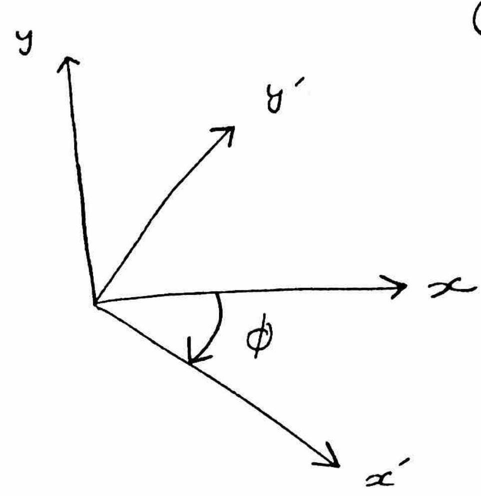
Problems on chapter Two



Problem 2.5

For plane stress state $\sigma_{xx} = 90$ MPa
$\sigma_{yy} = -10$ MPa and $\sigma_{xy} = 40$ MPa

If a new coordinate system is rotated by
$\phi = 30^\circ$ From x-axis in clock wise directi
find σ'_{xx} and σ'_{xy}

Solution

$$\sigma'_{xx'} = \sigma_{xx} \cos^2\theta + \sigma_{yy} \sin^2\theta + 2\sigma_{xy} \sin\theta \cos\theta$$

$$\sigma'_{xy} = -(\sigma_{xx} - \sigma_{yy}) \sin\theta\cos\theta + \sigma_{xy} (\cos^2\theta - \sin^2\theta)$$

$\theta = -30^\circ \Rightarrow \cos(-30) = 0.866$, $\sin(-30) = -0.5$

$\Rightarrow \sigma'_{xx} = 30.6$ MPa , $\sigma'_{xy} = 63.30$ MPa

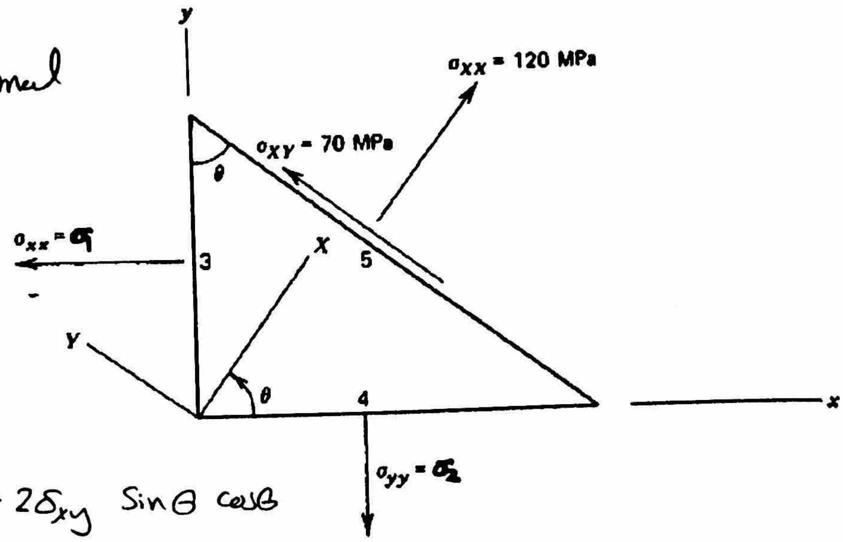
Problem 2.19

For the state of stress shown,

Find σ_{xx} and σ_{yy}.

Solution

σ_{XX} and σ_{XY} are transformed stress state



Thus

$$\sigma_{XX} = \sigma_{xx}\cos^2\theta + \sigma_{yy}\sin^2\theta + 2\sigma_{xy}\sin\theta\cos\theta$$

$$\sigma_{XY} = -(\sigma_{xx}-\sigma_{yy})\sin\theta\cos\theta + \sigma_{xy}(\cos^2\theta - \sin^2\theta)$$

$\sigma_{xy} = 0$ (No Shear)

$\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$

Then

$$120 = \sigma_{xx}\frac{9}{25} + \sigma_{yy}\frac{16}{25}$$

$$70 = -\sigma_{xx}\left(\frac{4}{5}\right)\left(\frac{3}{5}\right) + \sigma_{yy}\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$$

, $\sigma_{XY} = -\sigma_{xx}\sin\cos + \sigma_{yy}\sin\cos$

$$\Rightarrow \left.\begin{aligned} 9\sigma_{xx} + 16\sigma_{yy} &= 3000 \\ -12\sigma_{xx} + 12\sigma_{yy} &= 1750 \end{aligned}\right\}$$ Two equation, two unknowns

