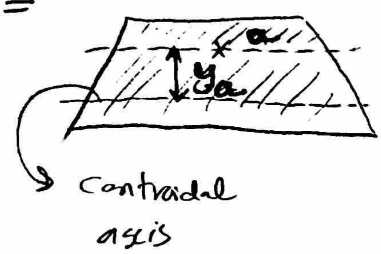
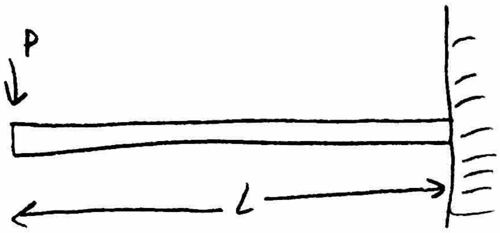
* Chapter Six shear stresses in Beams [6.1, 6.2, 6.3]

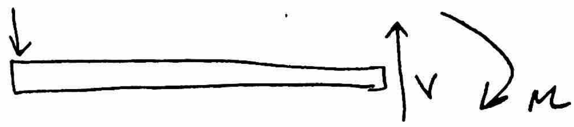
Normal stress at a

$$\sigma_a = \frac{-M y_a}{I}$$





F.B.D



$V = P$
$M = PL$

* shear stress at a

$$\tau = \frac{V Q_a}{I t}$$

V = Shear force [N]

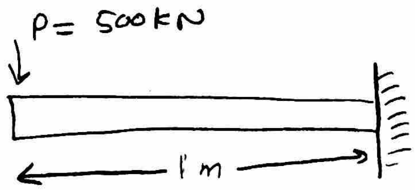
Q_a = First moment of area above or below point a $[m^3]$, $Q = A\,d$ (A → area; d → Distance between two centroids)

I : Moment of Inertia of the entire section $[m^4]$

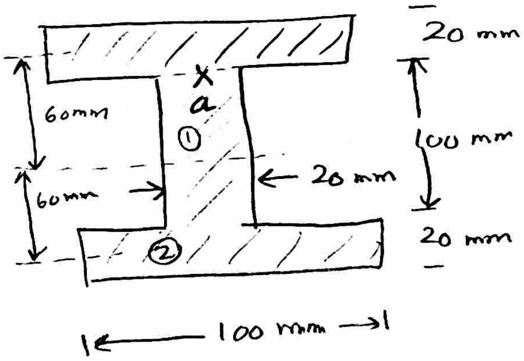
t : Smallest thickness [m]

Example

$I = 16.2 \times 10^{-6}\ m^4$



* Find shear stress at point (a)



$$\tau = \frac{V Q_a}{I t} = \frac{(500)(10^3)(12)(10^{-5})}{(16.2)(10^{-6})(20)(10^{-3})}$$

$$\boxed{\tau = 185.2\ MPa}$$

$V = P = 500kN$

$M = PL = 500\ kN.m$

Above a

$Q_a = A d = (20)(100)(60)$

$= 12 \times 10^4\ mm^3 = 12 \times 10^{-5}\ m^3$

$$\boxed{Q_a = 12 \times 10^{-5}\ m^3}$$

Below a

$Q_a = Q_1 + Q_2$

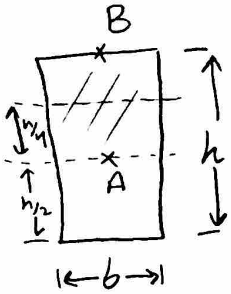
$Q_1 = A_1 d_1 = (20)(100)(0) = 0\ m^3$

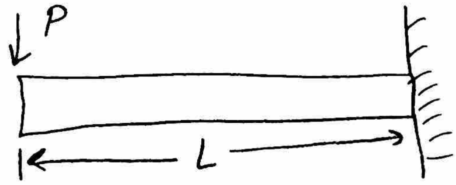
$Q_2 = A_2 d_2 = (20)(100)(60) = 12 \times 10^{4}\ mm^3$

