000 \times 10^2 \text{ m}$
<i>Zoom</i>	1 pixels/meter

7. Pull down the *Display Menu*. You will find the following options there:
 - Cartesian Components
 - Position Vector
 - Bounding Box
 - Point
 - Axes
 - Labels
8. Turn each of these on and off, and observe how the lab display is changed.
9. Use the *Rotation* control in the Control Window to look at the lab display from different perspectives.
10. Use the *Zoom* control in the Control Window to adjust the size of the lab display.
11. Observe the *Position Vector* as you change the coordinates of the point. It always goes from the origin to the point.

1.3.2 Experiment B: The X Coordinate

In this experiment, we will observe the result of varying the x coordinate of the point.

1. Start up the PEARLS *Cartesian Coordinates* experiment, or if it is already running, restore the default conditions by clicking the Defaults button (the button with the yellow arrow) or choose *Revert* from the *Simulation* menu.
2. Increase x with the slider of the X control. Continue until the maximum value is reached
 - (a) How does the position of the point change relative to the axes?
 - (b) Do y and z change as x changes?
3. Now change x to zero.



- (a) Where does the point go in relation to the axes?
- 4. Decrease x further, to make it negative.
 - (a) Where does the point go in relation to the axes?

1.3.3 Experiment C: The Y Coordinate

In this experiment, we will observe the result of varying the y coordinate of the point.

1. Start up the PEARLS *Cartesian Coordinates* experiment, or if it is already running, restore the default conditions by clicking the Defaults button (the button with the yellow arrow) or choose *Revert* from the *Simulation* menu.
2. Increase y with the slider of the Y control. Continue until the maximum value is reached
 - (a) How does the position of the point change relative to the axes?
 - (b) Do x and z change as y changes?
3. Now change y to zero.
 - (a) Where does the point go in relation to the axes?
4. Decrease y further, to make it negative.
 - (a) Where does the point go in relation to the axes?

1.3.4 Experiment D: The Z Coordinate

In this experiment, we will observe the result of varying the z coordinate of the point.

1. Start up the PEARLS *Cartesian Coordinate System* experiment, or if it is already running, restore the default conditions by clicking the Defaults button (the button with the yellow arrow) or choose *Revert* from the *Simulation* menu.
2. Increase z with the slider of the Z control. Continue until the maximum value is reached
 - (a) How does the position of the point change relative to the axes?
 - (b) Do x and y change as z changes?
3. Now change z to zero.
 - (a) Where does the point go in relation to the axes?
4. Decrease z further, to make it negative.
 - (a) Where does the point go in relation to the axes?

1.4 Review

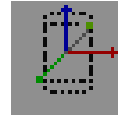
Please circle the correct answers to the following multiple-choice questions. You may, if you like, use PEARLS to help you find the correct answer.

1. Consider a point in space whose Y and Z coordinates are zero. Does this point lie on the X axis?
 - (a) yes
 - (b) no
2. How many numbers are required to specify the position of a point in space?
 - (a) one
 - (b) two
 - (c) three
3. In order for a point to be located at the origin (the intersection of the X , Y and Z axes) of a Cartesian coordinate system, what must be true?
 - (a) The X , Y , and Z coordinates must all be infinite
 - (b) The X , Y , and Z coordinates must all be zero
 - (c) The X and Z coordinates must be zero
4. What is the length of the position vector of a point whose coordinates are $(100, 0, 0)$?
 - (a) 0
 - (b) 10
 - (c) 100
 - (d) 10000
5. Consider the point $(0, 0, 45)$. Is this point located on the plane containing the X and Y axes?
 - (a) yes
 - (b) no
6. Consider the point $(0, 100, 0)$. What angle does the position vector make with the Y axis?
 - (a) 0°
 - (b) 10°
 - (c) 45°
 - (d) 90°



7. Consider the point $(0, 0, 100)$. What angle does the position vector make with the Y axis?
- (a) 0°
 - (b) 10°
 - (c) 45°
 - (d) 90°
8. Consider the point $(0, 100, 100)$. What angle does the position vector make with the Y axis?
- (a) 0°
 - (b) 10°
 - (c) 45°
 - (d) 90°
9. Do the position vectors $(100, 100, 0)$ and $(300, 300, 0)$ point in the same direction?
- (a) yes
 - (b) no
10. Do the position vectors $(50, 50, 0)$ and $(0, 50, 50)$ have the same length?
- (a) yes
 - (b) no

2 Cylindrical Coordinates



2.1 Introduction

In the previous assignment, we studied Cartesian coordinates. Cartesian coordinates are the most common and the simplest way to specify the location of a point in three dimensions.



