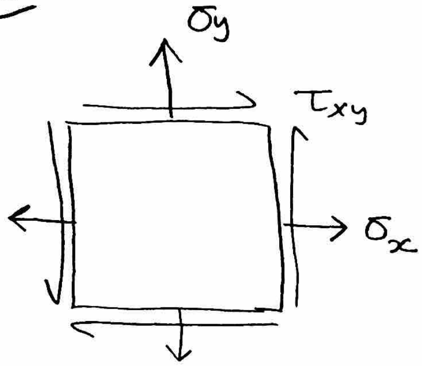
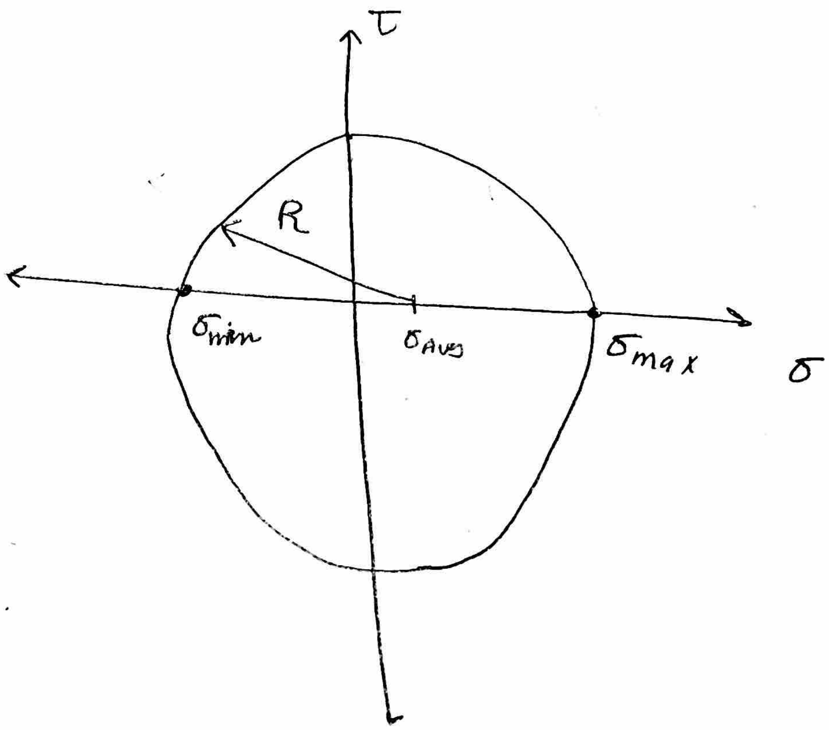
* Revision on Mohr's Circle

σ_x, σ_y and τ_{xy}



$$\sigma_{Avg} = \frac{\sigma_x + \sigma_y}{2}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$



$$\sigma_{max} = \sigma_{Avg} + R$$

$$\sigma_{min} = \sigma_{Avg} - R$$

principal planes (θ_p)

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Planes of max. shear (θ_s)

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

$$\Rightarrow \quad \tan 2\theta_p = \frac{-1}{\tan 2\theta_s} \quad \Rightarrow \quad \tan 2\theta_s \cdot \tan 2\theta_p + 1 = 0$$

only true if

$$2\theta_s = 2\theta_p \mp 90^\circ$$

$$\theta_s = \theta_p \mp 45^\circ$$

Example

For the stress state shown, Find:

