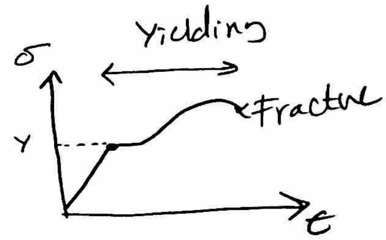
Chapter 15: Fracture mechanics

Fracture: Separation of a solid body into two or more pieces under action of stress.

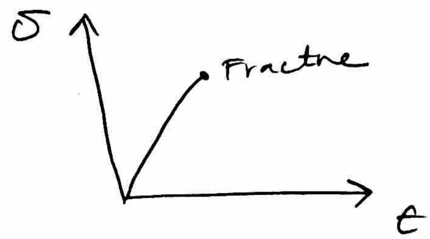
Fracture mechanics: Field of mechanics to study the fracture of solid bodies to cracks.

Types of failures, due to material behaviour

- Yielding → only for ductile materials



- Fracture → Ductile fracture (Yielding occurs prior to failure)
  - → Brittle fracture (Sudden fracture ⇒ without yielding)



Brittle fracture types

① Brittle fracture in brittle materials

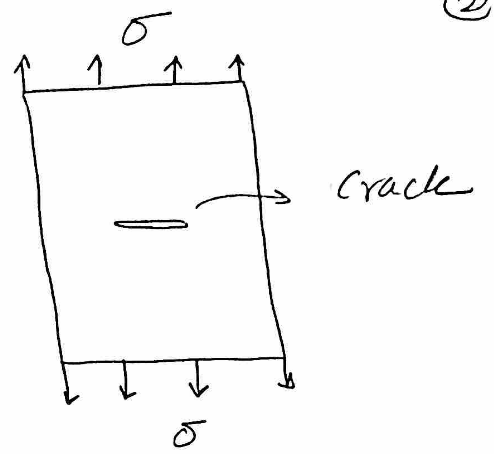
→ ② Brittle fracture due to cracks: Cracks → very high stress → No time for yielding → Brittle or sudden fracture

our main focus

③ Brittle fracture due to fatigue
" repeated loading:

* Cracks forms stress concentratio

* In theory we calculate stress on a line



$$\sigma_{crack} = \frac{P}{A}, \quad \text{for a line } A \cong 0$$

$\Rightarrow \sigma_{crack} \rightarrow$ infinity

$\Rightarrow$ Very high stress $\longrightarrow$ No time for yielding

$\rightarrow$ Brittle fracture

* Brittle fracture stages

① Crack initiation (start)

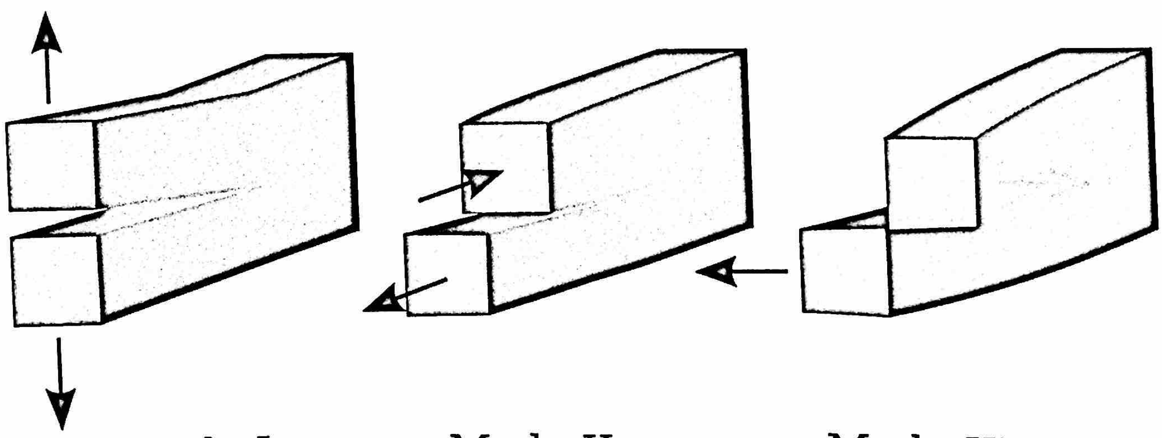
② Crack propagation (growth)

→ Slow (stable)

→ Fast (unstable)

Fracture Modes

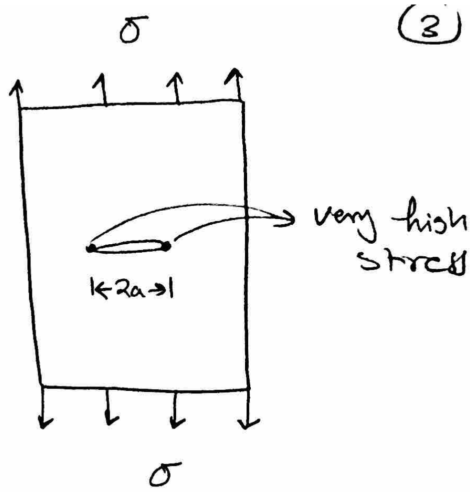
Mode I is the most Common



Mode I: Opening Mode

Mode II: Sliding Mode

Mode III: Tearing Mode



* Cracks presence leads to extremely high Elastic stress regions near the Crack tip.