There are times, however, when a different coordinate system should be used. For example, we do not usually specify a point near the Earth's surface with (x, y, z) coordinates. Instead, we use latitude, longitude and altitude. It isn't wrong to use x , y and z to specify positions on the Earth; it simply isn't convenient. A calculation can often be made much simpler by choosing a coordinate system that takes advantage of the symmetries in the problem.

Regardless what type of coordinate system we use, the number of coordinates that must be specified is three. We could use (x, y, z) or (latitude, longitude, altitude) or some other set of coordinates, but the number of coordinates is three. Furthermore, we can always write down equations that relate any two sets of coordinates to one another.

In this assignment we will be using cylindrical coordinates, which are a very good choice for problems with cylindrical symmetry. It would be natural to apply cylindrical coordinates to the study of water flow in a straight pipe, or to electrical current flow in a straight copper wire.

An important fact about cylindrical coordinates is that they are curvilinear. This means that, as we change the value of one of the coordinates (the angle) while holding the other two coordinates constant, a curve is traced out in space. This is quite different from Cartesian coordinates, where increasing, say, the x coordinate causes a straight line to be traced out in space.

2.2 Theory

Recall that, in the study of Cartesian coordinates, we represented a point in space by the set of numbers

$$(x, y, z).$$

In cylindrical coordinates, we replace this set with a new set,

$$(r, \theta, z).$$

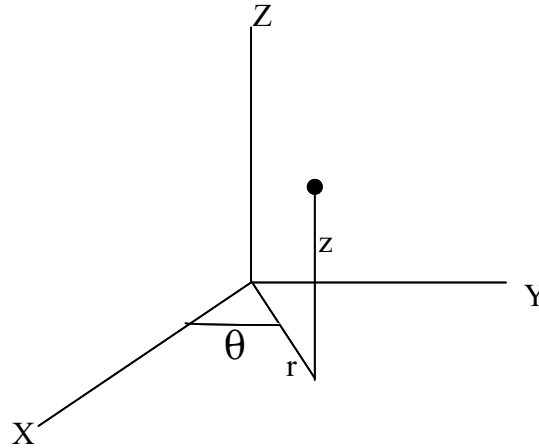


Figure 1: Cylindrical Coordinates

θ is the Greek letter theta.

In this system, r is the perpendicular distance from the point to the z axis (the length of the radius); z is the distance along the z axis, just as in Cartesian coordinates; and θ is the angle formed by the x axis and the radius to the point.

The theory of cylindrical coordinates consists of just three equations, which relate the cylindrical coordinates of a point to its Cartesian coordinates. Figure (1) illustrates the geometry.

The x and y coordinates of the point in the diagram can be obtained by first projecting the position vector onto the $x - y$ plane, creating a radius of a circle. The z component is the same in both systems. This procedure results in the equations

$$x = r \cos \theta \quad (3)$$

$$y = r \sin \theta \quad (4)$$

$$z = z \quad (5)$$

In some cases, the z coordinate does not enter into the calculation we wish to do. Then we need only equations 3 and 4, and we call r and θ the polar coordinates of the point.

The above equations show that, if we know the cylindrical coordinates (r, θ, z) of a point, we can calculate its Cartesian coordinates (x, y, z) , and vice versa.

2.3 Laboratory

2.3.1 Experiment A: Customizing the Display

The purpose of this experiment is to learn how the lab display can be modified for different purposes.

1. In the PEARLS window, select *Coordinate Systems*.
2. Now launch the *Cylindrical Coordinates* entry.
3. You will now see the laboratory window, with a Control Window at the left and a laboratory display at the right.
4. Adjust the window size as desired.
5. Move the laboratory display to a convenient location on your screen by dragging the display with your mouse.
6. The default settings for this experiment are:

<i>Radius</i>	1.304×10^2 m
<i>Theta</i>	5.753×10^1 degrees
<i>Z Component</i>	1.000×10^2 m
<i>X Component</i>	7.000×10^1 m
<i>Y Component</i>	1.100×10^2 m
<i>Z Component</i>	1.000×10^2 m
<i>Zoom</i>	1 pixels/meter

7. Pull down the Display Menu. You will find the following options there:
 - Cartesian Components
 - Cylindrical Coordinates
 - Position Vector
 - Cylinder
 - Point
 - Labels
 - Axes
8. Turn each of these on and off, and observe how the lab display is changed.
9. Use the Rotation Control at the bottom of the Control Window to look at the lab display from different perspectives.



