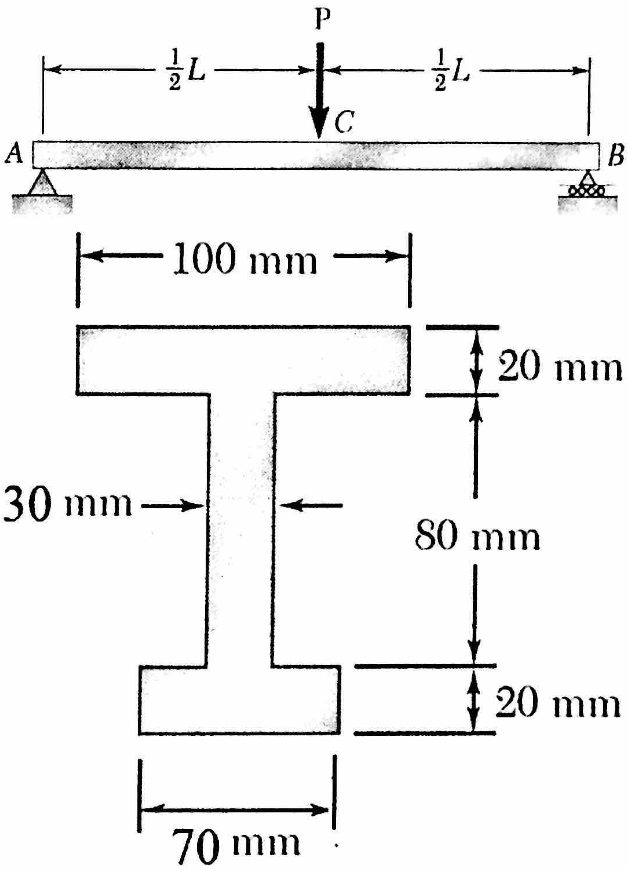
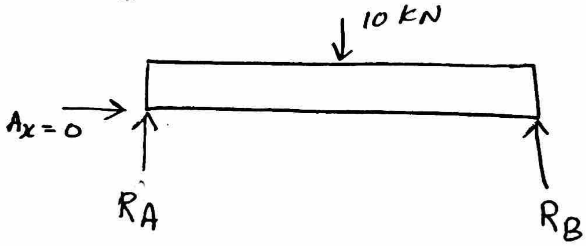
**Example**: Considering the beam with cross-section and the loading shown. If $L = 2\ m$ and $P = 10\ kN$ Answer the following:

1. Draw free body diagram of the beam and determine reactions at points ***A*** and ***B***.
2. Draw shear and moment diagrams. Also, identify the location of maximum shear force and maximum bending moment.
3. Calculate the maximum normal stress in the beam due to bending. Is it tension or compression?
4. Calculate the maximum shear stress in the beam due to shearing forces.




