

## Special Topics in Mechanical Engineering

### Suggested Problems – First Exam

**Problem 1:** For the single degree of freedom system driven by a harmonic force we discussed in the class. The governing equation is given by

$$m\ddot{x} + c\dot{x} + kx = f_o \sin \omega t$$

Where  $\omega$  is the driving (excitation) frequency. Given the initial conditions are  $x(0) = x_o$  and  $\dot{x}(0) = v_o$ . Combine the homogeneous and particular solutions and satisfy the initial conditions to obtain the complete solution for  $x(t)$ . Do so for all damping cases ( $\zeta < 1, \zeta = 1, \zeta > 1$ ).

**Problem 2:** For the single degree of freedom system driven by a harmonic base motion we discussed in the class. The governing equation is given by

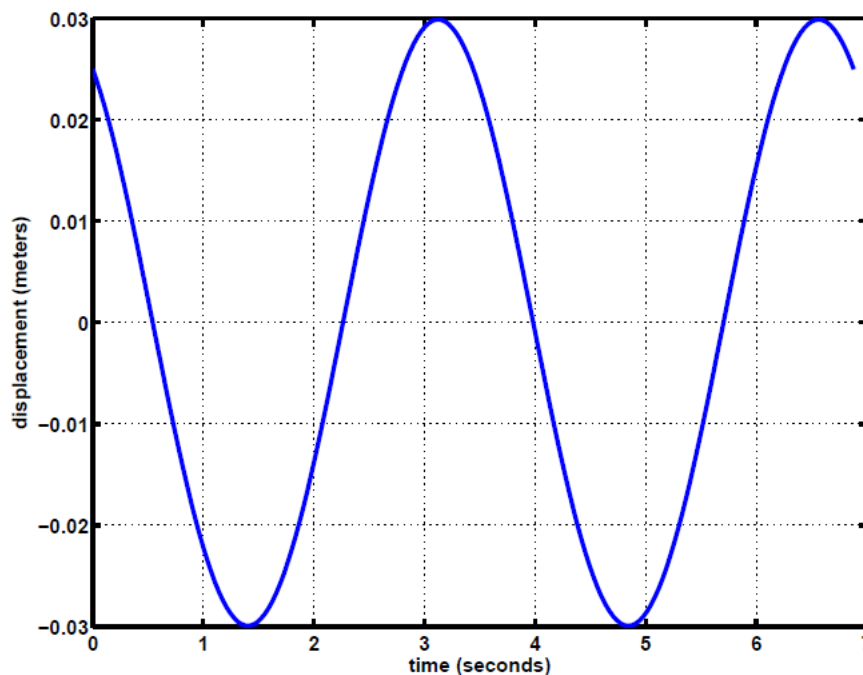
$$m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky$$

Where  $y(t) = Y \sin \omega t$  and  $\omega$  is the driving (excitation) frequency. Given the initial conditions are  $x(0) = x_o$  and  $\dot{x}(0) = v_o$ . Combine the homogeneous and particular solutions and satisfy the initial conditions to obtain the complete solution for  $x(t)$ . Do so for all damping cases ( $\zeta < 1, \zeta = 1, \zeta > 1$ ).

**Problem 3:** For the same problems above, if we used the complex numbers forcing form  $f(t) = f_o e^{i\omega t}$ . In class, we were able to show that  $x(t) = X e^{i\omega t}$  where  $X$  is a complex number that contains a magnitude and phase  $X = |X|e^{i\phi}$ . Use knowledge provided in the class to find the magnitude and phase for both excitations schemes of Problem 1 and Problem 2.

**Problem 4:** Solve Problems 4.20, 4.21 and 4.22 from textbook 2.

**Problem 5:** The response of a spring/mass system is shown below. Say as much as you can about the system.



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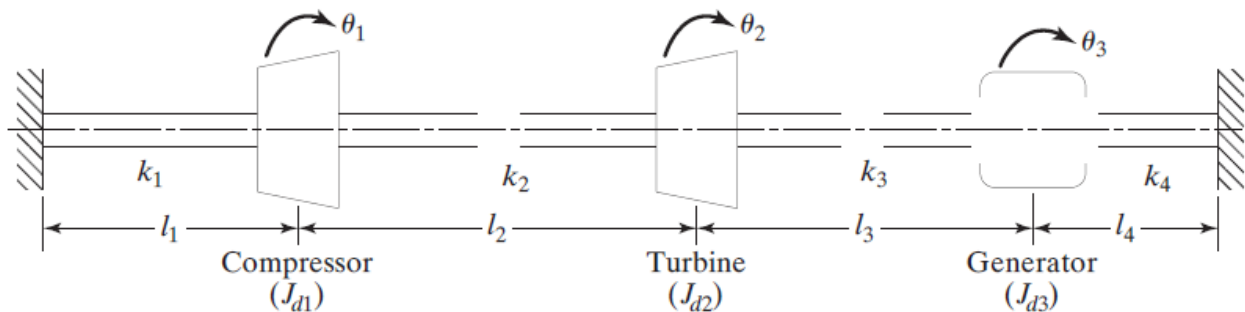
### Suggested Problems – Second Exam