2.3.2 Experiment B: The Radius

In this experiment, we will examine how things change when the radius r is adjusted.

1. Start up the PEARLS *Cylindrical Coordinates* experiment, or if it is already running, restore the default conditions by clicking the Defaults button (the button with the yellow arrow) or choose *Revert* from the *Simulation* menu.
2. Increase the radius r with the slider of the *Radius* control.
 - (a) How does the position of the point change relative to the axes?
 - (b) Does the z coordinate change as the radius changes?
 - (c) Do the x and y coordinates change as the radius changes?
 - (d) Now change the radius to zero.
 - (a) Where does the point go in relation to the axes?

2.3.3 Experiment C: The Z Coordinate

In this experiment, we will observe the result of varying the z coordinate of the point.

1. Start up the PEARLS *Cylindrical Coordinates* experiment, or if it is already running, restore the default conditions by clicking the Defaults button (the button with the yellow arrow) or choose *Revert* from the *Simulation* menu.
2. Increase Z with the slider of the Z control.
 - (a) How does the position of the point change relative to the axes?
 - (b) Does the radius change as Z changes?
 - (c) Do X and Y change as Z changes?
3. Now change Z to zero.
 - (a) Where does the point go in relation to the axes?
4. Decrease Z further, to make it negative.
 - (a) Where does the point go in relation to the axes?

2.3.4 Experiment D: The Angle Coordinate

In this experiment, we will observe the result of varying the theta coordinate of the point.

1. Start up the PEARLS *Cylindrical Coordinates* experiment, or if it is already running, restore the default conditions by clicking the Defaults button (the button with the yellow arrow) or choose *Revert* from the *Simulation* menu.
2. Change the angle theta θ with the slider of the *Theta* control.
 - (a) How does the position of the point change relative to the axes?
 - (b) Does the point trace out a straight line or a curve as theta changes?
 - (c) Do X and Y change as θ changes?
 - (d) Does Z change as θ changes?

2.3.5 Experiment E: Checking the Numbers

In this experiment, we will observe the result of varying the z coordinate of the point.

1. Start up the PEARLS *Cylindrical Coordinates* experiment, or if it is already running, restore the default conditions by clicking the Defaults button (the button with the yellow arrow) or choose *Revert* from the *Simulation* menu.
2. Set the radius r to 76.2 meters.
3. Set θ (theta) to 30 degrees.
4. Set Z to 60 meters.
5. Write down the Cartesian coordinates of the point, that is, the values of x , y and z .
6. Recall the equations (3), (4), (5).
7. With your calculator, compute $r \cos \theta$. Write down the value.
 - (a) Does your value agree with the one given by the X control?
8. With your calculator, compute $r \sin \theta$. Write down the value.
 - (a) Does your value agree with the one given by the Y control?
9. Repeat this check with another set of values for r , θ , and z .



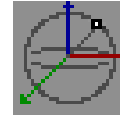
2.4 Review

Please circle the correct answers to the following multiple-choice questions. You may, if you like, use PEARLS to help you find the correct answer.

1. Consider a point in space whose r coordinate is zero. Does this point always lie on the Z axis?
 - (a) yes
 - (b) no
2. How many numbers are required to specify the position of a point in space in cylindrical coordinates?
 - (a) one
 - (b) two
 - (c) three
3. In order for a point to be located at the origin (the intersection of the X , Y , and Z axes) of a cylindrical coordinate system, what must be true?
 - (a) The angle θ must be zero
 - (b) z must equal infinity
 - (c) The z and r coordinates must be zero
4. What is the length of the position vector of a point whose z coordinate is 0 and whose r coordinate is 100?
 - (a) 0
 - (b) 10
 - (c) 100
 - (d) 10000
5. What is the angular coordinate (θ) of the point whose Cartesian coordinates are $x = 140$, $y = 140$, $z = 0$?
 - (a) 0 degrees
 - (b) 45 degrees
 - (c) 90 degrees
6. What is the angular coordinate (θ) of the point whose Cartesian coordinates are $x = 140$, $y = 0$, $z = 0$?
 - (a) 0 degrees
 - (b) 45 degrees