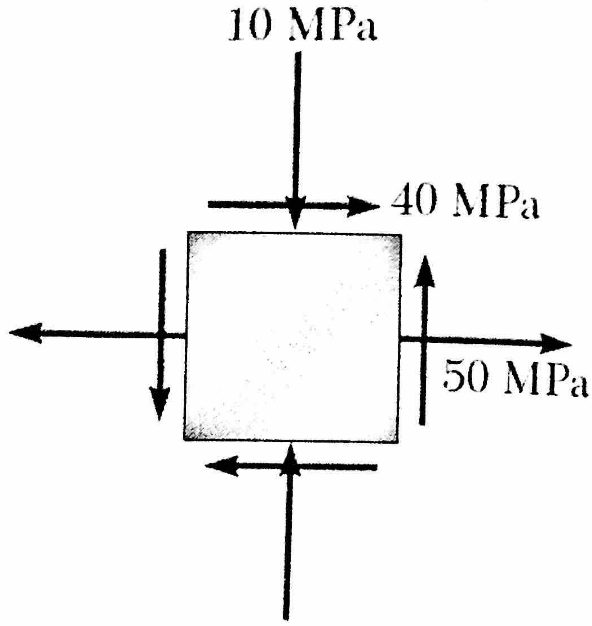
① Draw Mohr's Circle

② principal stress

③ max shear

④ Represent this stress state on Mohr's Circle

⑤ Principal planes and planes of max. shear.



Solution

① To draw Mohr's Circle, we need ① Center (σ_{Avg}) ② Radius (R)

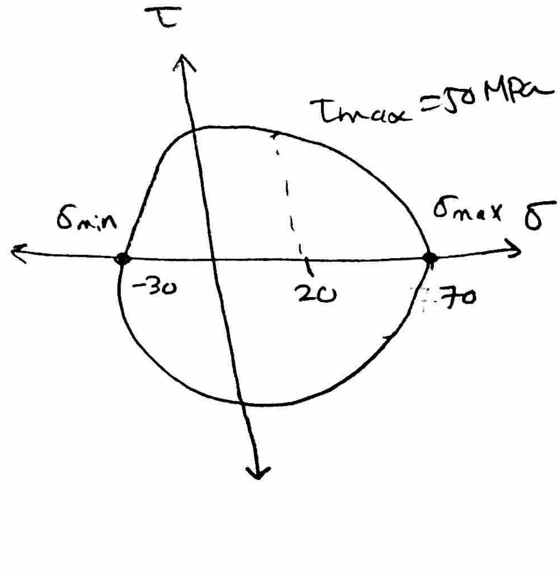
$$\sigma_{Avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{50 + {}^-10}{2} \Rightarrow \sigma_{Avg} = 20 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{50-(-10)}{2}\right)^2 + 40^2} \Rightarrow R = 50 \text{ MPa}$$

② Principal stresses

$$\sigma_{max} = \sigma_{Avg} + R = 20 + 50 = 70 \text{ MPa}$$

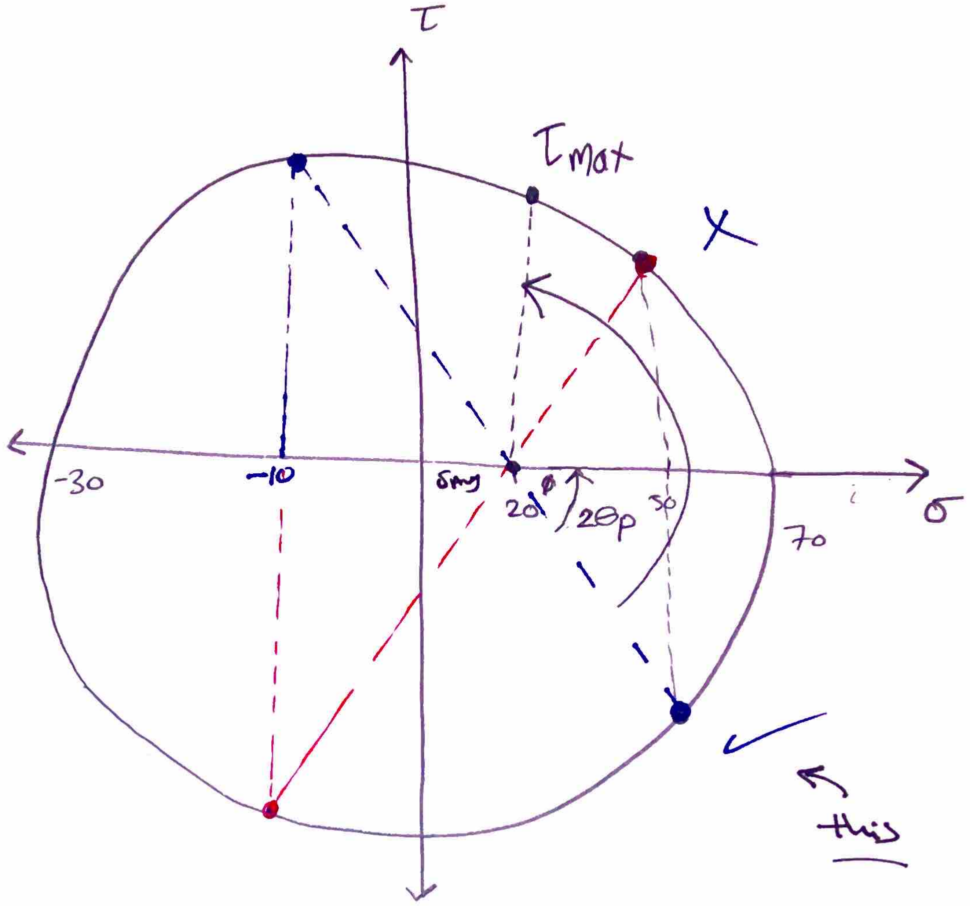
$$\sigma_{min} = \sigma_{Avg} - R = 20 - 50 = -30 \text{ MPa}$$



