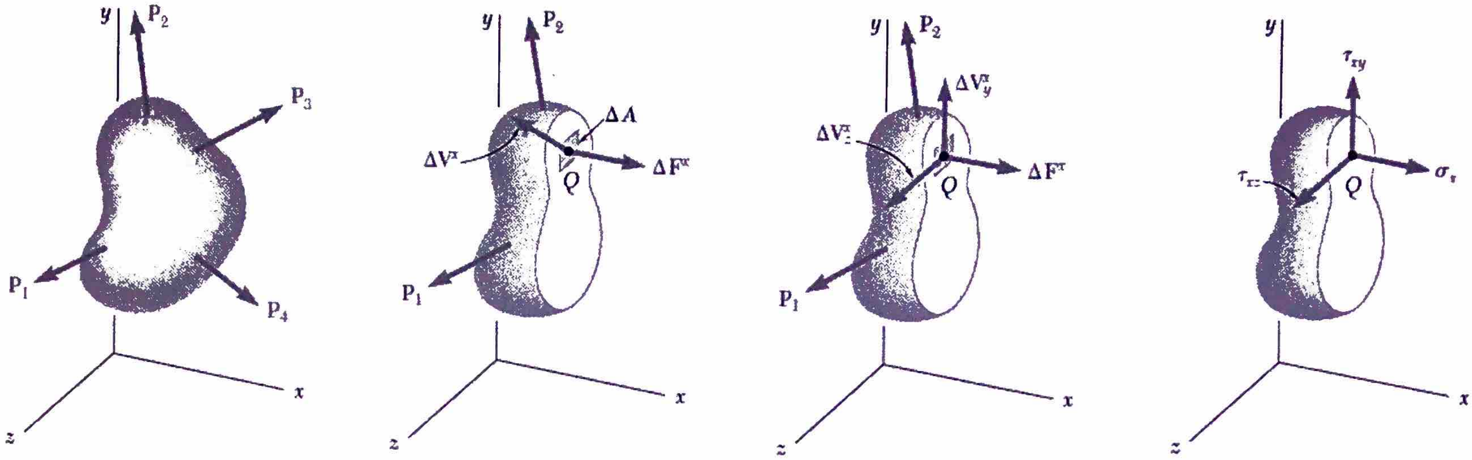
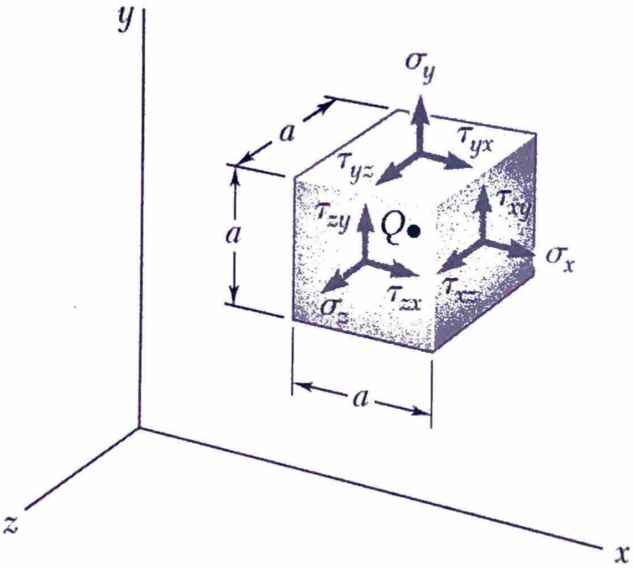
Chapter 7

* Transformations of stress and strain





$\sigma_{xx} = \sigma_x$

$\sigma_{yy} = \sigma_y$

$\sigma_{zz} = \sigma_z$

* stress notation

σ_{ij} or τ_{ij}

→ plane direction (normal to the plane)

