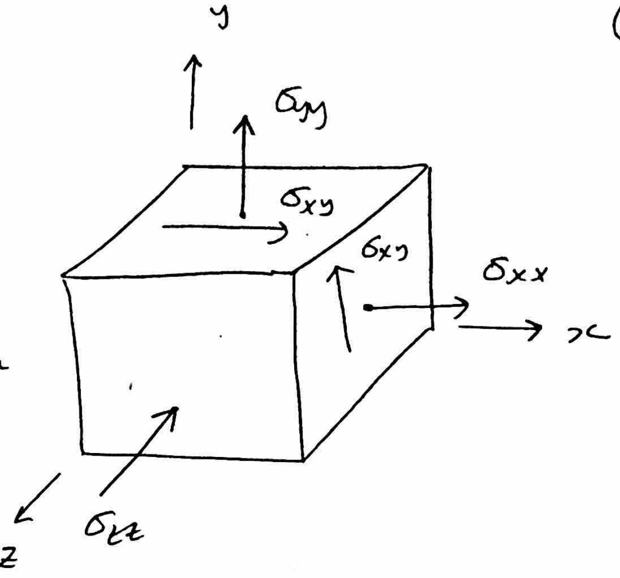
Example

$\sigma_{xx} = 7\ MPa$, $\sigma_{yy} = 2.1\ MPa$

$\sigma_{zz} = -2.8\ MPa$, $\sigma_{xy} = 1.4\ MPa$



$E_x = 14.7\ GPa$, $E_y = 1\ GPa$, $E_z = 0.735\ GPa$

$G_{xy} = 941\ MPa$, $G_{xz} = 1147\ MPa$, $G_{yz} = 103\ MPa$

$\nu_{xy} = 0.292$, $\nu_{xz} = 0.449$, $\nu_{yz} = 0.390$

Find ① Orientation of principal axes of stress (θ_p)

② strain components $[\epsilon]$

③ Orientation of principal axes of strain

Solution

① As, $\sigma_{xz} = \sigma_{yz} = 0 \Rightarrow \sigma_{zz}$ is a principal stress.

So, we just need to find θ_p ($x-y$) plane only

$$\tan 2\theta_p = \frac{2\sigma_{xy}}{\sigma_{xx} - \sigma_{yy}} = \frac{(2)(1.4)}{7 - 2.1} = 0.5714 \Rightarrow \theta_p = \frac{1}{2}\tan^{-1}(0.5714)$$

$\theta_p = 14.9^\circ$ or 104.9°

Or, we can find $\sigma_1, \sigma_2, \sigma_3$ from $\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0$

then find $\{V_{n1}\}, \{V_{n2}\}, \{V_{n3}\}$.

② [c]

Use Hooke's law

$$\epsilon_{xx} = \frac{1}{E_x}\sigma_{xx} - \overset{①}{\frac{\nu_{yx}}{E_y}}\sigma_{yy} - \overset{②}{\frac{\nu_{zx}}{E_z}}\sigma_{zz}$$

$$\epsilon_{yy} = -\frac{\nu_{xy}}{E_x}\sigma_{xx} + \frac{1}{E_y}\sigma_{yy} - \overset{③}{\frac{\nu_{zy}}{E_z}}\sigma_{zz}$$

$$\epsilon_{zz} = -\frac{\nu_{xz}}{E_x}\sigma_{xx} - \frac{\nu_{yz}}{E_y}\sigma_{yy} + \frac{1}{E_z}\sigma_{zz}$$

$$\epsilon_{xy} = \frac{1}{2G_{xy}}\sigma_{xy}, \quad \epsilon_{xz} = \frac{1}{2G_{xz}}\sigma_{xz}, \quad \epsilon_{yz} = \frac{1}{2G_{yz}}\sigma_{yz}$$

and $\overset{①}{\nu_{yx}} = \frac{E_y}{E_x}\nu_{xy}$, $\overset{②}{\nu_{zx}} = \frac{E_z}{E_x}\nu_{xz}$, $\overset{③}{\nu_{zy}} = \frac{E_z}{E_y}\nu_{yz}$

$$\Rightarrow \epsilon_{xx} = \frac{1}{E_x}\sigma_{xx} - \left(\frac{\cancel{E_y}}{E_x}\right)\left(\frac{\nu_{xy}}{\cancel{E_y}}\right)\sigma_{yy} - \left(\frac{\cancel{E_z}}{E_x}\right)\left(\frac{\nu_{xz}}{\cancel{E_z}}\right)\sigma_{zz}$$

$$= \frac{7\times10^6}{14.7\times10^9} - (0.292)\left(\frac{2.1\times10^6}{14.7\times10^9}\right) - (0.449)\left(\frac{-2.8\times10^6}{14.7\times10^9}\right) \Rightarrow \boxed{\epsilon_{xx} = 520\times10^{-6}}$$

$$\Rightarrow \epsilon_{yy} = -\frac{\nu_{xy}}{E_x}\sigma_{xx} + \frac{\sigma_{yy}}{E_y} - \left(\frac{\cancel{E_z}}{E_x}\right)\left(\frac{\nu_{xz}}{\cancel{E_z}}\right)\sigma_{zz}$$