- (c) 90 degrees
7. Consider two points whose r and z coordinates are the same, but whose angular coordinates differ by 180 degrees. How do the directions of their position vectors compare?
 - (a) they point in the same direction
 - (b) they point in opposite directions
 8. If the x coordinate of a point is varied, keeping y and z constant, does the point trace out a curve or a straight line?
 - (a) straight line
 - (b) curve
 9. If the θ coordinate of a point is varied, keeping r and z constant, does the point trace out a curve or a straight line?
 - (a) straight line
 - (b) curve
 10. What is the x coordinate of a point with $\theta = 90$ degrees?
 - (a) 0
 - (b) 10
 - (c) 100

3 Spherical Coordinates



3.1 Introduction

Thus far, our study of coordinate systems has included the Cartesian and cylindrical systems. We now extend our study to include spherical coordinates.

Recall from the earlier assignments that Cartesian coordinates are a convenient choice when we are interested in locating a point in a box-like geometry. Then we saw that cylindrical coordinates are useful when locating points in a cylindrical geometry, such as inside a straight pipe with circular cross section.

Spherical coordinates are most useful when the geometry under consideration is itself spherical, or nearly so. For example, the Earth's shape deviates only slightly from that of a sphere, and so spherical coordinates are applicable. In fact, the system of latitude, longitude and altitude used by navigators is closely related to the spherical coordinate system physicists use. Again we find that three numbers are required to specify a location, since we are in a three-dimensional world.

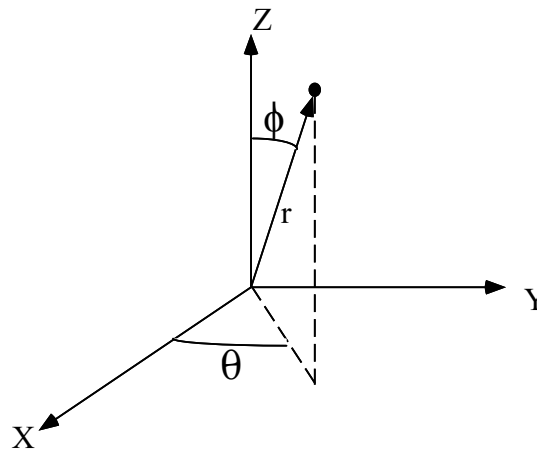


Figure 2: Spherical Coordinates

We will see that spherical coordinates are another example of a curvilinear system, like cylindrical coordinates. If one coordinate of a point is varied while



holding the other two coordinates fixed, a curve is traced out in the general case.

3.2 Theory

Recall that, in the study of Cartesian coordinates, we represented a point in space by the set of numbers

$$(x, y, z)$$

In spherical coordinates, we replace this set with a new set,

$$(r, \theta, \phi).$$

θ is the Greek letter theta, and ϕ is the Greek letter phi. θ and ϕ are angles, measured in radians or degrees, while r is a distance measured in centimeters or meters.

If we know the spherical coordinates (r, θ, ϕ) of a point, we can use the equations below to calculate the Cartesian coordinates (x, y, z) .

$$x = r \sin \phi \cos \theta \quad (6)$$

$$y = r \sin \phi \sin \theta \quad (7)$$

$$z = r \cos \phi \quad (8)$$

Note that the variable r occurs in both cylindrical and spherical coordinates, but with different meanings. In cylindrical coordinates r is the distance of a point from the z axis, while in spherical coordinates, r is the distance from the origin.

3.3 Laboratory

3.3.1 Experiment A: Customizing the Display

The purpose of this experiment is to learn how the lab display can be modified for different purposes.

1. In the PEARLS window, select *Coordinate Systems*.
2. Now launch the *Spherical Coordinates* entry.
3. You will now see the laboratory window, with a Control Window at the left and a laboratory display at the right.
4. Adjust the window size as desired.
5. Move the laboratory display to a convenient location on your screen by dragging the display with your mouse.



6. The default settings for this experiment are:

<i>Radius</i>	1.643×10^2 m
<i>Theta</i>	5.753×10^1 degrees
<i>Phi</i>	5.251×10^1 degrees
<i>X Component</i>	7.000×10^1 m
<i>Y Component</i>	1.100×10^2 m
<i>Z Component</i>	1.000×10^1 m
<i>Zoom</i>	1 pixels/meter

7. Pull down the Display Menu. You will find the following options there:

- Cartesian Components
- Spherical Coordinates
- Radius Vector
- Sphere
- Point
- Labels
- Axes

8. Turn each of these on and off, and observe how the lab display is changed.

9. Use the *Rotation* control at the bottom of the Control Window to look at the lab display from different perspectives.