$$Q_a = 12 \times 10^{-5}\ m^3$$

Example

Find shear stress at A and B





At point A

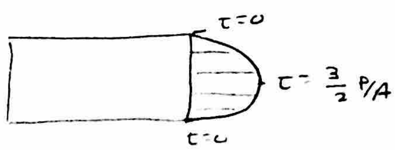
$V = P$

$M = PL$

$$\tau_A = \frac{VQ}{It}$$

$$= \frac{P\,(bh^2/8)}{\frac{1}{12}(bh^3)(b)} \quad , \quad Q = b\frac{h}{2} \cdot \frac{h}{4} = \frac{bh^2}{8}$$

$$\boxed{\tau_A = \frac{3}{2}\frac{P}{A}}$$



At point B

$$\tau_B = \frac{VQ}{It} = \frac{P\,(0)}{\frac{1}{12}(bh^3)(b)} \quad , \quad Q = Ad = (b)(h)(0)$$

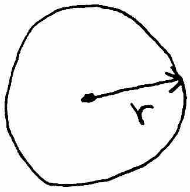
$$\boxed{\tau_B = 0}$$

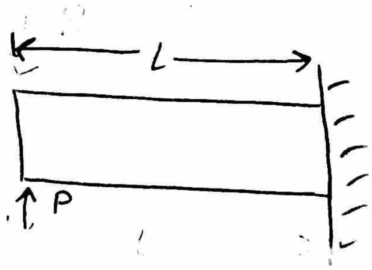
Shear is max at centroid and zero at top/bottom surface

	Bending	Shear
Stress	σ	τ
Max	top/bottom	centroid
min (0)	centroid	top/bottom

Example

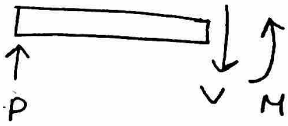
Find ① Max. shear stress
② Max. Normal stress





Solution

① Max. shear stress is at centroid



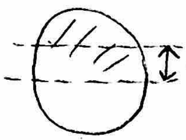
$M = PL$

$$\tau_{max} = \frac{VQ_{max}}{It}$$

$V = P$
$t = 2r$

$$\boxed{I = \frac{\pi}{4} r^4}$$

$$Q_{max} = Ad = \left(\frac{\pi}{2} r^2\right)\left(\frac{4}{3\pi} r\right)$$



$d = \bar{y} = \frac{4}{3\pi} r$

$$Q_{max} = \frac{2}{3} r^3$$

$$\Rightarrow \quad \tau_{max} = \frac{P\left(\frac{2}{3} r^3\right)}{\left(\frac{\pi}{4} r^4\right)(2r)} \Rightarrow \tau_{max} = \frac{4}{3\pi} \cdot \frac{P}{r^2}$$

$$\boxed{\tau_{max} = \frac{4}{3} \frac{P}{A}}$$ at centroid

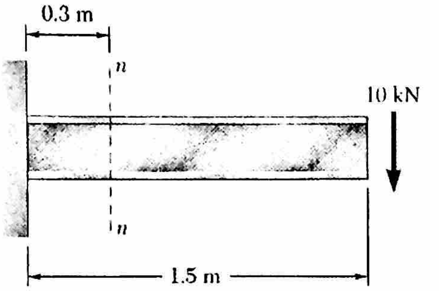
② Max. Normal stress is at top/bottom

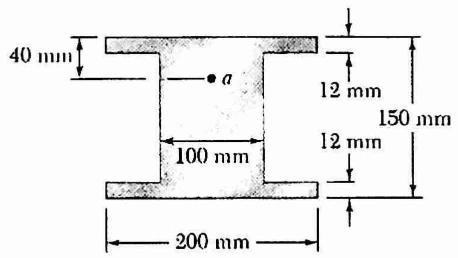
$$\sigma_{max} = \frac{+My_{max}}{I} = \frac{PLr}{\frac{\pi}{4} r^4} = \frac{4}{\pi} \cdot \frac{PL}{r^3} = \frac{4}{\pi r^2} \cdot \frac{PL}{r}$$

$$\sigma_{max} = \frac{4PL}{rA}$$

top / bottom

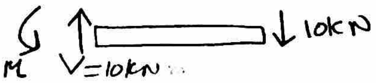
$I = 39.58 \times 10^{-6}\ m^4$





PROBLEM 6.10

For the beam and loading shown, consider section n-n and determine (a) the largest shearing stress in that section, (b) the shearing stress at point a.

