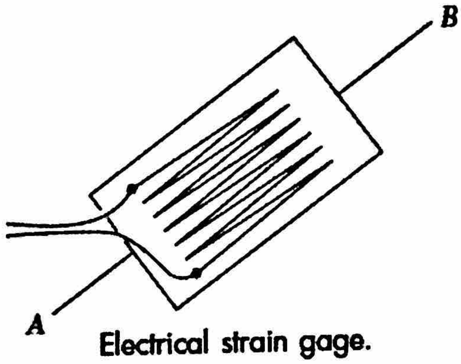
2.9 Strain measurements and strain Rosettes

* In Practical applications, it is important to measure strains of loaded structures to, consequently, compute stresses and build safe designs.

* Strain gages are the most common tool in strain measurements.

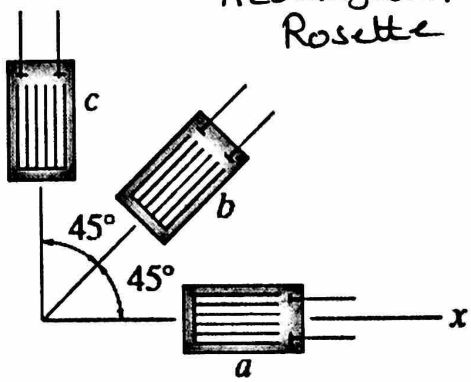


Electrical strain gage.

* For strain measurements, commonly we require three extensional strain gages in three different Directions to be attached at a point.

* Strain Rosettes are common form of such strain measurement configurations In which strain gages are separated by an equal angle θ

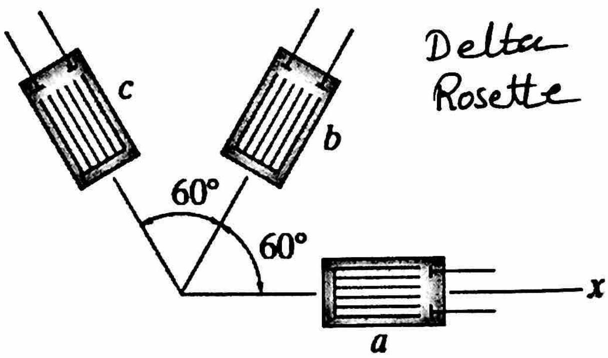
Rectangular Rosette



45° strain rosette

* Two common strain Rosette types
  ① Rectangular Rosette $\theta = 45°$
  ② Delta Rosette $\theta = 60°$

Delta Rosette



60° strain rosette

* To compute strains from measurements, we require strain transformation

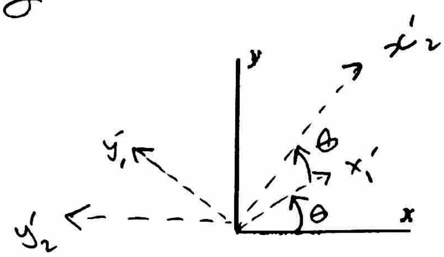
* plane strain state

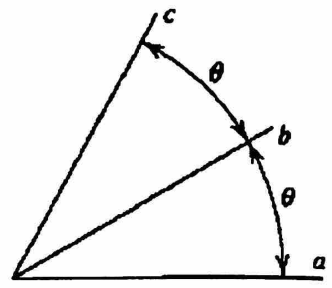
Example

For strain Rosette shown,

If it measures ϵ_a, ϵ_b and ϵ_c

- Find ϵ_{xx}, ϵ_{yy} and ϵ_{xy}





Solution

(1)

$$\epsilon_{xy} = \epsilon_a$$

(2)

$$\epsilon_b = \epsilon_{x_1' x_1'} = \epsilon_{xx} \cos^2\theta + \epsilon_{yy} \sin^2\theta + 2\,\epsilon_{xy} \sin\theta \cos\theta$$

(3)

$$\epsilon_c = \epsilon_{x_2' x_2'} = \epsilon_{xx} \cos^2 2\theta + \epsilon_{yy} \sin^2 2\theta + 2\epsilon_{xy} \sin 2\theta \cos 2\theta$$

Matrix Form

$$\begin{bmatrix} 1 & 0 & 0 \\ \cos^2\theta & \sin^2\theta & \sin\theta\cos\theta \\ \cos^2 2\theta & \sin^2 2\theta & \sin 2\theta \cos 2\theta \end{bmatrix} \begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} \epsilon_a \\ \epsilon_b \\ \epsilon_c \end{Bmatrix}$$

$$[A]\{x\} = \{c\} \quad \Rightarrow \quad \{x\} = [A]^{-1}\{c\}$$

$$\epsilon_{xx} = \epsilon_a$$

$$\epsilon_{yy} = \frac{(\epsilon_a - 2\epsilon_b)\, 4 \sin 4\theta + 2\epsilon_c \sin 2\theta}{4 \sin^2\theta \, \sin 2\theta}$$

$$\epsilon_{xy} = \frac{2\epsilon_a(\sin^2\theta \cos^2 2\theta - \sin^2 2\theta \cos^2\theta) + 2(\epsilon_b \sin^2 2\theta - \epsilon_c \sin^2\theta)}{4 \sin^2\theta \, \sin 2\theta}$$