10. Use the *Zoom* control to adjust the size of the lab display.

3.3.2 Experiment B: The Radius

In this experiment, we will examine how things change when the radius r is adjusted.

1. Start up the PEARLS Spherical Coordinate System experiment, or if it is already running, restore the default conditions by clicking the Defaults button (the button with the yellow arrow) or choose *Revert* from the *Simulation* menu.
2. Increase the radius r with the slider of the *Radius* control.
 - (a) How does the position of the point change relative to the axes?
 - (b) Do the x , y and z coordinates change as the radius changes?
3. Now change the radius r to zero.
 - (a) Where does the point go in relation to the axes?

3.3.3 Experiment C: The Theta Coordinate

In this experiment, we will observe the result of varying the θ coordinate of the point.

1. Start up the PEARLS *Spherical Coordinates* experiment, or if it is already running, restore the default conditions by clicking the Defaults button (the button with the yellow arrow) or choose *Revert* from the *Simulation* menu.
2. Increase θ with the slider of the *Theta* control.
 - (a) How does the position of the point change relative to the axes?
 - (b) Do x , y and z change as θ changes?
3. Now change θ to zero.
 - (a) Where does the point go in relation to the axes?
4. Decrease θ further, to make it negative.
 - (a) Where does the point go in relation to the axes?

3.3.4 Experiment D: The Phi Coordinate

In this experiment, we will observe the result of varying the ϕ coordinate of the point.

1. Start up the PEARLS *Spherical Coordinates* experiment, or if it is already running, restore the default conditions by clicking the Defaults button (the button with the yellow arrow) or choose *Revert* from the *Simulation* menu.
2. Increase ϕ with the slider of the *Phi* control.
 - (a) How does the position of the point change relative to the axes?
 - (b) Do x , y and z change as ϕ changes?
3. Now change ϕ to zero.
 - (a) Where does the point go in relation to the axes?

3.3.5 Experiment E: The Cartesian Coordinates

In this experiment, we examine the connection between the cylindrical and Cartesian coordinates of a point.

1. Start up the PEARLS *Spherical Coordinates* experiment, or if it is already running, restore the default conditions by clicking the Defaults button (the button with the yellow arrow) or choose *Revert* from the *Simulation* menu.



2. Change x to its maximum value of 200 meters.
3. Slowly adjust the x component to smaller values, until the minimum is reached.
 - (a) How did the radius behave as the adjustment was carried out?
 - (b) How did the angles θ , ϕ behave as the adjustment was carried out?
 - (c) Did the z coordinate change as x changed?
4. Repeat the above procedure for the y and z coordinates.

3.3.6 Experiment F: Checking the Numbers

In this experiment, we will observe the result of varying the z coordinate of the point.

1. Start up the PEARLS *Spherical Coordinates* experiment, or if it is already running, restore the default conditions by clicking the Defaults button (the button with the yellow arrow) or choose *Revert* from the *Simulation* menu.
2. Set the radius r to 145 meters.
3. Set θ (theta) to 32.4 degrees.
4. Set ϕ (phi) to 70.2 degrees.
5. Write down the Cartesian coordinates of the point, that is, the values of x , y and z .
6. Recall the equations (6), (7), (8).
7. With your calculator, compute $r \sin \phi \cos \theta$. Write down the value.
 - (a) Does your value agree with the one given by the X control?
8. With your calculator, compute $r \sin \phi \sin \theta$. Write down the value.
 - (a) Does your value agree with the one given by the Y control?
9. With your calculator, compute $r \cos \phi$. Write down the value.
 - (a) Does your value agree with the one given by the Z control?
10. Repeat this check with another set of values for r , θ , and z .

3.4 Review

Please circle the correct answers to the following multiple-choice questions. You may, if you like, use PEARLS to help you find the correct answer.