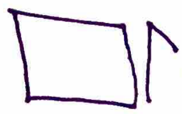
③ max shear

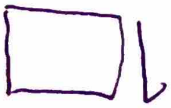
$$\tau_{max} = R = 70 \text{ MPa}$$

④ Stress state on Mohr's Circle



- Rule


$\Rightarrow$ below ($\sigma +$)


$\Rightarrow$ Above ($\sigma -$)

$$\tan\phi = \frac{\tau_{xy}}{\sigma_x - \sigma_{Avg}}$$

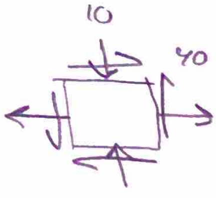
$$\sigma_x - \sigma_{avg} = \sigma_x - \frac{\sigma_x + \sigma_y}{2}$$

$$= \frac{2\sigma_x}{2} - \frac{\sigma_x}{2} + \frac{\sigma_y}{2}$$

$$= \frac{\sigma_x}{2} - \frac{\sigma_y}{2} = \frac{\sigma_x - \sigma_y}{2}$$

$$\tan\phi = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \leftarrow \tan 2\theta_p \Rightarrow \phi = 2\theta_p$$

⑤ principal planes



$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(40)}{50-(-10)} = \frac{80}{60} = \frac{4}{3}$$

$$2\theta_p = 53.13^0$$

$$\theta_p = 26.57^0 \quad \text{CCW}$$

Principal shear

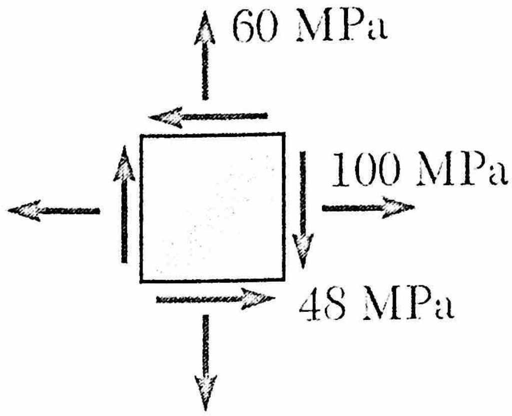
$$2\theta_s = 2\theta_p + 90^0 \Rightarrow 2\theta_s = 53.13 + 90^0 = 143.13$$

$$\theta_s = 71.57^0 \quad \text{CCW}$$

Example

For stress state shown

1. Draw Mohr's Circle
2. principal stresses
3. max. shear
4. Represent this stress state on Mohr's Circle
5. principal planes (θ_p) and planes of max. shear (θ_s)



