Exp. # 2

Strain Measurement with Strain Gauges

Objective:

To learn and practice the techniques of strain gauge measurement by applying strain gages to a beam which is simply supported, and to calculate the state of stress at two different points, Poisson's ratio and modulus of elasticity E of the tested beam.

Theory:

The electrical strain gauge is a tool which translates small changes in dimensions and consequent electrical resistance into an equivalent change of strain.

A strain indicator is usually provided in order to give accurate measurements of such a strain. Due to their small size, strain gauges can be used on small surface in any direction.

The electrical strain gauge measurement is based on the simple fact that the electrical resistance of a conductor changes once the length of the conductor changes. If the resistance of a conductor is (R_o) when its length is (l_o), then its resistance will change by (ΔR) when its length changes by (Δl).

The physical relationship between strain and the change of resistance is linear. See figure 1.

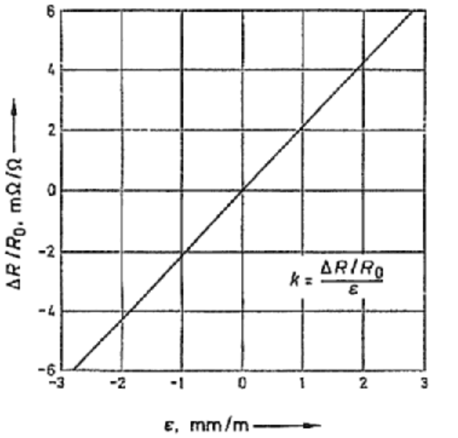


Figure 1: Characteristic for a metal strain gage and the definition of the gage factor k.

A strain gage's sensitivity is expressed by the ratio of the relative change of resistance to the strain and it is represented by the symbol k:

$$k = \frac{\Delta R / R_o}{\Delta l / l_o} = \frac{\Delta R / R_o}{\varepsilon} = \frac{\Omega / \Omega}{m / m} \qquad \text{........................ [1]}$$

And it is a unit less quantity.

The circuit diagram of the *Wheatstone Bridge*:

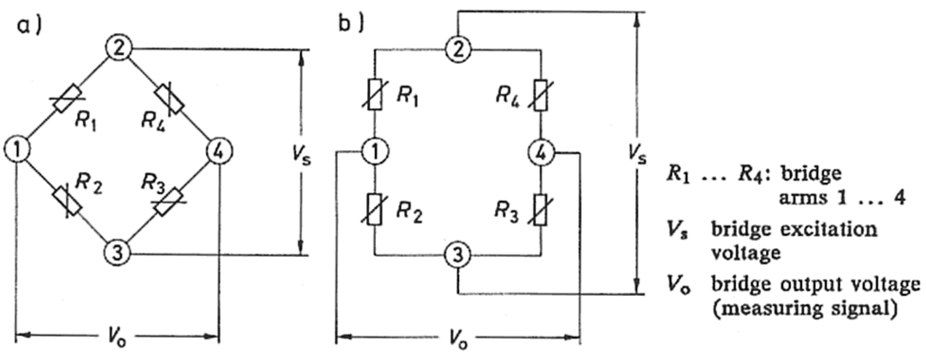


Figure 2: Different representations of the Wheatstone Bridge circuit.

Figure 2 shows two different illustrations of the Wheatstone bridge which are however electrically identical; Figure 2-a shows the usual rhombus type of representation which Wheatstone used; Figure 2-b is a representation of the same circuit which is more clear for the electrically untrained person.

The four arms or branches of the bridge circuit are formed by the resistances R_1 to R_4, the corner points 2 and 3 of the bridge designate the connections for the bridge excitation voltage V_s; the bridge output voltage V_o, the measurement signal, is available on the corner points 1 and 4.

The bridge excitation is usually an applied, stabilized direct or alternating voltage V_s. Sometimes a current supply is used.

The principle of the *Wheatstone Bridge* Circuit:

If a supply voltage V_s is applied to the two bridge supply points 2 and 3 then this is divided up in the two halves of the bridge R_1, R_2 and R_4, R_3 as a ratio of the corresponding bridge resistances, i.e. each half of the bridge forms a voltage divider, see figure 3.

The following treatment of the bridge circuit assumes that the source resistance R_6 of the voltage supply is negligibly small and that the internal resistance of the instrument for measuring the bridge output voltage is very high and does not cause any load on the bridge circuit. This method of treatment is acceptable.

