# Torsion test

Objective

To determine the behavior of materials when subjected to torsion, and to obtain some of their mechanical properties.

Introduction

In many applications, such as axles, coil springs, and derives shafts; an engineering material must have good resistance to stresses induced by twisting (TORSION). The stress resulting from such torsion load can be determined by means of the torsion test. This test resembles the tension test in that a load deflection curve is also development (which is transformed to a shear-strain curve).

In a torsion test, a solid or hollow cylindrical specimen is twisted and the resultant deformation, measured as the angle through which the bar is twisted. The test then consists of measuring the angle of twist, Φ (rad) at selected increments of torque, T (N.m). Expressing Φ as the angular deflection curve per unit gage length, one is able to plot a T-Φ curve that is analogous to the load deflection curve of the torsion test.

To be useful for engineering purpose, its necessary to convert this T-Φ curve to the shear stress τ, and shear strain γ.
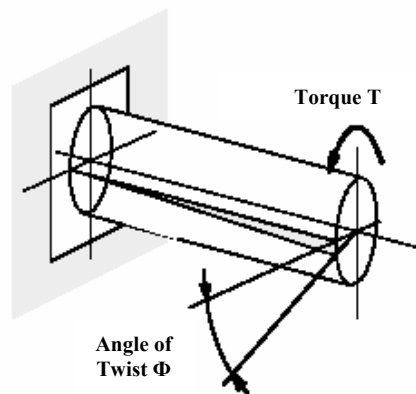
Theory

To obtain a relationship between the internal torque and the stresses it sets up in members with circular and tubular cross sections, it is important to make few assumptions:

1. A plane section of material perpendicular to the axis of a circular member remains plane after the torque is applied (note that this is not true for large deformations).
2. In a circular member subject to torque, shearing strains vary linearly from the central axis.
3. Shearing stress is proportional to shearing strain.

Consider a bar, or shaft, of circular cross-section, twisted by torque T acting at its ends (fig. 1a). A rotation at one end of the bar relative to the other end will occur. The rotation angle of the cross section, Φ is known as the angle of twist.

Also, there is a longitudinal distortion formed along the length of the shaft at angle, γ.

(a)   (b)
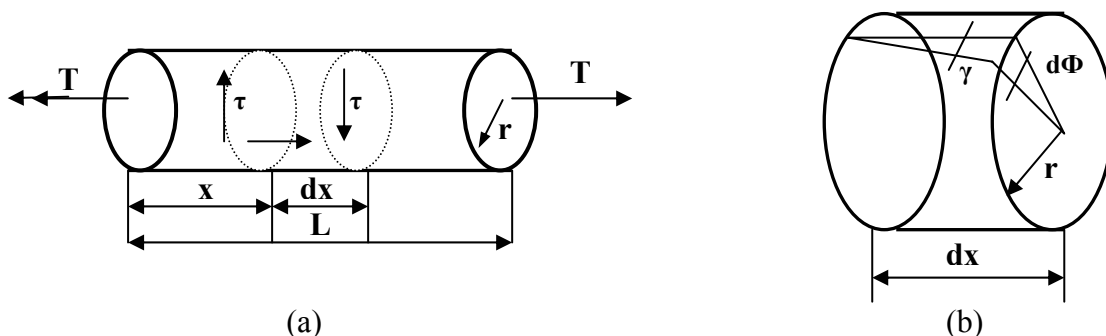
Fig 1: Circular bar in pure torsion

If we take a longitudinal section of length dx (fig. 1b), we find

$$\gamma = r\frac{d\phi}{dx} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(1)$$

In pure torsion, the rate of change $\dfrac{d\phi}{dx}$ is constant. This constant value is defined as θ, Where, $\theta = \dfrac{\phi}{l}$

then:

$$\gamma = r\theta = r\frac{\phi}{l} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots. (2)$$

For linear elastic material, the shear stresses τ in the bar is proportional to the shear strain γ by Hook's law in shear, that is:

$$\tau = G\gamma \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (3)$$

Where G is the modulus of elasticity (modulus of rigidity).

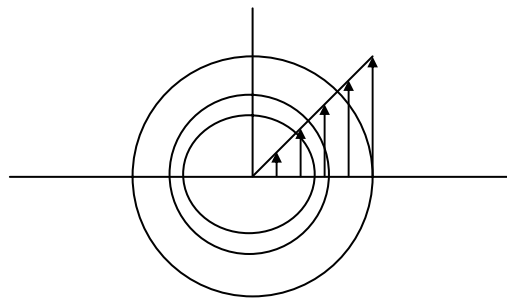Also the shear stress distribution is uniform across the section as shown in figure 2.