Solution

① Free-Body-Diagram

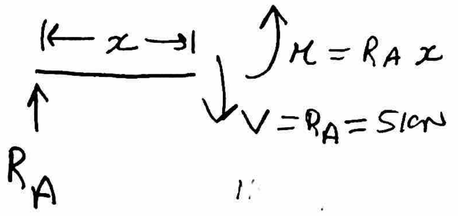


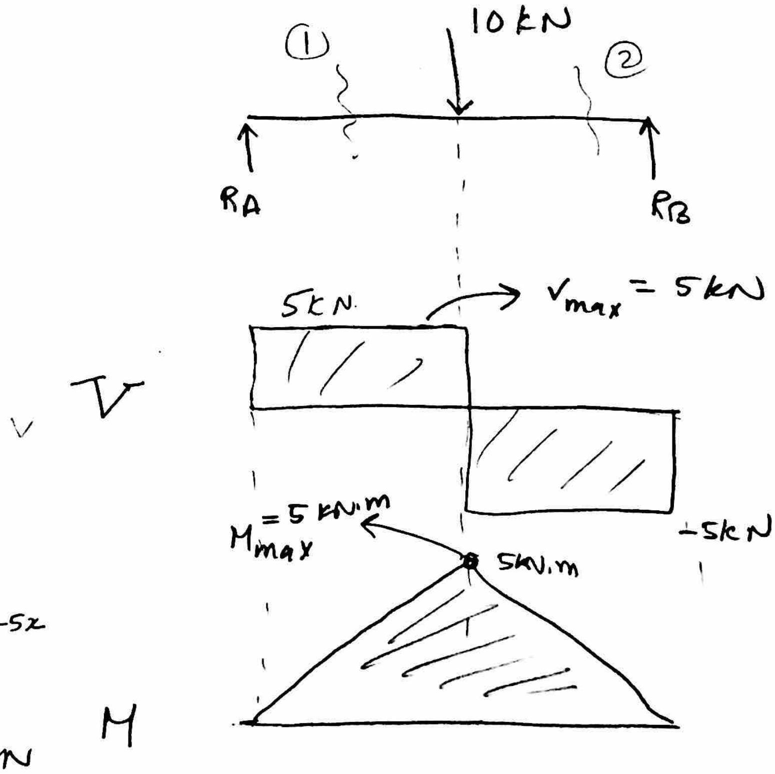

$+\uparrow \Sigma F = 0 \Rightarrow \boxed{R_A + R_B = 10\ kN}$

$+\circlearrowleft \Sigma M_A = 0 \Rightarrow (-10)(1) + R_B(2) = 0 \Rightarrow \boxed{R_B = 5\ kN} \Rightarrow \boxed{R_A = 5\ kN}$

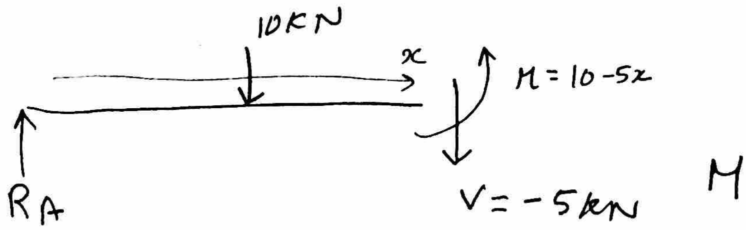
② Shear-Moment Diagrams

①

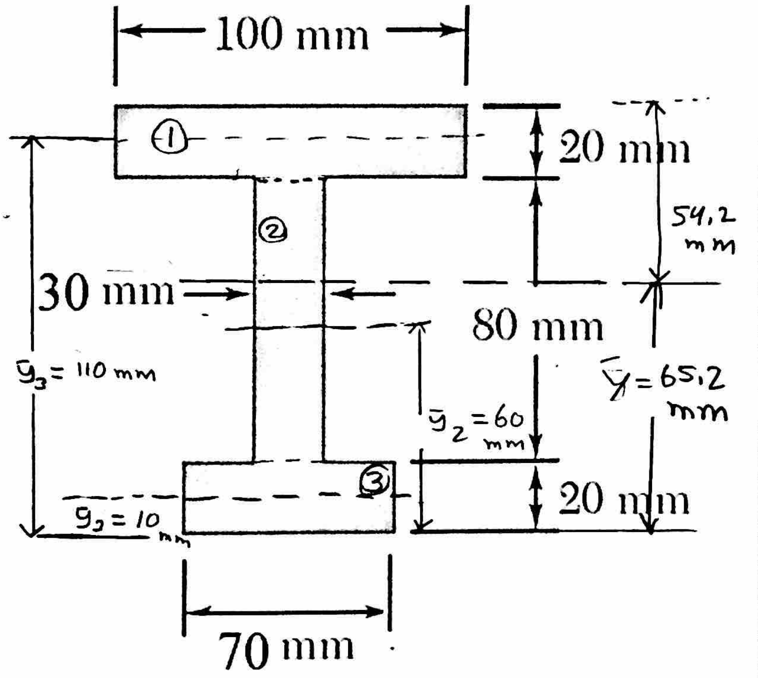



②



③ Max Normal Stress due to bending.