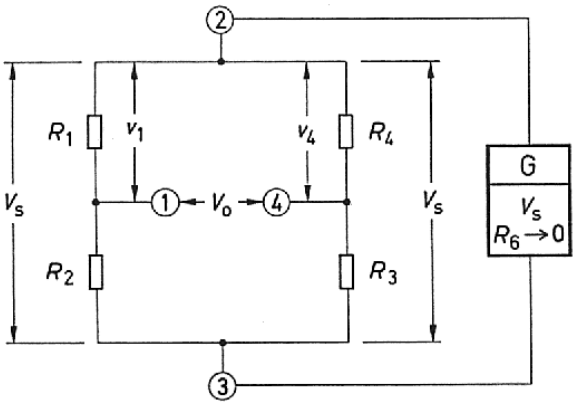


Figure 3: Principle of the voltage-fed Wheatstone bridge circuit.

The partial voltage v_1 on bridge node 1 can be calculated as:

$$v_1 = \frac{R_1}{R_1 + R_2} . V_s \qquad \text{[2]}$$

And the partial voltage v4 on bridge node 4 is:

$$v_4 = \frac{R_4}{R_3 + R_4} . V_s \qquad \text{[3]}$$

The difference between the two partial voltages are the bridge output voltage V_o:

$$V_o = V_s \left(\frac{R_1}{R_1 + R_2} - \frac{R_4}{R_3 + R_4} \right) = V_s (v_1 - v_4) \qquad \text{[4]}$$

If the unbalance in the bridge is defined as the relative output voltage Vo/Vs, then equation (4) appears in the form

$$\frac{V_o}{V_s} = \frac{R_1}{R_1 + R_2} - \frac{R_4}{R_3 + R_4} \qquad \text{[5-a]}$$

Or

$$\frac{V_o}{V_s} = \frac{R_1 . R_3 - R_2 . R_4}{(R_1 + R_2).(R_3 + R_4)} \qquad \text{[5-b]}$$

There are two conditions where $V_o = 0$;

- When all bridge resistors are equal value, $R_1 = R_2 = R_3 = R_4 = R$
- When the resistor ratios in the two halves of the bridge are the same, ($\frac{R_1}{R_2} = \frac{R_4}{R_3}$), in this case ($\frac{V_o}{V_s} = 0$) and the bridge circuit is balanced.

If the bridge resistors R_1 to R_4 change their value by an amount ΔR, then the bridge circuit becomes unbalanced, and an output voltage is present between points 1 and 4. Equation (5-a) becomes:

$$\frac{V_o}{V_s} = \frac{R_1 + \Delta R_1}{R_1 + \Delta R_1 + R_2 + \Delta R_2} - \frac{R_4 + \Delta R_4}{R_3 + \Delta R_3 + R_4 + \Delta R_4} \qquad \text{……………………. [6]}$$

All bridge conditions can be calculated with equation 6, irrespective of the basic resistance R that the individual arms of the bridge possess. However, in strain gage techniques all the arms of the bridge should have the same resistance; at least the two halves of the bridge R_1, R_2, R_3 and R_4 must have the same resistances. Variations due to the tolerance on the strain gage resistance do not affect the measurement accuracy. Even differences of 5% in the resistances of R_1 and R_2 produce errors of less than 0.1%.

In strain-gauge techniques the amounts by which the resistance changes in the metal strain gages are very small and of the order of about 10^{-3}. It is therefore usual to use the approximation below, which provides sufficiently accurate results for practical requirements, instead of equation 6.

$$\frac{V_o}{V_s} = \frac{1}{4}\left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4}\right) \qquad \text{……………………. [7]}$$

The approximate formula also shows that the relative change of resistance of each arm of the bridge is the governing factor in balancing the bridge and not the absolute change of resistance.

Also

$$\frac{\Delta R}{R} = k.\varepsilon \qquad \text{……………………. [8]}$$

Where equation 7 in the form:

$$\frac{V_o}{V_s} = \frac{k}{4}\left(\varepsilon_1 - \varepsilon_2 + \varepsilon_3 - \varepsilon_4\right) \qquad \text{……………………. [9]}$$

The equations 7 & 9 assume that all the resistances in the bridge change. This situation occurs for example in transducers or with test objects performing a similar function. In experimental stress analysis this is hardly ever the case and usually only some of the arms of the bridge contain active strain gages, the remainder being made up of bridge completion resistors. Designations for the various forms such as Quarter Bridge, Half Bridge, Double Quarter or Diagonal Bridge and full bridge are commonplace. Figure 4 illustrates the different forms.