* This high stress is often measured by "Stress intensity factor K".

* Mode I K_I, Mode II K_{II}, mode III K_{III}

* We will mainly focus on Mode I and K_I.

<u>In general</u>

$$K_I = \sigma \sqrt{\pi a}$$

σ: Average Stress

a: Crack Length

Unit $= Pa \cdot m^{1/2}$

$= (\text{stress})(\text{length})^{1/2}$

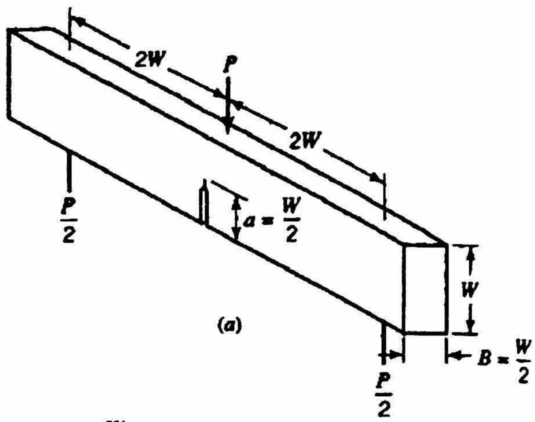
* In order to fracture to occur K_I should equal to fracture toughness (K_{IC}).

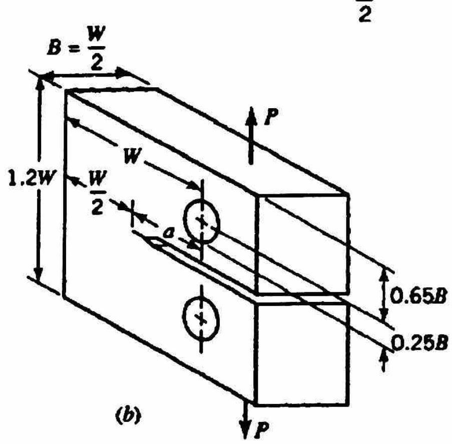
For fracture $K_I = K_{IC}$

* Fracture toughness is a material property and it is measured experimentally.

* Fracture toughness is material ability to resist brittle fracture.

- Fracture toughness is often measured using fracture toughness tests
  - → Bending Fracture toughness (a)
  - → Tensile Fracture toughness (b)





when Crack initiation happens

$K_I = K_{IC} \rightarrow$ Fracture toughness

$K_I = \sigma\sqrt{\pi a}$, $\sigma = \frac{P}{A}$, $\sigma = \frac{Mc}{I}$

★ Stress intensity factor

As in stress concentration factor, the stress intensity factor is obtain through tabulated and plotted data.

In general

$$K_I = \sigma\sqrt{\pi a} \cdot f(\lambda)$$

σ: Nominal stress

a: Crack length

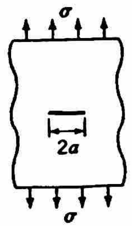
$f(\lambda)$: Geometric factor

Table 15.2 textbook

(5)

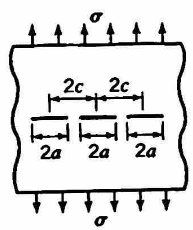
TABLE 15.2 Stress Intensity Factors K_I

Case 1: Infinite sheet with through-thickness crack and uniform tension at infinity. Griffith's crack



$$K_I = \sigma\sqrt{\pi a}$$

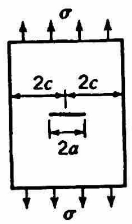
Case 2: Periodic array of through-thickness cracks in infinite sheet with uniform tension at infinity



$$K_I = \sigma\sqrt{\pi a}\,f(\lambda);\ \lambda = \frac{a}{c}$$

λ	$f(\lambda)$
0.1	1.00
0.2	1.02
0.3	1.04
0.4	1.08
0.5	1.13
0.6	1.21

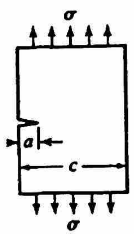
Case 3: Central crack in finite-width strip subjected to uniform tension at infinity