Fig. 2: Elastic shear stress distribution.

Considering the very thin circumferential ring shown, the torque resisted by this ring is given by:

$$dT = \tau\, dA \times a = 2\pi a^2 \tau\, da \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots. (4)$$

Since the distribution is linear, the shear stress τ at any radius a is related to the maximum shear stress τ_{max} at r thus; $\tau = \tau_{max}\dfrac{a}{r}$, substituting into equation 4 to give:

$$dT = 2\pi\frac{\tau_{max}}{r}a^3 da \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots. (5)$$

Integrating over the entire cross-sectional area, the total external torque is

$$T = \frac{2\pi}{r}\tau_{max}\int_0^r a^3 da = \frac{\pi}{2}\tau_{max} r^3 \quad \dots\dots\dots\dots\dots\dots\dots\dots (6)$$

Solving equation 6 for τ_{max}

$$\tau_{max} = \frac{2T}{\pi r^3} = \frac{T.r}{J} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (7)$$

Where J is the polar moment of inertia given by $\dfrac{1}{2}\pi r^4$ for solid circular shaft.

When the metal starts to deform plastically, the shear stress distribution is no longer linear. The torque at a very thin ring of radius a is again given by equation 2 so the external torque resisted across the section is then

$$T = 2\pi\int_0^a \tau a^2 da \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (8)$$

The shear strain γ at any radius is still valid, and given by $\gamma = \dfrac{a\phi}{L}$. Thus $a = \dfrac{L}{\phi}\gamma$, and $da = \dfrac{L}{\phi}d\gamma$

Substitute the last equation into equation 8

$$T = 2\pi \int_0^{\gamma_{max}} \tau \left(\frac{L\gamma}{\phi} \right)^2 \frac{L}{\phi} d\gamma \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (9)$$

The shear stress at any radius (a) is also a function of γ only, then:

$$T\phi^3 = 2\pi L^3 \int_0^{\gamma_{max}} f(\gamma) d\gamma \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (10)$$

Differentiating both sides of equation 8 with respect to Φ:

$$\frac{d}{d\phi} (T\phi^3) = 2\pi L^3 \tau \gamma_{max} \frac{d\gamma_{max}}{d\phi} \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (11)$$

But: $\dfrac{d\gamma_{max}}{d\phi} = \dfrac{r}{L}$. Substituting in equation 11 we get:

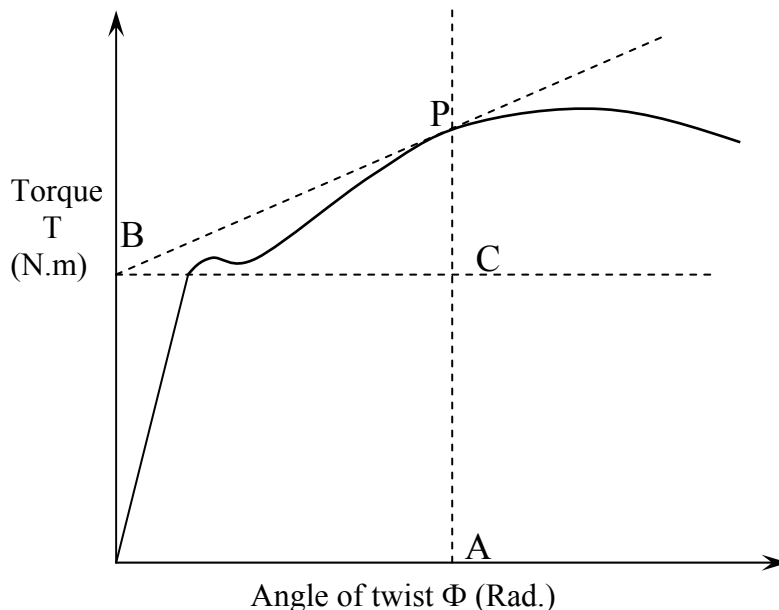$$3T + \phi \frac{dT}{d\phi} = 2\pi\tau r^3 \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (12)$$

Solving for the shear stress:

$$\tau = \frac{1}{2\pi r^3} \left(\phi \frac{dT}{d\phi} + 3T \right) \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (13)$$

Refer to figure 3. At the typical point P at which it is desired to obtain the shear stress, we observe that:

$$\phi = BC, \frac{dT}{d\phi} = \frac{PC}{BC}, T = AP \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (14)$$

Thus,

$$\tau = \frac{1}{2\pi r^3} \left(BC \frac{PC}{BC} + 3AP \right) = \frac{PC + 3AP}{2\pi r^3} \dots \dots \dots \dots \dots \dots \dots \dots (15)$$


Torque T (N.m)

Angle of twist Φ (Rad.)