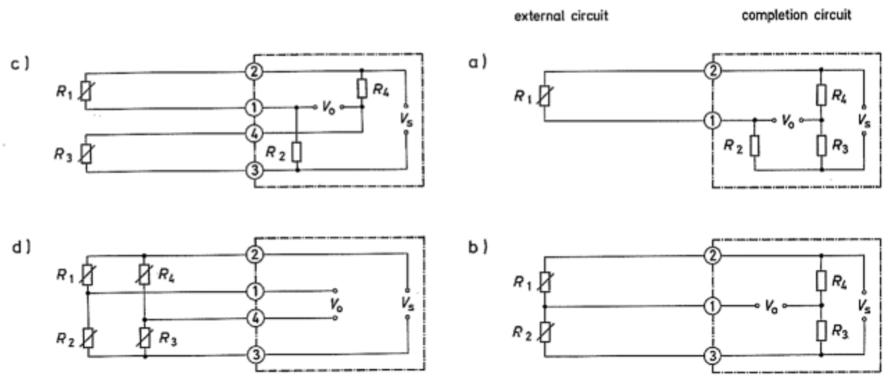


Figure 4:
Forms of the Wheatstone bridge circuit used in strain gauge techniques:

a) Quarter Bridge.
b) Half Bridge.
c) Bouble Quarter or Diagonal Bridge.
d) Full bridge.

The measurement system:

The strains measured with strain gauges are normally very small. Consequently the changes of resistance are also very small and cannot be measured directly, say with an ohmmeter. The strain gauge must therefore be included in a measurement system where precise determination of the strain gauge's change of resistance is possible.

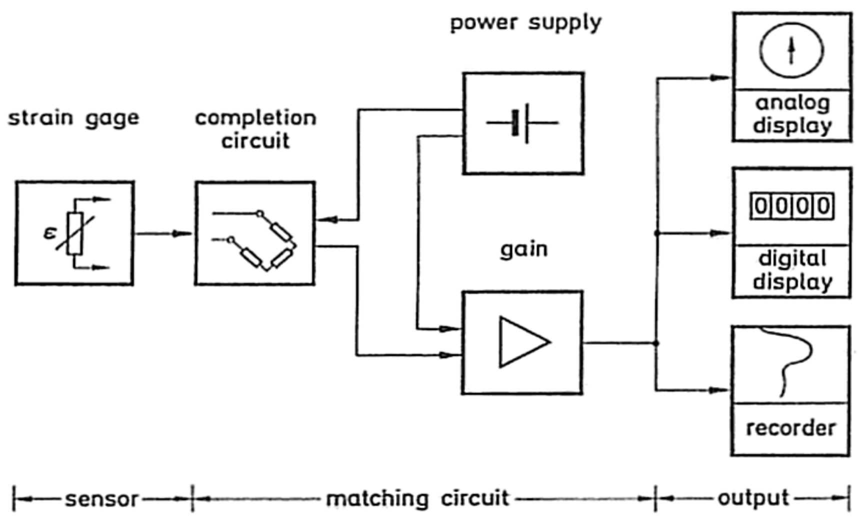


Figure 5: Diagram of a measurement system for measuring strains with a strain gage.

