* Quick Review on Matrix operations

① Matrix Notation

$A_{mn} \Rightarrow$ $m \times n$ is the size of the matrix

$m \Rightarrow$ # of rows

$n \Rightarrow$ # of Column

$$A_{32} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$ 3 rows, 2 columns

If $m = 1 \Rightarrow A_{1n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \end{bmatrix}$, row vector

If $n = 1 \Rightarrow A_{m1} = \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}$ Column vector

If $m = n \Rightarrow$ Squared matrix $A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \\ a_{m1} & & a_{mn} \end{bmatrix}$

If $A_{mn} = A_{nm} \Rightarrow$ Symmetric matrix

$$\begin{bmatrix} 5 & 1 & 2 \\ 1 & 3 & -7 \\ 2 & -7 & 4 \end{bmatrix}$$

Diagonal matrix : Square matrix will all non-diagonal elements are $= 0$

$$\begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & \ddots & a_{nn} \end{bmatrix}$$

Identity matrix (I)

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & \vdots \\ \vdots & & \ddots & \\ 0 & \cdots & & 1 \end{bmatrix}$$

* Upper triangular Matrix [U] : Matrix will all elements below diagonal are zero

$$[U] = \begin{bmatrix} 6 & -3 & -9 \\ 0 & 7 & 3 \\ 0 & 0 & 2 \end{bmatrix}$$

* Lower triangular Matrix [L] : Matrix will all elements above diagonal are zero

$$[L] = \begin{bmatrix} 8 & 0 & 0 \\ 3 & -7 & 0 \\ 1 & 2 & 5 \end{bmatrix}$$

② Matrix Operation

- Addition / subtraction

$$[A] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$[C] = [A] \pm [B]$$

$$[B] = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$[C] = \begin{bmatrix} c_{11} & c_{12} \\ a_{21} & c_{22} \end{bmatrix} = \begin{bmatrix} a_{11} \pm b_{11} & a_{12} \pm b_{12} \\ a_{21} \pm b_{21} & a_{22} \pm b_{22} \end{bmatrix}$$

- Additions only are commutative

$$[A] + [B] = [B] + [A] \quad , \quad [A] - [B] \neq [B] - [A]$$

- Addition's only are associative

$$([A] + [B]) \pm [C] = [A] + ([B] \pm [C])$$

* Multiplication

- By a constant

$$[A] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \text{ constant } (C), \quad C[A] = [A]C = \begin{bmatrix} a_{11}C & a_{12}C \\ a_{21}C & a_{22}C \end{bmatrix}$$

- Product of two matrices

$$[C] = [A][B], \quad [C]_{m \times L} = [A]_{m \times n} [B]_{n \times L}$$

size of new matrix

$$[A] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad [B] = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$[C] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \Rightarrow \text{ row } \times \text{ column}$$

$$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

* $[A][B] \neq [B][A]$

* Matrix Determinant (D)

2x2

$$[A] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad D(A) = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$D = a_{11}a_{22} - a_{12}a_{21}$$

3x3 Matrix

$$[A] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = +a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$D = a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

* Matrix Inverse $[A]^{-1}$

$$[A][A]^{-1} = [A]^{-1}[A] = [I]$$

2x2 $\quad [A] = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow [A]^{-1} = \frac{1}{D} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$$= \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

* Transpose $[A]^T$

A_{mn}, $[A]^T = A_{nm}$

$[A] = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $[A]^T = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$, $\left[[A]^T\right]^T = [A]$

$[A] = \begin{bmatrix} a & b & c & d \end{bmatrix}$, $[A]^T = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$