(Fig.3 Determination of τ in the plastic range)

Once the shear stress – strain curve is obtained, we can easily evaluate several engineering properties:

- The *Modulus of Rigidity,* **(G)** is the slope of the $\tau - \gamma$ curve in the elastic range and is compatible to Young's Modulus found in the tension test.


Shear Stress

Slope = G

Shear Strain
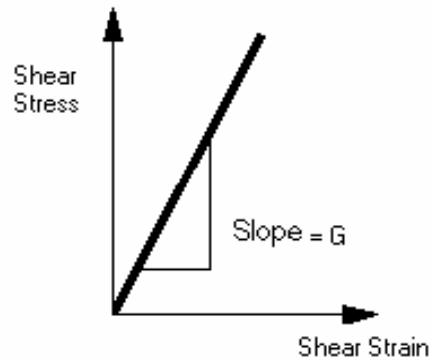
- The *Modulus of Resilience* is the area under the elastic portion of the $\tau - \gamma$ curve, which represents the energy absorbed by the material in the elastic region.

Shear-strain

Yield

Elatic

Modulus of resiliense=
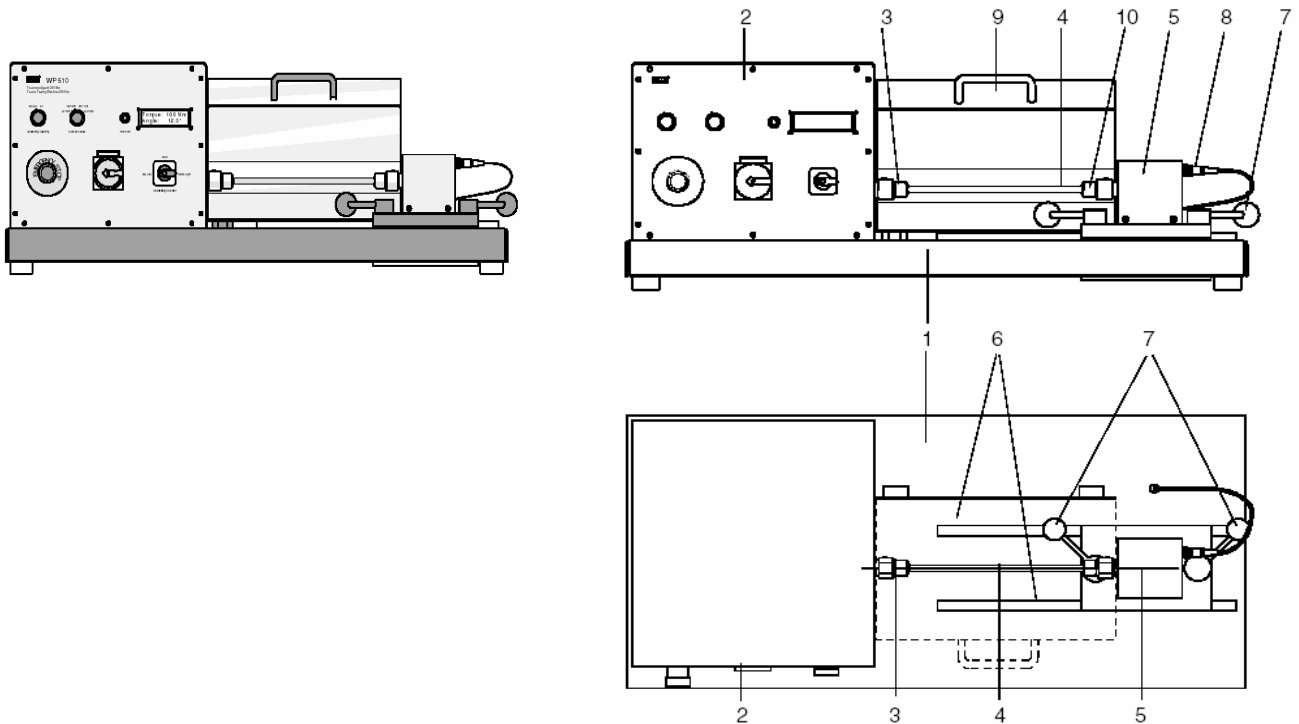0.5*(shear-strin)^2/(G)
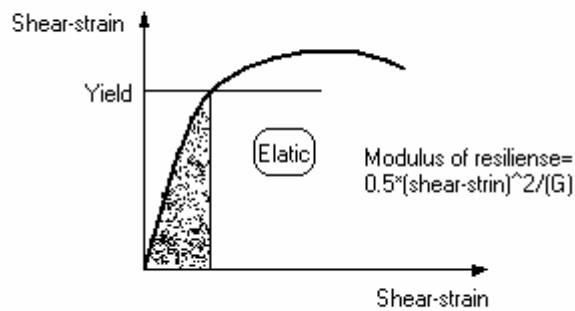
Shear-strain