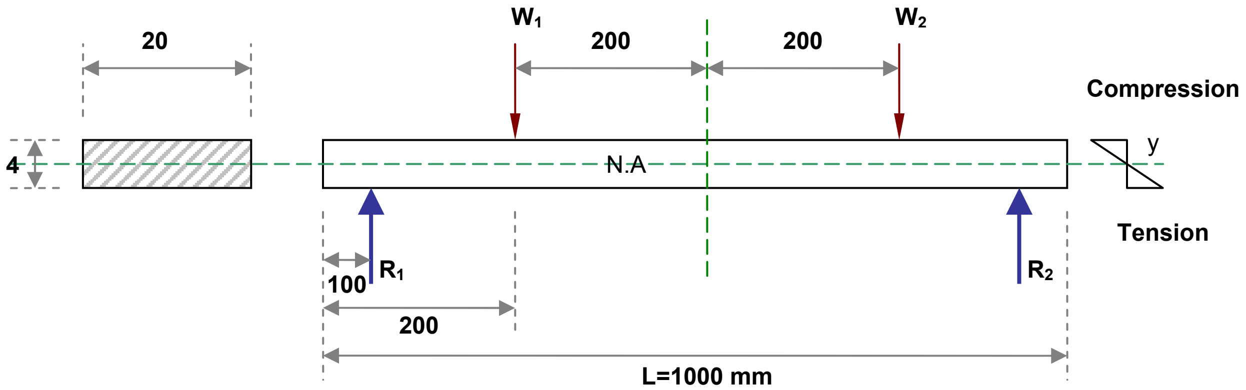


Figure 6: Free body diagram.

The relation for the normal bending stresses in beams is very well developed. As for shown in figure 6 the bending stress σ is directly proportional to the distance y from the neutral axis and the bending moment M.

$$\sigma = -\frac{My}{I} \qquad \text{.......................... [10]}$$

Where I is the second moment of area about the z - axis. It is customary to designate c = y_{max}. to omit the negative sign, and to write:

$$\sigma = -\frac{Mc}{I} \qquad \text{.......................... [11]}$$

Where it is understood that the above equation gives the maximum stress.

If the strain gauge techniques are used to measure strain. Then using Hook's law, the stress state at a point can be calculated after the state of strain has been measured. We define the principal strains as the strains in the direction of the principal stresses. The general form of Hook's law is

$$\sigma_1 = E\varepsilon_1(1-\nu) + \nu E(\varepsilon_2 + \varepsilon_3)/1-\nu-2\nu^2$$
$$\sigma_2 = E\varepsilon_2(1-\nu) + \nu E(\varepsilon_1 + \varepsilon_3)/1-\nu-2\nu^2$$
$$\sigma_3 = E\varepsilon_3(1-\nu) + \nu E(\varepsilon_1 + \varepsilon_2)/1-\nu-2\nu^2$$

where σ_1, σ_2, σ_3 are the principal stresses, ε_1, ε_2, ε_3, are the principal strains, E is the modulas of elasticity and v is the Poison's ratio.

For the case of uniaxial type of stress, Hook's Law reduces to the form:

$$\sigma_1 = E\varepsilon_1$$
$$\sigma_2 = 0$$
$$\sigma_3 = 0$$

And the Poisson's ratio for the beam is given by:

$$\nu = \frac{Lateral\ Strain}{Axial\ Strain}$$

Strain Measurement:

The governing Equations of the strain reading, in a half bridge setup, of the indicator is given by:

$$\text{Voltage read in mV} = \frac{Ge}{k_R} = \frac{G}{k_R}.\frac{V}{4}.\left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2}\right) = \frac{GV}{4}.\frac{k}{k_R}\left(\varepsilon_1^J - \varepsilon_2^J\right) \qquad \text{.......................... [12]}$$

Where:

k is the real gauge factor of the gauges.

k_R is the value of the dividing factor set as gauge factor on the bridge.

(**GV/4**) = 1000 for the EI 616 Bridge.

Note that we set k_R = k in the experiment, consequently all readings are in terms of 10^{-6}.

See figure 7 for comparison of half bridge with full bridge setup.

Apparatus & equipments needed:

- Beam Apparatus.
- Two half bridge Strain Gauges fixed on lower & upper faces of the beam.
- Digital strain indicator as shown in the figure below (EI 616 STRAIN BRIDGE).