**Problem 1 to 4:** Solve Problems 2.15 – 2.18 From Textbook 1.

**Problem 5:** The arrangement of the compressor, turbine, and generator in a thermal power plant is shown below. This arrangement can be considered as a torsional system where  $J_{di}$  denote the mass moments of inertia of the three components (compressor, turbine, and generator), and  $k_i$  represent the torsional spring constants of the shaft between the components. Derive the equations of motion of the system using Lagrange's equations by treating the angular displacements  $\theta_i$  of the components as generalized coordinates. Given that:

$$T = \frac{1}{2}J_{d1}\dot{\theta}_1^2 + \frac{1}{2}J_{d2}\dot{\theta}_2^2 + \frac{1}{2}J_{d3}\dot{\theta}_3^2$$

$$V = \frac{1}{2}k_1\theta_1^2 + \frac{1}{2}k_2(\theta_2 - \theta_1)^2 + \frac{1}{2}k_3(\theta_3 - \theta_2)^2 + \frac{1}{2}k_4\theta_3^2$$



**Problem 6:** For a multidegrees-of-freedom system expressed as

$$[m]\{\ddot{x}\} + [c]\{\dot{x}\} + [k]\{x\} = \{0\}$$

Where  $[m] = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$ ,  $[c] = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$ ,  $[k] = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}$ . The first and second natural frequencies of this system are  $\omega_1 = 3 \text{ rad/sec}$  and  $\omega_2 = 20 \text{ rad/sec}$  and the damping ratios are  $\zeta_1 = 0.1$  and  $\zeta_2 = 0.2$ . Given that  $m_1 = 5 \text{ kg}$ ,  $m_2 = 7 \text{ kg}$ ,  $k_1 = 80 \text{ N/m}$  and  $k_2 = 20 \text{ N/m}$ . Find the  $[c]$  matrix.

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### Suggested Problems – Final Exam

**Problem 1:** The cantilever beam is defined as a beam that is fixed at  $x = 0$  and completely free at  $x = L$ . For this sort of beam, find the general equation of the natural frequencies, mode shapes and the free vibration response. Use the derivation process used in the class.

**Problem 2:** The free-free beam is defined as a beam that is free at  $x = 0$  and  $x = L$ . For this sort of beam, find the general equation of the natural frequencies, mode shapes and the free vibration response. Use the derivation process used in the class.

**Problem 3:** The fixed-fixed beam is defined as a beam that is fixed at  $x = 0$  and  $x = L$ . For this sort of beam, find the general equation of the natural frequencies, mode shapes and the free vibration response. Use the derivation process used in the class.

**Problem 4:** Experimental Modal Analysis (EMA) is a well-known measurement technique. Say as much as you can about this technique including:

- What quantities are being measured using EMA?
- What are the basic concepts of EMA?
- What are the tools and equipment that are used in EMA?
- How can we find damping ratios?

**Note:** Problem 1 to 3. Write out by hand the equations and procedures used to solve these problems. You can use the equation of motion and boundary conditions provided in the class for the types above. Hint: Look at Sections 11.5.2, 11.5.3 and 11.5.5 of textbook 1 for more details.

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## Suggested Problems

### Chapter 11 (Textbook 1)

### Chapters 9 and 10 (Textbook 2)

#### Chapter 11 – Textbook 1

**Problem 1:** Follow the same procedure presented in the class to find the natural frequencies and mode shapes of beams with the following boundary conditions:

- Fixed at both ends (fixed-fixed).
- Free at both ends (free-free)
- Fixed at one end and pinned on the other (fixed-pinned).
- Fixed at one end and free on the other (cantilevered beam).

#### Chapter 09 – Textbook 2

**Review Questions:** 9.1 to 9.5.

#### Chapter 10 – Textbook 2

**Review Questions:** 10.1 to 10.5.

**Note:** For chapter 9 and 10 of textbook 2 questions, only answer the questions for the sections were studied in the class.