→ stress direction

3-dimensional stress state

$+\circlearrowleft \sum M = 0$

$\tau_{xy} = \tau_{yx}$

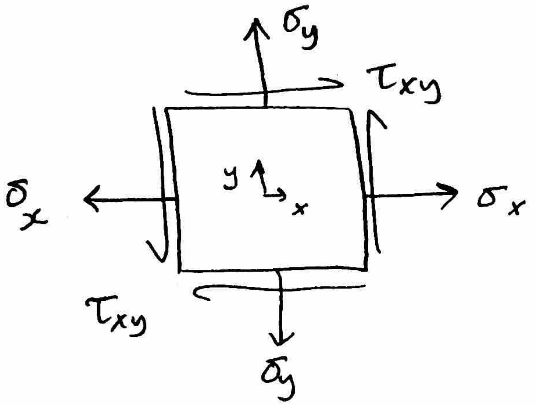
$\tau_{xz} = \tau_{zx}$

$\tau_{yz} = \tau_{zy}$

$$\Rightarrow \quad \sigma = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix}$$

only 6 components

* plane stress state $(\sigma_z = \tau_{xz} = \tau_{zx} = \tau_{yz} = \tau_{zy} = 0)$



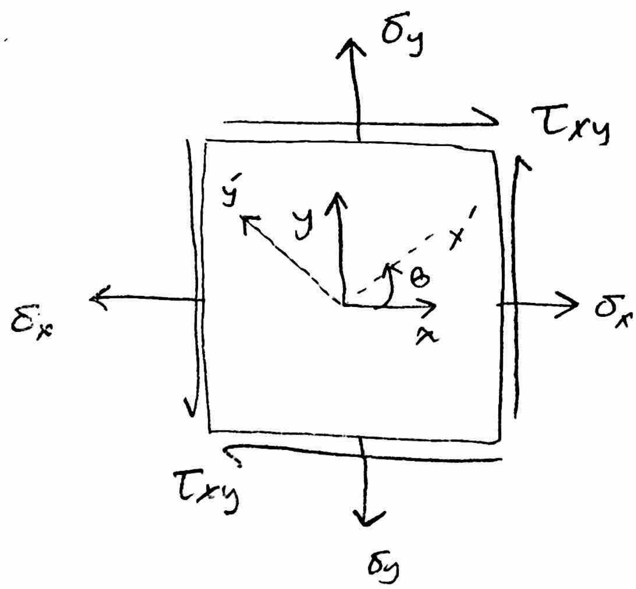
$$\sigma = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix}$$

* stress Transformation

σ_x, σ_y, τ_{xy}

now, we have:

$\sigma_{x'}$, $\sigma_{y'}$, $\tau_{x'y'}$



- To Find $\sigma_{x'}$, $\sigma_{y'}$, $\tau_{x'y'}$,

  $\Sigma F = 0$

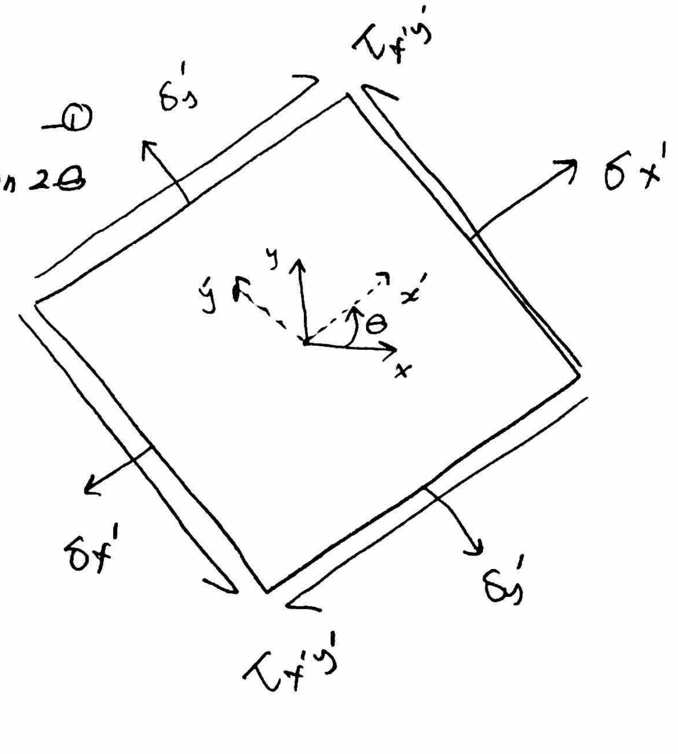
  $\Sigma M = 0$

$\Rightarrow$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \quad -①$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta \quad -②$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \quad -③$$