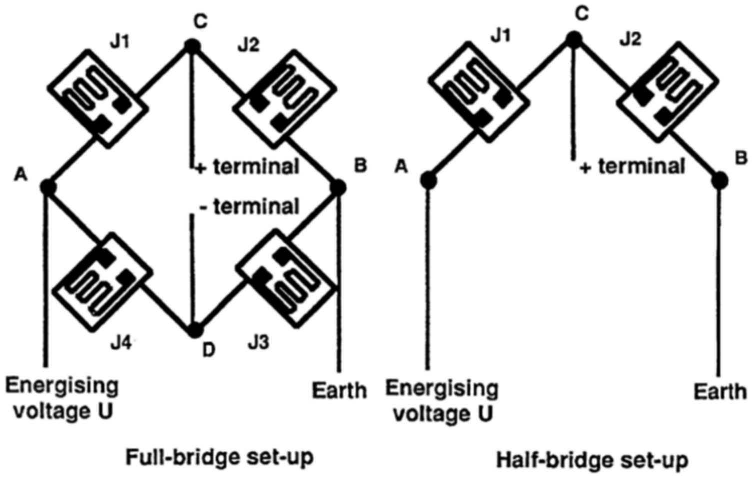
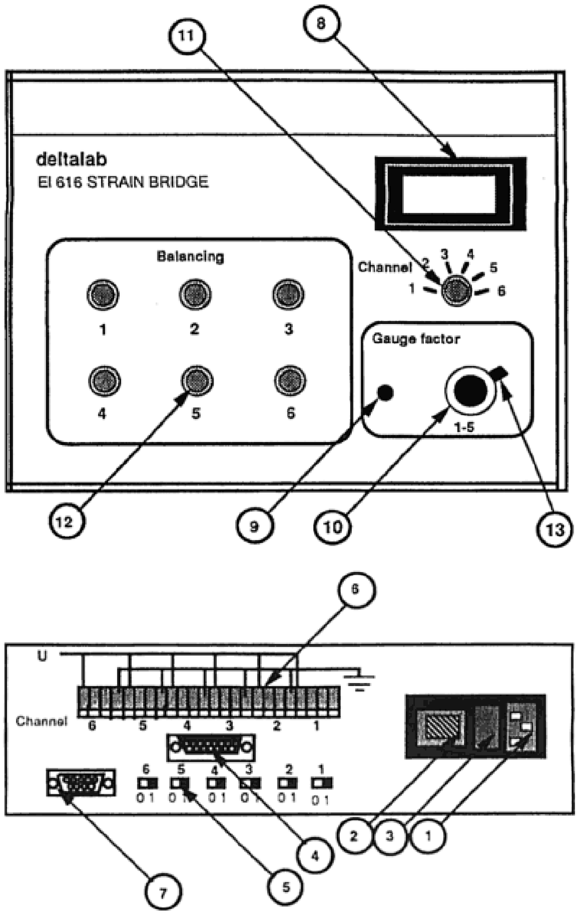


Figure 7: for comparison of half bridge with full bridge setup.

Figure 8: Digital strain indicator, EI 616 STRAIN BRIDGE. The first figure for the front panel and the second one is for the rear panel. Illustration of the function of each component is indicated as follows:



1. Mains Power socket.
2. On-Off switch.
3. Fuse.
4. 15-pin cannon socket for gauge bridge connection.
5. Half Bridge- Full Bridge selector for each measurement channel.
6. Pressure contacts for connecting individual strain bridges.
7. Analogue output via 9-pin cannon socket.
8. 20 000-point digital display.
9. Push-button for setting gauge factor.
10. Gauge factor adjustment potentiometer.
11. Channel selector switch.
12. Potentiometers for initial balancing of bridge gauges.
13. Gauge factor potentiometer lock.

Procedure

1. Set up the beam apparatus with two load hangers.
2. Two linear, half circuits strain gauges to be connected on the top and bottom of the beam at midspan.
3. Insert the gauge factor, and zero the gauge readings.
4. Apply loads to the hangers, and take readings of the channels of the strain gauges indicator.
5. Repeat the experiment at different applied loads.

Results and analysis:

1. Fill the experimental results at table below.
2. Calculate the bending moment (M) at the midspan.
3. Calculate the axial stress (σ).
4. Plot (σ) against (ε_{axial}), find E and compare it with the theoretical value.
5. Plot ε_{axial} against $\varepsilon_{lateral}$, find v and compare it with the theoretical value obtained from 6.
6. Obtain the value of Poison's ratio v for the type of steel beam from strength of material and books.
7. State 5 applications of strain measurement using strain gauges.
8. State 5 specific sources of error in this experiment.

Rod material:	....	Cross section dimensions:	....
Length:	....	Distance of W from center:	....
Cross Section Type:	....	Ends condition:	....

Table 1: experiment parameters.

No.	W_1 (N)	W_2 (N)	Channel 1 reading $\varepsilon_{lateral}$	Channel 2 reading ε_{axial}	Bending moment (M)	Stress (σ)
1	*0*	*5*				
2	*5*	*5*				
3	*5*	*10*				
4	*10*	*10*				
5	*10*	*15*				
6	*15*	*15*				

Table 2: experiment data and results.