$$K_I = \sigma\sqrt{\pi a}\,f(\lambda);\ \lambda = \frac{a}{c}$$

λ	$f(\lambda)$
0.1	1.01
0.2	1.03
0.3	1.06
0.4	1.11
0.5	1.19
0.6	1.30

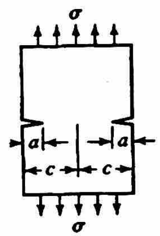
Case 4: Single-edge crack in finite-width sheet



$$K_I = \sigma\sqrt{\pi a}\,f(\lambda);\ \lambda = \frac{a}{c}$$

λ	$f(\lambda)$
$0 (c \to \infty)$	1.12
0.2	1.37
0.4	2.11
0.5	2.83

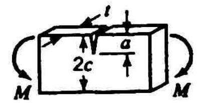
Case 5: Double-edge crack in finite-width sheet



$$K_I = \sigma\sqrt{\pi a}\,f(\lambda);\ \lambda = \frac{a}{c}$$

λ	$f(\lambda)$
$0 (c \to \infty)$	1.12
0.2	1.12
0.4	1.14
0.5	1.15
0.6	1.22

Case 6: Edge crack in beam in bending



$$K_I = \sigma\sqrt{\pi a}\,f(\lambda)$$

$$\lambda = \frac{a}{2c}$$

$$\sigma = \frac{3M}{2tc^2}$$

λ	$f(\lambda)$
0.1	1.02
0.2	1.06
0.3	1.16
0.4	1.32
0.5	1.62
0.6	2.10

Example Find K_I for the following

(1) For case 1

$\sigma = 10$ MPa , $a = 10$ mm

(2) Case 2 $\sigma = 10$ MPa, $a = 10$ mm , $c = 20$ mm

(3) Case 3 $\sigma = 10$ MPa , $a = 10$ mm, $c = 20$ mm

(4) Case 4 $\sigma = 10$ MPa , $a = 10$ mm , $c = 1000$ mm

(5) Case 5 $\sigma = 10$ MPa, $a = 12$ mm , $c = 30$ mm

(6) Case 6 $M = 10$ N.m , $t = 10$ mm , $c = 10$ mm

$a = 10$ mm

Solution

① $K_I = \sigma\sqrt{\pi a} \Rightarrow K_I = 10\times10^6\sqrt{\pi\, 0.01} = 1.772 \text{ MPa}\cdot m^{1/2}$

② $K_I = \sigma\sqrt{\pi a}\cdot f(\lambda)$, $f(\lambda)$ at $\lambda = \frac{a}{c}$

$\lambda = \frac{10}{20} \Rightarrow \lambda = 0.5 \rightarrow f(\lambda) = 1.13$

$K_I = 10\times10^6\sqrt{\pi(0.01)} \times (1.13) \Rightarrow K_I = 2.0 \text{ MPa } m^{1/2}$

③ $K_I = \sigma\sqrt{\pi a}\cdot f(\lambda)$, $\lambda = \frac{a}{c} = \frac{10}{20} = 0.5 \rightarrow f(\lambda) = 1.19$

$K_I = (10)(10)^6\sqrt{\pi(0.01)}\,(1.19) = 2.11 \text{ MPa } m^{1/2}$

④ $K_I = \sigma\sqrt{\pi a}\, f(\lambda)$, $\lambda = \frac{a}{c} = \frac{10}{1000} = 0.01 \approx 0 \Rightarrow f(\lambda) = 1.12$

$K_I = (10)(10^6)\sqrt{\pi(0.01)}\,(1.12) = 1.985 \text{ MPa } m^{1/2}$

⑤ $K_I = \sigma\sqrt{\pi a}\cdot f(\lambda)$, $\lambda = \frac{a}{c} = \frac{12}{30} = 0.4 \Rightarrow f(\lambda) = 1.14$

$K_I = (10)(10^6)\sqrt{\pi(0.012)}\cdot(1.14) = 2.213 \text{ MPa } m^{1/2}$

⑥ $K_I = \sigma\sqrt{\pi a}\cdot f(\lambda)$, $\sigma = \frac{3M}{2tc^2} = \frac{(3)(10)(1\ )}{(2)(0.01)(0.01)^2} = 15 \text{ MPa}$

$\lambda = \frac{a}{2c} = \frac{10}{(2)(10)} = \frac{10}{20} = 0.5$

$\Rightarrow f(\lambda) = 1.62$

$K_I = (15)(10^6)\sqrt{\pi(0.01)}\cdot(1.62) = 4.31 \text{ MPa } m^{1/2}$