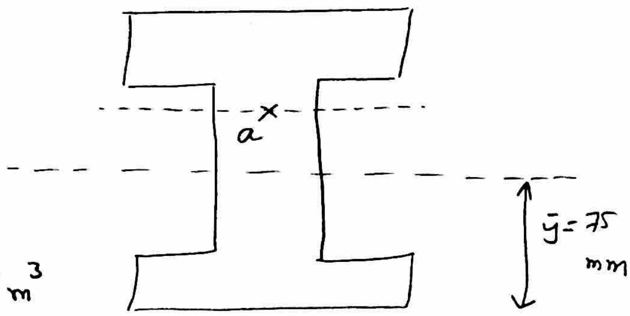


<u>Solution</u>

at (a)

$$\tau_a = \frac{VQ_a}{It} = \frac{(10)(10^3)\,(302.8)(10^{-6})}{(39.58)(10^{-6})\,(100)(10^{-3})}$$

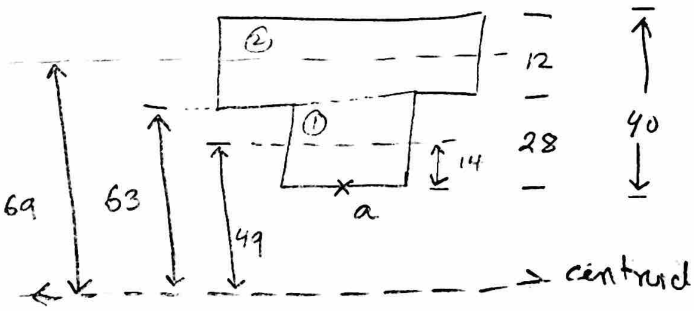
$$\tau_a = 765\ kPa$$



$$Q_a = Q_1 + Q_2 = 302.8 \times 10^3\ mm^3 = 302.8 \times 10^{-6}\ m^3$$

$$Q_1 = A_1 d_1 = (28)(100)(49)$$
$$= 137.2 \times 10^3\ mm^4$$

$$Q_2 = A_2 d_2 = (200)(12)(69)$$
$$= 165.6 \times 10^3\ mm^3$$



<u>max shear</u> ⇒ at <u>centroid</u>

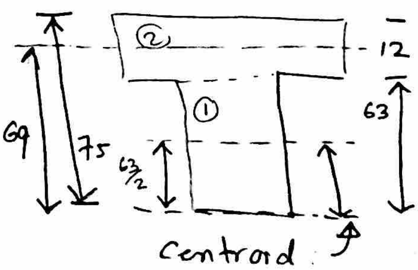
$$\tau_{max} = \frac{VQ_{max}}{It} = \frac{(10)(10^3)\,(364.05 \times 10^{-6})}{(39.58 \times 10^{-6})(100)(10^{-3})}$$

$$\boxed{\tau_{max} = 920\ kPa}$$

$$Q_{max} = Q_1 + Q_2 = 364.05 \times 10^{-6}\ m^4$$

$$Q_1 = A_1 d_1 = (63)(100)\left(\frac{63}{2}\right) = 198.45 \times 10^3\ mm^3$$

$$Q_2 = (200)(12)(69) = 165.6 \times 10^3\ mm^3$$



Example: 6.01 Self-study
and included in your exams

In chapter 6, we only need sections
6.1, 6.2 and 6.3