Chapter 5: Design for Vibration suppression

In this Chapter, we will only discuss 5.3 vibration Absorbers and this will be the end of the class.

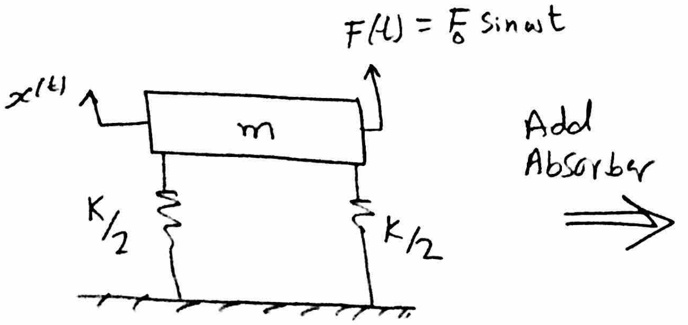
* 5.3 Vibration Absorbers :-

* Vibration absorbers are commonly used to prevent a primary devices vibration

* Vibration absorbers are a second mass-spring system that are attached to the primary device

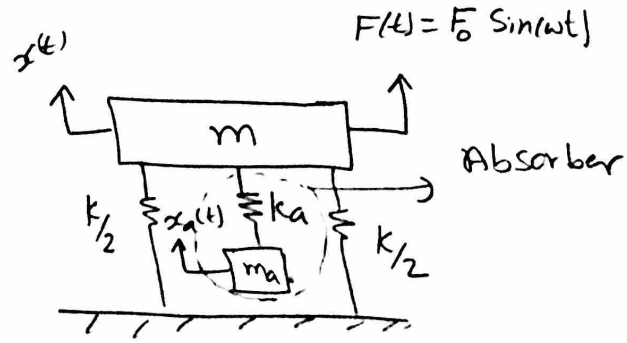
* The major effect of adding a second spring-mass system is to change from a single-degree-of-freedom system to a two-degree-of-freedom system with two natural frequencies.

* The values of the mass (m_a) and stiffness (k_a) of the absorber are choosen such that the motion of the original mass (m) is reduced



Primary system (SDOF)

Figure 1



system with absorber (MDOF)

Figure 2

The equation of motion of the 2DOF system of figure 2, is:-

$$[m]\{\ddot{x}\} + [k]\{x\} = \{F(t)\} \quad -(1)$$

<u>where</u>

$[m] = \begin{bmatrix} m & 0 \\ 0 & m_a \end{bmatrix}$ (m: device mass, ma: absorber mass) , $[k] = \begin{bmatrix} k+k_a & -k_a \\ k_a & k_a \end{bmatrix}$, k: device stiffness, ka: Absorber stiffness

$\{\ddot{x}\} = \begin{Bmatrix} \ddot{x}(t) \\ \ddot{x}_1 \end{Bmatrix}$ $\ddot{x}(t)$: Device Acceleration, $\ddot{x}_a(t)$: Absorber acceleration, $\{x(t)\} = \begin{Bmatrix} x(t) \\ x_a(t) \end{Bmatrix}$ Device motion, Absorber motion

$\{F(t)\} = \begin{Bmatrix} F_0 \sin\omega t \\ 0 \end{Bmatrix}$ Applied force on device

<u>To solve this equation</u>

$\{x(t)\} = \begin{Bmatrix} x(t) \\ x_a(t) \end{Bmatrix} = \begin{Bmatrix} X \sin\omega t \\ X_a \sin\omega t \end{Bmatrix}$ → substitute in eq(1) and re-arrange

$$\underbrace{\begin{bmatrix} k+k_a-m\omega^2 & -k_a \\ -k_a & k_a-m_a\omega^2 \end{bmatrix}}_{[A]} \underbrace{\begin{Bmatrix} X \\ X_a \end{Bmatrix}}_{\{x\}} \sin\cancel{\omega}t = \underbrace{\begin{Bmatrix} F_0 \\ 0 \end{Bmatrix}}_{\{F\}} \sin\cancel{\omega}t$$