1. Consider a point in space whose r coordinate is zero. Does this point always lie at the origin?
 - (a) yes
 - (b) no
2. How many numbers are required to specify the position of a point in space in cylindrical coordinates?
 - (a) one
 - (b) two
 - (c) three
3. What is the x coordinate of a point whose ϕ (phi) coordinate is 180 degrees?
 - (a) 0
 - (b) infinity
 - (c) 100
4. What is the ϕ coordinate of a point whose z coordinate is zero?
 - (a) 0 degrees
 - (b) 45 degrees
 - (c) 90 degrees
 - (d) 135 degrees
5. Consider two points whose θ and r coordinates are the same, but whose ϕ coordinates differ by 180 degrees. How do the directions of their position vectors compare?
 - (a) they point in the same direction
 - (b) they point in opposite directions
6. If the x coordinate of a point is varied, keeping y and z constant, does the point trace out a curve or a straight line?
 - (a) straight line
 - (b) curve



7. If the θ (theta) coordinate of a point is varied, keeping r and ϕ constant, does the point trace out a curve or a straight line?
- (a) straight line
 - (b) curve
8. What is the x coordinate of a point with $\theta = 90$ degrees?
- (a) 0
 - (b) 10
 - (c) 100
9. What is the z coordinate of a point with $\phi = 90$ degrees?
- (a) 0
 - (b) 10
 - (c) 100
10. If a point's x and y coordinates are equal, what is its θ coordinate?
- (a) 0 degrees
 - (b) 45 degrees
 - (c) 90 degrees
 - (d) 135 degrees

Part IX
Answers

Answers: Cartesian Coordinates

- 1.3.2 2a The point moves parallel to the X axis, away from the origin.
 2b y and z remain fixed as x changes. We say that y and z are independent of x .
 3a The point is now on the $Y - Z$ plane.
 4a The point moves in a direction opposite to the arrow of the X axis.
- 1.3.3 2a The point moves parallel to the Y axis, away from the origin.
 2b x and z remain fixed as y changes.
 3a The point is now on the $X - Z$ plane.
 4a The point moves in a direction opposite to the arrow of the Y axis.
- 1.3.4 2a The point moves parallel to the Z - axis, away from the origin.
 2b x and y remain fixed as z changes.
 3a The point is now on the $X - Y$ plane.
 4a The point moves in a direction opposite to the arrow of the Z axis.

Answers: Cylindrical Coordinates

- 2.3.2 2a The point moves away from the Z axis and the origin.
 2b The z coordinate does not change. We say z is independent or r .
 2c x and y both change as r changes. We say that x and y depend on r .
 2a The point now lies on the Z axis. Its x and y coordinates are now zero.
- 2.3.3 2a The point moves away from the $X - Z$ plane. It moves parallel to the Z axis.
 2b The radius does not change as z changes.
 2c x and y do not change as z changes.
 3a The point is now on the $X - Y$ plane.
 4a The point moves in a direction opposite to the arrow of the Z axis.
- 2.3.4 2a The point moves around the Z axis.
 2b The point traces out a curve (a circle) as the angle is changed.
 2c Yes, x and y change as θ changes.
 2d No, z does not change as θ changes.
- 2.3.5 7a $x = r \cos \theta = 76.2 \cos 30^\circ = 66$ meters
 8a $y = r \sin \theta = 76.2 \sin 30^\circ = 38$ meters

Answers: Spherical Coordinates

- 3.3.2 2a The point moves away from the origin.
 2b Yes, the (x, y, z) coordinates change.
 3a The point is now located at the origin (the intersection of the x, y, z axes).
- 3.3.3 2a The point moves in a circle around the Z axis.
 2b x and y change, but z does not.
 3a The point is now located in the $X - Z$ plane.
 4a The point moves into the region where its y coordinate is zero or negative.
- 3.3.4 2a The point moves towards the $X - Y$ plane until $\phi = 90^\circ$, then passes through to the other side of the plane.
 2b $x, y,$ and z all change.
 3a The point is now located on the Z axis.
- 3.3.5 3a ϕ started out with a value of 66.3° . As x decreased, ϕ decreased to a minimum value of 47.7° , then increased back to its original value of 66.3° .
 3b The radius started out with a value of 249 meters. As x decreased, the radius decreased to a minimum value of 149 meters, then increased back to its original value of 249 meters. θ increased continuously from 28.8° to 151° .
 3c No, the z coordinate did not change.
- 3.3.6 5 The cartesian coordinates are $(115, 73.1, 49.1)$.
 7a $r \sin \phi \cos \theta = 145 \sin 70.2^\circ \cos 32.4^\circ = 115.2$. This agrees with the value of x to three significant figures.
 8a $r \sin \phi \sin \theta = 145 \sin 70.2^\circ \sin 32.4^\circ = 73.102$. This agrees with the value of y to four significant figures.
 9a $r \cos \phi = 145 \cos 70.2^\circ = 49.12$ This agrees with the value of z to three significant figures.