The *Modulus of Ruptur* is the total area under the $\tau - \gamma$ curve, which represents the total energy absorbed by the material before fracture.

Shear-strain

$$Toughness = \int_{(\varepsilon = 0)}^{f}$$

Tough-2/3*(Max-shear-stress)*(Maxshear-strain)
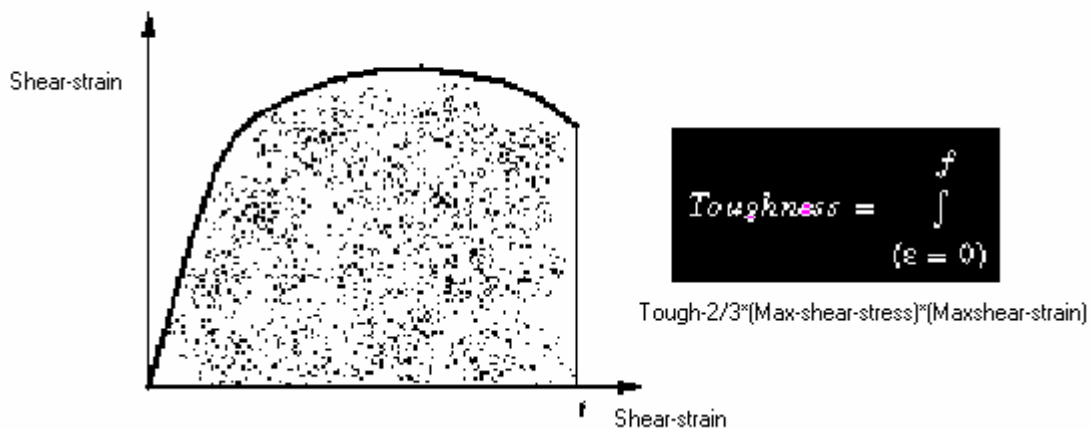
Shear-strain

- The yield shear stress, the ultimate shear stress, the fracture shear stress.
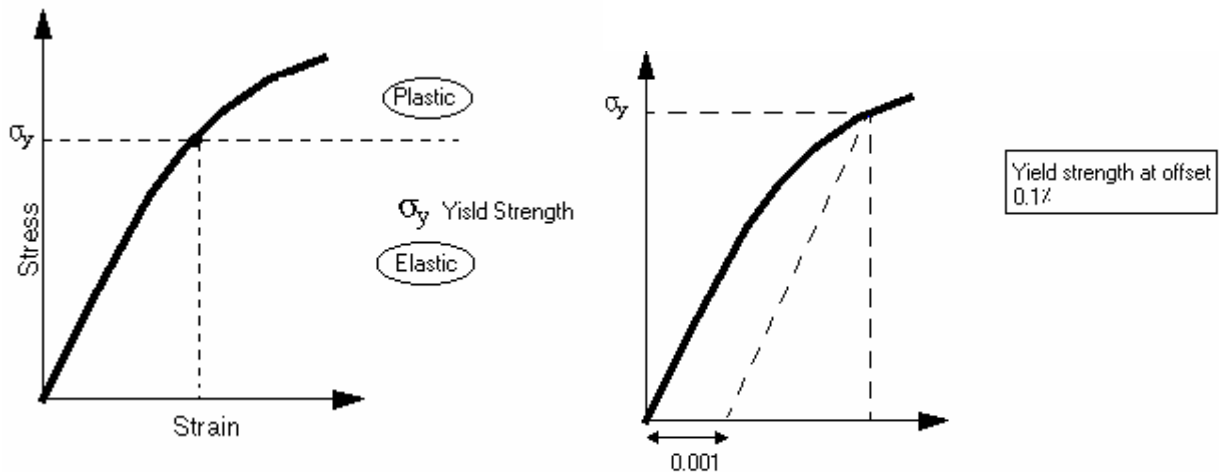