$$= -(0.292)\left(\frac{7\times10^6}{14.7\times10^9}\right) + \left(\frac{2.1\times10^6}{1\times10^9}\right) - (0.449)\left(\frac{-2.8\times10^6}{14.7\times10^9}\right) \Rightarrow \boxed{\epsilon_{yy} = 3053\times10^{-6}}$$

$$\Rightarrow \epsilon_{zz} = -(0.449)\left(\frac{7\times10^6}{14.7\times10^9}\right) - (0.39)\left(\frac{2.1\times10^6}{1\times10^9}\right) + \left(\frac{-2.8\times10^6}{735\times10^6}\right) \Rightarrow \boxed{\epsilon_{zz} = -4844\times10^{-6}}$$

$$\epsilon_{xy} = \frac{1}{(2)(941\times10^6)} \cdot 1.4\times10^6 \Rightarrow \boxed{\epsilon_{xy} = 744.\times10^{-6}}$$

$$\boxed{\epsilon_{xz} = \epsilon_{yz} = 0}$$

3 θ_p for strain

AS, we did in part (1), Since $\epsilon_{xz} = \epsilon_{yz} = 0$, then

$\epsilon_{zz} = -4884 \times 10^{-6}$ is a principal strain

So, we only need to find θ_p in (x-y plane)

$$\tan 2\theta_p = \frac{2\,\epsilon_{xy}}{\epsilon_{xx} - \epsilon_{yy}} = \frac{(2)(744 \times 10^{-6})}{520 \times 10^{-6} - 3053 \times 10^{-6}} = -0.5875$$

$$\theta_p = \frac{1}{2} \tan^{-1}(-0.5875)$$

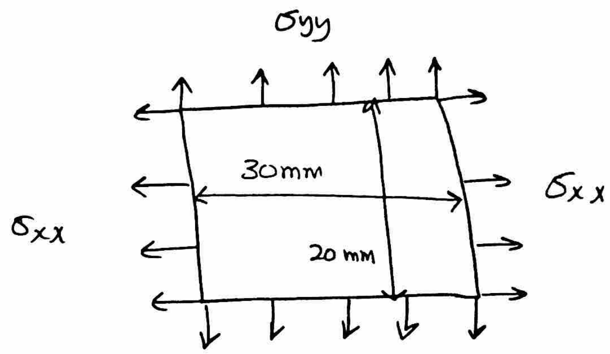
$$\theta_p = -15.22^\circ \quad \text{or} \quad 74.78^\circ$$

Example $E = 200\,GPa$, $\nu = 0.30$, $\alpha = 13.7 \times 10^{-6}\ /C^\circ$

$\sigma_{xx} = 50\ MPa$, $\sigma_{yy} = 70\ MPa$

and Heat from $T_1 = 25°C$

to $T_2 = 125°C$



Find $[\epsilon]$

$$\epsilon_{xx} = \frac{1}{E}\left(\sigma_{xx} - \nu(\sigma_{yy} + \cancel{\sigma_{zz}}^{\,0})\right) + \alpha \Delta T$$

$$\epsilon_{yy} = \frac{1}{E}\left(\sigma_{yy} - \nu(\sigma_{xx} + \cancel{\sigma_{zz}}^{\,0})\right) + \alpha \Delta T$$

$\Delta T = 125 - 25$
$= 100\,°C$

$$\epsilon_{zz} = \frac{1}{E}\left(\cancel{\sigma_{zz}}^{\,0} - \nu(\sigma_{yy} + \sigma_{xx})\right) + \alpha \Delta T$$

$$\epsilon_{xy} = \frac{1}{2G}\sigma_{xy}, \quad \epsilon_{xz} = \frac{1}{2G}\sigma_{xz}, \quad \epsilon_{yz} = \frac{1}{2G}\sigma_{yz}$$

$$\epsilon_{xx} = \frac{\sigma_{xx}}{E} - \nu\frac{\sigma_{yy}}{E} + \alpha \Delta T = \frac{50 \times 10^6}{200 \times 10^9} - (0.3)\left(\frac{70 \times 10^6}{200 \times 10^9}\right) + 13.7 \times 10^{-6}(100)$$

$$= 1.515 \times 10^{-3}$$

$$\epsilon_{yy} = \frac{\sigma_{yy}}{E} - \nu\frac{\sigma_{xx}}{E} + \alpha \Delta T = \frac{70 \times 10^6}{200 \times 10^9} - (0.3)\frac{50 \times 10^6}{200 \times 10^9} + 13.7 \times 10^{-6}(100)$$

$$= 1.645 \times 10^{-3}$$

$$\epsilon_{zz} = -\frac{\nu}{E}(\sigma_{xx} + \sigma_{yy}) + \alpha \Delta T = \frac{-0.3}{200 \times 10^9}(50 \times 10^6 + 70 \times 10^6) + 13.7 \times 10^{-6}(100)$$

$$= 1.19 \times 10^{-3}$$

$$[\epsilon] = \begin{bmatrix} 1.515 & 0 & 0 \\ 0 & 1.645 & 0 \\ 0 & 0 & 1.19 \end{bmatrix} \times 10^{-3}$$

$$\epsilon_{xy} = \epsilon_{xz} = \epsilon_{yz} = 0$$