Transformed stress state

Take eq(1) and eq(3)

$$\left(\sigma_{x'} - \frac{\sigma_x+\sigma_y}{2}\right)^2 = \left(\overbrace{\frac{\sigma_x-\sigma_y}{2}}^{A}\cos 2\theta + \overbrace{\tau_{xy}}^{B}\sin 2\theta\right)^2$$

$$\left(\tau_{x'y'}\right)^2 = \left(-\overbrace{\frac{\sigma_x-\sigma_y}{2}}^{A}\sin 2\theta + \tau_{xy}\cos 2\theta\right)^2 \quad +$$

$$\left(\sigma_{x'} - \underbrace{\frac{\sigma_x+\sigma_y}{2}}_{\sigma_{Avg}}\right)^2 + \left(\tau_{x'y'}\right)^2 = \underbrace{\boxed{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}}_{R^2} \leftarrow \text{How?}$$

$$\left(\sigma_{x'} - \sigma_{Avg}\right)^2 + \left(\tau_{x'y'}\right) = R^2 \longleftarrow \text{Equation of a circle}$$

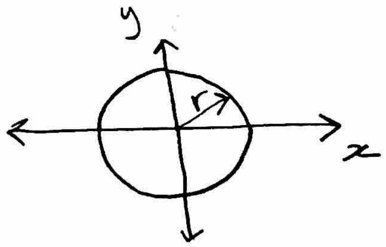
$$\left(A\cos 2\theta + B\tau_{xy}\sin 2\theta\right)^2 = A^2\cos^2 2\theta + 2AB\sin 2\theta\cos 2\theta + B^2\sin^2 2\theta$$

$$\left(-A\sin 2\theta + B\cos 2\theta\right)^2 = A^2\sin^2 2\theta - 2AB\sin 2\theta\cos 2\theta + B^2\cos^2 2\theta \quad +$$

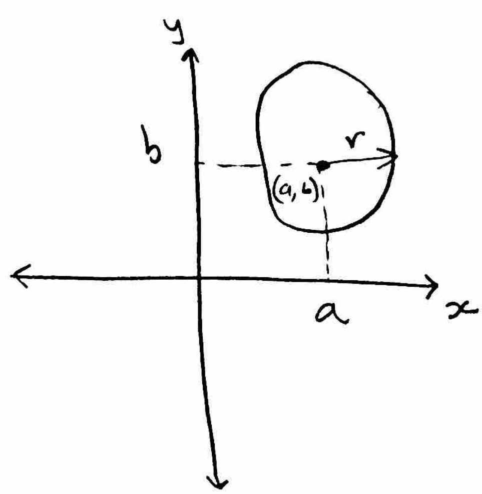
$$A^2\underbrace{\left(\sin^2 2\theta + \cos^2 2\theta\right)}_{=1} + B^2\underbrace{\left(\sin^2 2\theta + \cos^2 2\theta\right)}_{=1}$$