Apparatus:

The testing unit consists of the following components: base plate (1), drive unit (2) with geared motor to generate the testing moment. The moment is transferred to the testing rod (4) via a square drive (3/4") and standard socket spanners (3). The rotational angle sensor, the electronic system for the recording and display of the measured values, and the drive motor control are located in the drive unit housing. A frequency converter is used to adjust the speed of the drive motor. The other end of the testing rod (4) is fixed to the support (5) with torque measurement device. The support (5) can be shifted on guide rails (6) and can be braced with two clamp levers (7), to allow for the testing of samples of different lengths. The torque is measured using a metering shaft equipped with a strain gauge. The shaft has ball bearings on the sample side to avoid measuring errors resulting from friction. The electrical power is connected to the basic unit via a cable with a 5-pin plug (8). The base plate (1) is reinforced with box sections in order to ensure a high degree of torsion rigidity and low inherent distortion. This ensures that a high degree of precision during torsion measurement has been reached in conjunction with the high-resolution optoelectronic torsion sensor. A transparent protective hood (9) protects against flying fragments. This can occur when especially hard and brittle materials fracture.

All display and operating elements required for conducting the test are arranged on the front plate of the drive unit. The unit is turned on with the main switch (1). The unit can be stopped and de-energized with the emergency stop switch (2) at any time. The motor control switch (3) switches the drive motor in both rotational directions. The motor stops in the middle position, and rotates to the left in the left position and to the right in the right position. The switch has two positions. The first, non-locking position permits jogging operation; the motor stops when the switch is released. The motor runs continuouslyin the second, locking position. The speed selection switch (4) has 4 different deformation speeds: 50°/min, 100°/min, 200°/min and 500 °/min.
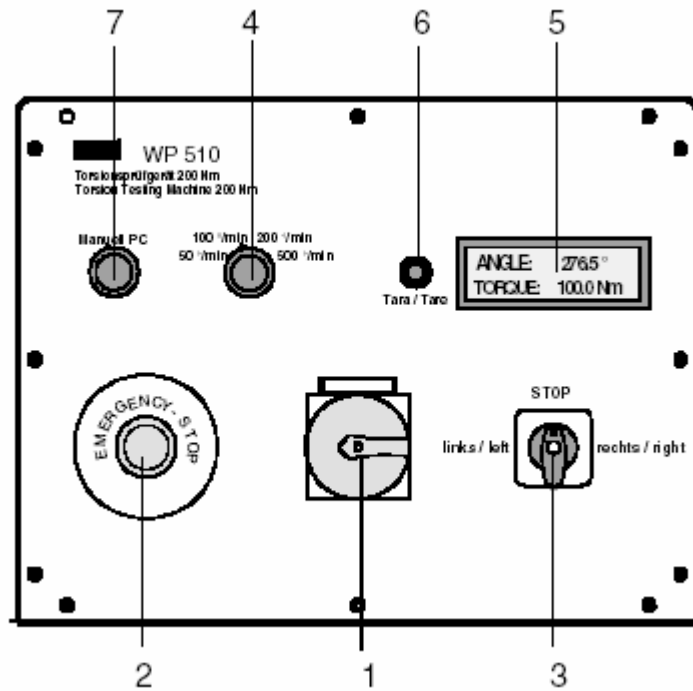
The LCD display (5) shows the current testing moment (torque) in Nm and the angle of rotation in degrees. Before the test run the displayed values can be set to zero with the tare key (6). The operating switch (7) is used to select between manual operation at the unit or remote control via a PC. Special control software is required for remote control with the PC.