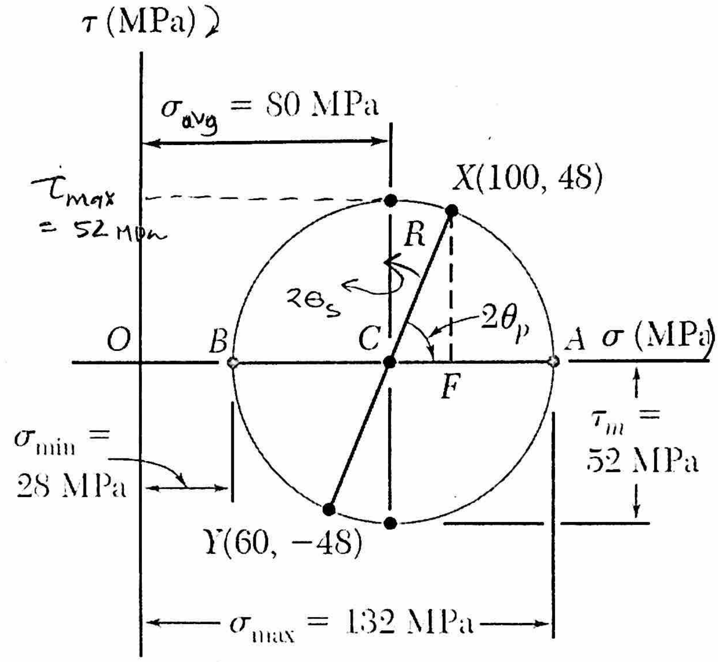
Solution

① Mohr's Circle → center (σ_{Avg}), Radius (R)

$$\sigma_{Avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{100+60}{2} = 80 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{100+60}{2}\right)^2 + 48^2} = 52 \text{ MPa}$$



② principal stresses

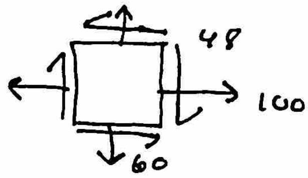
$$\sigma_{max} = \sigma_{Avg} + R = 80 + 52 = 132 \text{ MPa}$$

$$\sigma_{min} = \sigma_{Avg} - R = 80 - 52 = 28 \text{ MPa}$$

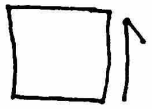
③ Max. shear

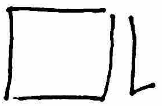
$$\tau_{max} = R = 52 \text{ MPa}$$

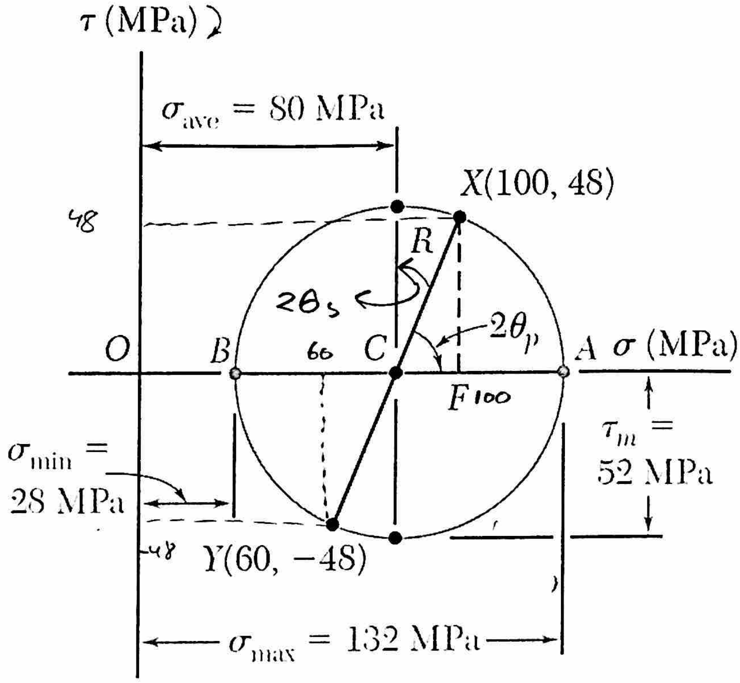
④ Represent state on MC



- Rule

 $\Rightarrow$ Below (σ+)

 $\Rightarrow$ Above (σ+)



⑤ principal planes (θ_p)

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(48)}{100 - 60}$$

$\Rightarrow 2\theta_p = 67.38^\circ \Rightarrow \theta_p = 33.69^\circ$ clock wise (cw).