$$A^2 + B^2 = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2$$

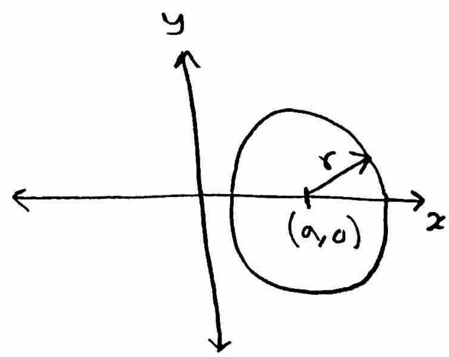
$x^2+y^2=r^2$



$(x-a)^2+(y-b)^2=r^2$



$(x-a)^2+y^2=r^2$

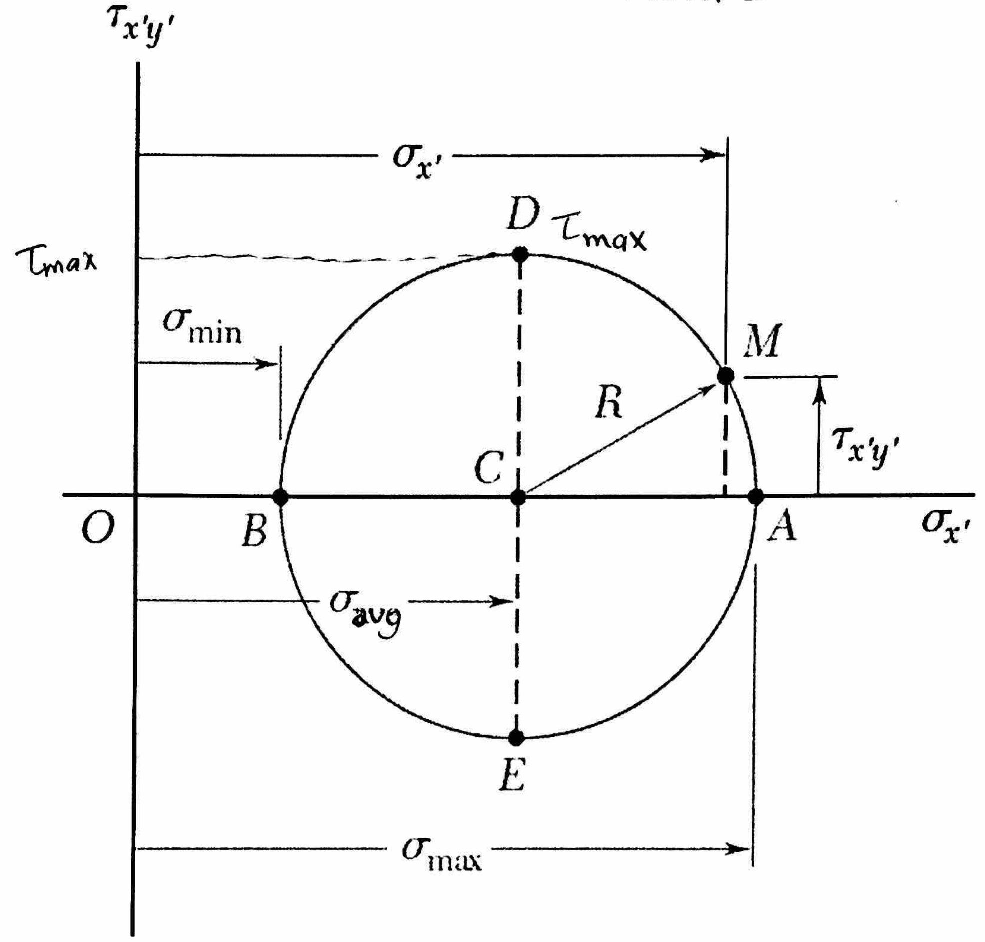


Same as

$$\Big(\underset{x}{\underset{\Uparrow}{\sigma_{x'}}} - \sigma_{Avg}\Big)^2 + \underset{y}{\underset{\Downarrow}{\tau_{x'y'}^2}} = R^2$$

$$(\sigma_{x'} - \sigma_{Avg})^2 + \tau_{x'y'}^2 = R^2$$

Mohr's Circle



center $(\sigma_{Avg}, 0)$: $\sigma_{Avg} = \dfrac{\sigma_x + \sigma_y}{2}$

Radius (R) : $R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

At points A and B

$\sigma_A = \sigma_{max} = \sigma_{Avg} + R$ max. principal stress

$\sigma_B = \sigma_{min} = \sigma_{Avg} - R$ min Principal stress

σ_{max} and σ_{min} ⇒ Principal stresses at which $\tau = 0$

At point D

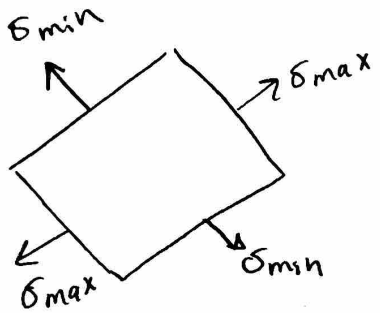
$\tau = \tau_{max} = R$

Question (1) What is θ makes $\tau_{x'y'} = 0$?

Answer: Eq(3) $\Rightarrow$ $\tau_{x'y'} = -\dfrac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$

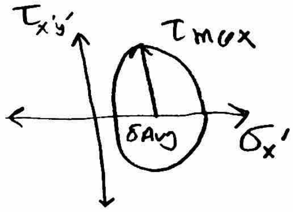
$$\tau_{x'y'} = 0 = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\Rightarrow \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$



θ_p : principal planes

Question (2): What θ makes the max shear ?



Answer: Eq(1) $\Rightarrow$ $\underset{\sigma_{Avg}}{\underset{\parallel}{\sigma_x}} = \overbrace{\boxed{\dfrac{\sigma_x + \sigma_y}{2}}}^{\sigma_{Avg}} + \dfrac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$

$$\cancel{\sigma_{Avg}} = \cancel{\sigma_{Avg}} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\Rightarrow \tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

θ_s → plane of max shear