$\sigma_{xx} = 26.67$ MPa

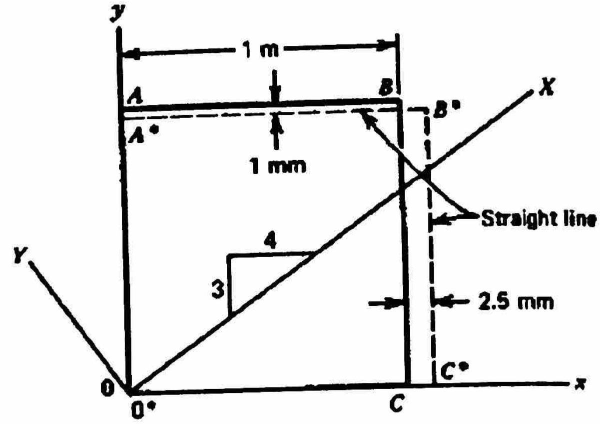
$\sigma_{yy} = 172.5$ MPa

Problem 2.36

For the figure shown, we have a plane strain state ($\epsilon_{zz} = \epsilon_{zx} = \epsilon_{zy} = 0$)

Determine

1. Displacement fields u and v
2. ϵ_{xx}, ϵ_{yy} and ϵ_{xy}
3. For the new coordinate system (X, Y) find transformed strain state.



Solution

① Displacement fields

u

$1\,m \longrightarrow 2.5\ mm$

$x \longrightarrow u$

$\Rightarrow u = (2.5)(10^{-3})\,x$

$\boxed{u = 0.0025\,x}$

v

$1\,m \longrightarrow -1\ mm$

$y \longrightarrow v$

$\Rightarrow v = (-1) \times 10^{-3}\ y$

$\boxed{v = -0.001\,y}$

② strain fields

$$\epsilon_{xx} = \frac{\partial u}{\partial x} = 0.0025, \quad \epsilon_{yy} = \frac{\partial v}{\partial y} = -0.001\,y$$

$$\epsilon_{xy} = \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) = 0$$

③ Transformed state of strain

$\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

$$\epsilon_{XX} = \epsilon_{xx}\cos^2\theta + \epsilon_{yy}\sin^2\theta + \cancelto{0}{2\epsilon_{xy}\sin\theta\cos\theta}$$

$$= (0.0025)\left(\frac{3}{5}\right)^2 + \bar{0}.001\left(\frac{4}{5}\right)^2 \Rightarrow \epsilon_{XX} = 0.00124$$

$$\epsilon_{YY} = \epsilon_{xx}\sin^2\theta + \epsilon_{yy}\cos^2\theta - \cancelto{0}{2\epsilon_{xy}\sin\theta\cos\theta} \Rightarrow \epsilon_{YY} = 0.00026$$

$$\epsilon_{XY} = -(\epsilon_{xx} - \epsilon_{yy})\sin\theta\cos\theta + \cancelto{0}{\epsilon_{xy}}(\cos^2\theta - \sin^2\theta) \Rightarrow \epsilon_{XY} = \bar{0}.000168$$

Problem 2.47

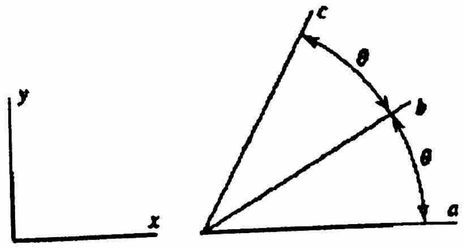
For a delta Rosette Shown ($\theta = 60°$)

If $\epsilon_a = 2450$, $\epsilon_b = 1360$, $\epsilon_c = -1310$

Find

① Principal strains ϵ_1 and ϵ_2

② principal planes



Solution

①

$$\epsilon_{1,2} = \frac{\epsilon_a + \epsilon_b + \epsilon_c}{3} \pm \frac{1}{3}\sqrt{(2\epsilon_a - \epsilon_b - \epsilon_c)^2 + 3(\epsilon_b - \epsilon_c)^2}$$

$$\epsilon_{1,2} = \frac{2450 + 1360 + {}^{-}1310}{3} \pm \frac{1}{3}\sqrt{(2\times 2450 - 1360 - ({}^{-}1310))^2 + 3(1360 - ({}^{-}1310))^2}$$

$$= 833.33 \pm 2233.81$$

$$\epsilon_1 = 3067.14$$

$$\epsilon_2 = 1400.48$$

②

$$\tan 2\theta_p = \frac{\sqrt{3}\,(\epsilon_b - \epsilon_c)}{2\epsilon_a - \epsilon_b - \epsilon_c} = \frac{\sqrt{3}\,(1360 - ({}^{-}1310))}{(2)(2450) - 1360 - ({}^{-}1310)}$$

$$\tan 2\theta_p = 0.9535 \Rightarrow 2\theta_p = \tan^{-1}(0.9535)$$

$$2\theta_p = 43.64° \Rightarrow \boxed{\theta_p = 21.82°}$$