- planes of max shear

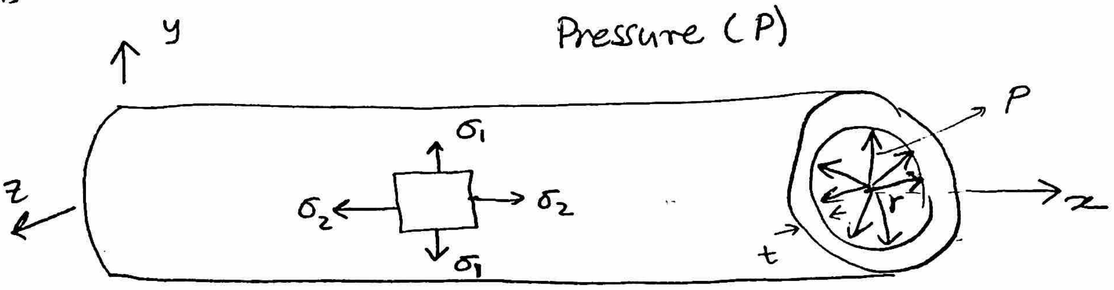
$$2\theta_s = 90 - 2\theta_p \Rightarrow$$
$$= 90 - 67.38$$

$2\theta_s = 22.62^\circ \Rightarrow \theta_s = 11.31^\circ$ CCW

* Thin-walled pressure vessels

① Cylinders

Inner radius (r)
Thickness (t)
Pressure (P)



$\sigma_1 \Rightarrow$ Hoop stress $\quad \sigma_1 = \dfrac{Pr}{t}$

$\sigma_2 \Rightarrow$ Longitudinal stress $\quad \sigma_2 = \dfrac{Pr}{2t}$

$$\boxed{\sigma_1 = 2\sigma_2}$$

Draw Mohr's Circle

$$\sigma_{Avg} = \frac{\sigma_2 + \sigma_1}{2} = \frac{\sigma_2 + 2\sigma_2}{2} = \frac{3}{2}\sigma_2$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \cancel{\tau_{xy}^2}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \sqrt{\left(\frac{\sigma_2 - \sigma_1}{2}\right)^2} = \sqrt{\left(\frac{\sigma_2 - 2\sigma_2}{2}\right)^2}$$

$$\Rightarrow \boxed{\frac{1}{2}\sigma_2 = R}$$

Principal stresses $\Rightarrow$ $\sigma_{max} = \sigma_{Avg} + R$

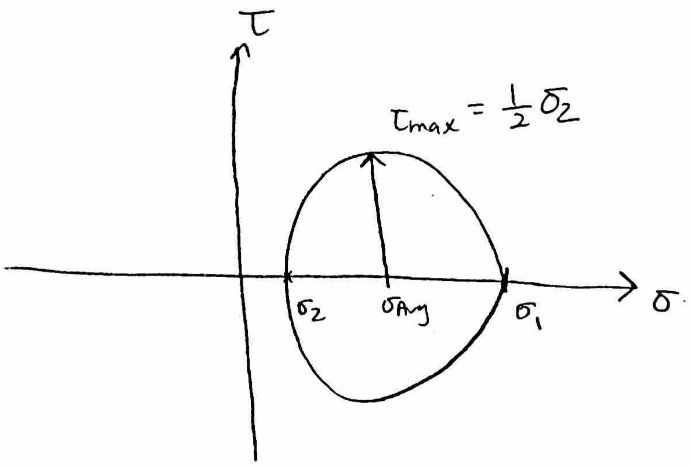
$= \frac{3}{2}\sigma_2 + \frac{1}{2}\sigma_2 =$

$= 2\sigma_2 = \sigma_1$

$\sigma_{min} = \sigma_{Avg} - R$

$= \frac{3}{2}\sigma_2 - \frac{1}{2}\sigma_2$

$= \sigma_2$



$\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y} = 0 \Rightarrow \theta_p = 0$

$\tan 2\theta_s = -\dfrac{\sigma_x - \sigma_y}{2\tau_{xy}} = \infty$

$\Rightarrow 2\theta_s = 90$

$\theta_s = 45$