For Delta Rosette $\theta = 60^\circ$

$$\epsilon_{xx} = \epsilon_a$$

$$\epsilon_{yy} = \frac{2(\epsilon_b + \epsilon_c) - \epsilon_a}{3}$$

$$\epsilon_{xy} = \frac{\epsilon_b - \epsilon_c}{\sqrt{3}}$$

For Rectangular Rosette $\theta = 45^\circ$

$$\epsilon_{xx} = \epsilon_a$$

$$\epsilon_{yy} = \epsilon_c$$

$$\epsilon_{xy} = \epsilon_b - \frac{1}{2}(\epsilon_a + \epsilon_c)$$

Problem 2.45 textbook

For a rectangular rosette, use 2D Mohr's circle to find
① principal strains
② principal strain plane

Solution

① Principal strains

$$\epsilon_{1,2} = \epsilon_{avg} \pm R$$

$$\epsilon_{avg} = \frac{1}{2}(\epsilon_{xx} + \epsilon_{yy})$$

$$R = \sqrt{\frac{1}{4}(\epsilon_{xx} - \epsilon_{yy})^2 + \epsilon_{xy}^2}$$

$$\epsilon_{Avg} = \frac{1}{2}\left(\epsilon_a + \epsilon_c\right)$$

$$R = \sqrt{\frac{1}{4}(\epsilon_a - \epsilon_c)^2 + \left(\epsilon_b - \frac{1}{2}(\epsilon_a + \epsilon_c)\right)^2} = \frac{1}{2}\sqrt{(\epsilon_a - \epsilon_c)^2 + (2\epsilon_b - \epsilon_a + \epsilon_a)^2}$$

$$\epsilon_1 = \frac{\epsilon_a + \epsilon_c}{2} + R$$

$$\epsilon_2 = \frac{\epsilon_a + \epsilon_c}{2} - R$$

② Principal strain plane

$$\tan 2\theta_p = \frac{2\epsilon_{xy}}{\epsilon_{xx} - \epsilon_{yy}} = \frac{2\epsilon_b - \epsilon_a - \epsilon_c}{\epsilon_a - \epsilon_c}$$

Problem 2.46 – textbook

For a delta rosette, use 2D Mohr's Circle to find

1. Principal strains
2. Principal strain plane

Solution

① Principal Strains

$$\epsilon_{avg} = \frac{1}{2}\left(\epsilon_{xx} + \epsilon_{yy}\right)$$

$$\epsilon_{1,2} = \epsilon_{Avg} \pm R \qquad R = \sqrt{\frac{1}{4}\left(\epsilon_{xx} - \epsilon_{yy}\right)^2 + \epsilon_{xy}^2}$$

$$\epsilon_{Avg} = \frac{1}{2}\left(\epsilon_a + \frac{2(\epsilon_b + \epsilon_c) - \epsilon_a}{3}\right)$$

$$= \frac{1}{2}\left(\epsilon_a + \frac{2}{3}\epsilon_b + \frac{2}{3}\epsilon_c - \frac{1}{3}\epsilon_a\right) = \frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{3}\epsilon_a + \frac{\cancel{2}}{3}\epsilon_b + \frac{\cancel{2}}{3}\epsilon_c\right)$$

$$\epsilon_{Avg} = \frac{\epsilon_a + \epsilon_b + \epsilon_c}{3}$$

$$R = \sqrt{\left(\frac{\epsilon_a - \frac{2}{3}\epsilon_b - \frac{2}{3}\epsilon_c + \frac{1}{3}\epsilon_a}{2}\right)^2 + \frac{1}{3}\left(\epsilon_b - \epsilon_c\right)^2}$$

$$= \sqrt{\left(\frac{\frac{\cancel{4}^{2}}{3}\epsilon_a - \frac{\cancel{2}}{3}\epsilon_b - \frac{\cancel{2}}{3}\epsilon_c}{\cancel{2}}\right)^2 + \frac{1}{3}\left(\epsilon_b - \epsilon_c\right)^2} = \frac{1}{3}\sqrt{\left(2\epsilon_a - \epsilon_b - \epsilon_c\right)^2 + 3\left(\epsilon_b - \epsilon_c\right)^2}$$

$$\epsilon_1 = \frac{1}{3}\left(\epsilon_a + \epsilon_b + \epsilon_c\right) + \frac{1}{3}\left(\left(2\epsilon_a - \epsilon_b - \epsilon_c\right)^2 + 3\left(\epsilon_b - \epsilon_c\right)\right)^{1/2}$$

$$\epsilon_2 = \frac{1}{3}\left(\epsilon_a + \epsilon_b + \epsilon_c\right) - \frac{1}{3}\left(\left(2\epsilon_a - \epsilon_b - \epsilon_c\right)^2 + 3\left(\epsilon_b - \epsilon_c\right)\right)^{1/2}$$

2 Principal Strain planes

$$\tan 2\theta_p = \frac{2\epsilon_{xy}}{\epsilon_{xx} - \epsilon_{yy}} = \frac{\frac{2}{\sqrt{3}}(\epsilon_b - \epsilon_c)}{\epsilon_a - \frac{2}{3}\epsilon_a - \frac{2}{3}\epsilon_b + \frac{\epsilon_a}{3}}$$

$$\tan 2\theta_p = \frac{\sqrt{3}(\epsilon_b - \epsilon_c)}{2\epsilon_a - \epsilon_b - \epsilon_c}$$