The power supply connection (230 V / 50 Hz) and an interface socket are located on the back of the drive unit to link the PC. A button on the back panel is used to reset the overheating protection of the drive.The following block diagram provides an overview of the measuring and control technology of the test unit.
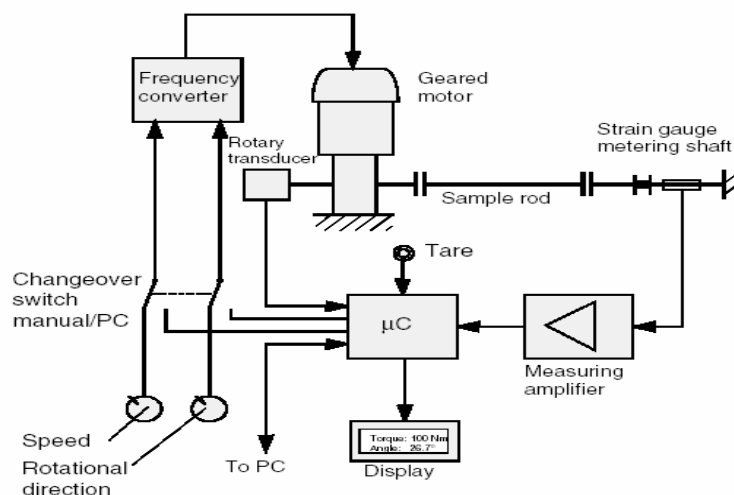

Fig. 2.3   Block diagram - Torsion testing unit

Procedure:

- *Installing the sample*
    1. Release the clamping lever (1) on the torque measuring device (2) and push back.
    2. Place sample (3) into the socket spanner (4) on the drive side.
    3. Turn sample with the motor until it fits into the socket spanner (5) on the torque measuring device.
    4. Push torque measuring device (2) forwards again. Ensure that the samples have an axial play of around 2-3 mm.
    5. Brace the torque measuring device with clamping levers (1).
    6. Shut the protective hood (6).
    7. Carefully pre-stress the sample in jogging operation until there is no more play and the torque display moves.
    8. Set display for torsional moment and angle to zero with the tare function.
- *Stressing the sample:*
    1. Twisting the sample using the motor control switch at steps of 2-5°.
    2. Read the torsion moment from the display after every angular step. Record together with the displayed torsion.
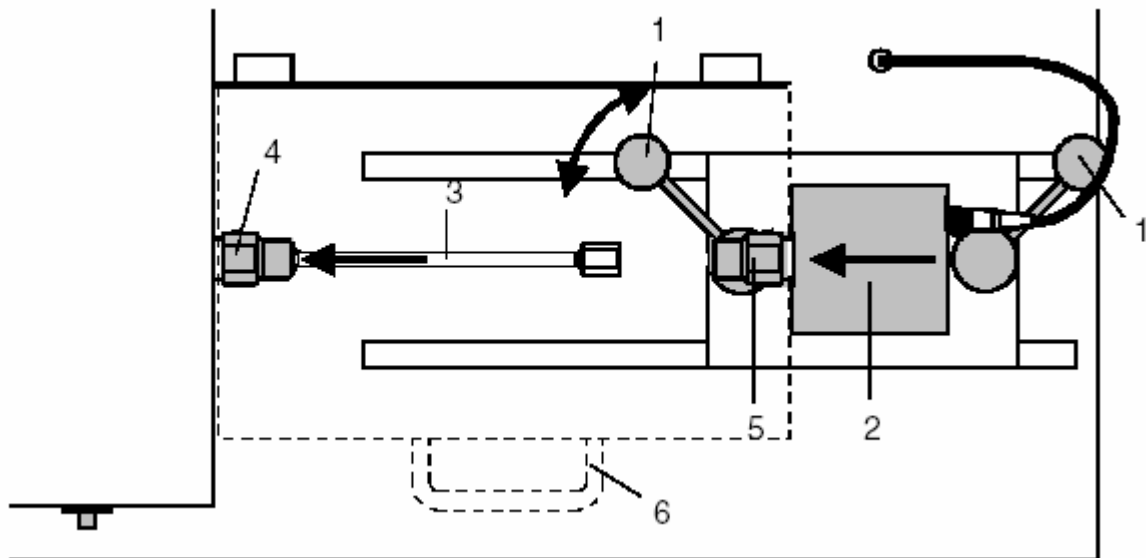
Results and analysis

1. Plot graph of torque (T) versus angle of rotation (Φ).
2. Calculate shear stress and shear strain in both plastic and elastic ranges.
3. Plot a $\tau - \gamma$ curve. Find the following:
    a. The proportional limit.
    b. Yield strength at an offset 0.1%.
    c. Modulus of Rigidity.
    d. Modulus of resilience.
    e. Modulus rupture.
4. The total angle of twist.
5. Draw the shape of fracture and explain.

DATA SHEET

Material:
Diameter D:
Gage length L:

Angle of twist (φ) Degree	Applied torque(T) N.m	Angle of twist (φ) Degree	Applied torque(T) N.m