$$[A]\{x\} = \{F\}$$

$$\Rightarrow \{x\} = [A]^{-1}\{F\}$$

$$\Rightarrow \begin{Bmatrix} X \\ X_a \end{Bmatrix} = \frac{1}{(k+k_a-m\omega^2)(k_a-m_a\omega^2)-k_a^2} \begin{Bmatrix} (k_a-m_a\omega^2)F_0 \\ k_a F_0 \end{Bmatrix}$$

$$X = \frac{(k_a - m_a\omega^2)F_o}{(k + k_a - m\omega^2)(k_a - m_a\omega^2) - k_a^2} \quad \text{— eq (2.a)}$$

$$X_a = \frac{k_a F_o}{(k + k_a - m\omega^2)(k_a - m_a\omega^2) - k_a^2} \quad \text{eq (2.b)}$$

The goal of adding the absorber is to make the displacement of the primary mass is zero ($X = 0$) ← Eq (2.a)

So, $(k_a - m_a\omega^2)F_o = 0 \Rightarrow \omega^2 = \dfrac{k_a}{m_a}$

apply $k_a = m_a\omega^2$ in equation (2.b), so

$X_a = -\dfrac{F_o}{k_a}$, so, $x_a(t) = X_a \sin(\omega t) \Rightarrow x_a(t) = -\dfrac{F_o}{k_a}\sin(\omega t)$

Now, let's define the ratio between absorber mass to the primary mass as:-

$$\mu = \frac{m_a}{m} \quad \text{— Eq (3)}$$

and $\omega_p = \sqrt{\dfrac{k}{m}}$ Natural Frequency of the primary device without attaching the absorber

$\omega_a = \sqrt{\dfrac{k_a}{m_a}}$ natural Frequency of absorber before adding it to the device.

→ apply in eq (2.a)

$$\frac{k_a}{k} = \frac{\omega_a^2 m_a}{\omega_p^2 m} = \frac{\omega_a^2}{\omega_p^2}\mu \Rightarrow \boxed{\frac{k_a}{k} = \mu\beta^2}$$

$$\beta^2 = \frac{\omega_a^2}{\omega_p^2} \text{ or } \beta = \frac{\omega_a}{\omega_p}$$

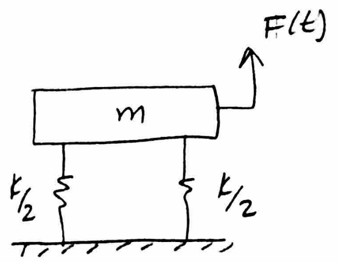
$$\frac{XK}{F_0} = \frac{1 - \omega^2/\omega_n^2}{\left[1 + \mu\beta^2 - \left(\frac{\omega}{\omega_p}\right)^2\right]\left[1 - \left(\frac{\omega}{\omega_a}\right)^2 - \mu\beta^2\right]}$$

where ω_n = natural Frequency of the system with absorber

where

$$\left(\frac{\omega_n}{\omega_a}\right)^2 = \frac{1 + \beta^2(1+\mu)}{2\beta^2} \pm \frac{1}{2\beta^2}\sqrt{\beta^4(1+\mu)^2 - 2\beta^2(1-\mu) + 1}$$

Example :-



If $m = 73.16$ kg

$K = 2600$ N/m

$F(t) = 13 \sin(18.85t)$

Design the absorber (Find k_a and m_a) to have $X_a = 2 \times 10^{-3}$ m

Solution

stiffness +ve

$$X_a = -\frac{F_0}{k_a} \Rightarrow k_a = \left|\frac{F_0}{X_a}\right| = \frac{13}{2 \times 10^{-3}} \Rightarrow \boxed{k_a = 6500 \text{ N/m}}$$

also,

$$\omega_a^2 = \frac{k_a}{m_a} \Rightarrow m_a = \frac{k_a}{\omega_a^2} = \frac{6500}{(18.85)^2}$$

$$\Rightarrow \boxed{m_a = 18.29 \text{ kg}}$$