- We need to find centroid location and moment of Inertia

* Centroid

$$\bar{Y} = \frac{\sum_{i=1}^{3} A_i \bar{y}_i}{\sum_{i=1}^{3} A_i} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3}{A_1 + A_2 + A_3}$$

$$= \frac{(100)(20)(110) + (30)(80)(60) + (70)(20)(10)}{(100)(20) + (30)(80) + (70)(20)}$$

$\bar{Y} = 65.2$ mm

* Moment of Inertia

$$I = I_1 + I_2 + I_3$$

$$= (4.1 + 1.3 + 4.3) \times 10^6$$

$$= 9.7 \times 10^{+6}\ \text{mm}^4 = 9.7 \times 10^{-6}\ \text{m}^4$$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2$$

$$= \frac{1}{12}(100)(20)^3 + (100)(20)(110 - 65.3)^2$$

$$= 4.1 \times 10^{+6}\ \text{mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2$$

$$= \frac{1}{12}(30)(80)^3 + (30)(80)(60 - 65.2)^2$$

$$= 1.3 \times 10^6\ \text{mm}^4$$

$$I_3 = \frac{1}{12} b_3 h_3^3 + A_3 d_3^2$$

$$= \frac{1}{12}(70)(20)^3 + (70)(20)(65.2 - 10)^2$$

$$= 4.3 \times 10^{+6}\ \text{mm}^4$$

max normal stress is at the bottom surface $y = -65.2 \times 10^{-3}$ m

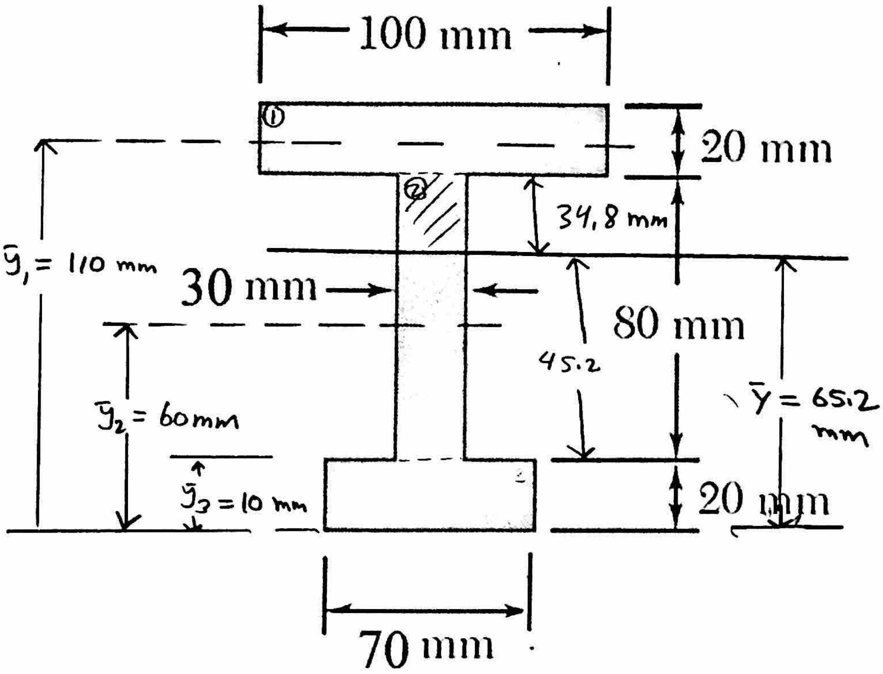
$$\sigma_{max} = \frac{-M_{max}\, y}{I} = -\frac{(5 \times 10^3)(-65.2)(10^{-3})}{9.7 \times 10^{-6}}$$

$$\sigma_{max} = 33.61\ \text{MPa}$$

(Tension)

④ Max shear stress due to Shear Force

max shear is located at centroid




$$\tau_{max} = \frac{V_{max}\, Q_{max}}{I\, t}$$

$$= \frac{(5)(10^3)\,(107.5)(10^{-6})}{(9.7)(10^{-6})\,(30)(10^{-3})}$$

$$\tau_{max} = 1.85 \text{ MPa}$$

$$Q_{max} = Q_1 + Q_2$$

$$= (89.3 + 18.2)\,10^{+3}$$

$$= 107.5 \times 10^{3} \text{ mm}^3$$

$$Q_{max} = 107.5 \times 10^{-6} \text{ m}^3$$

$$Q_1 = A_1 d_1 = (100)(20)(110 - 65.2)$$

$$= 89.3 \times 10^{3} \text{ mm}^3$$

$$Q_2 = (30)(34.8)\left(\frac{34.8}{2}\right)$$

$$= 18.2 \times 10^{3} \text{ mm}^3$$

This Example is very Important!