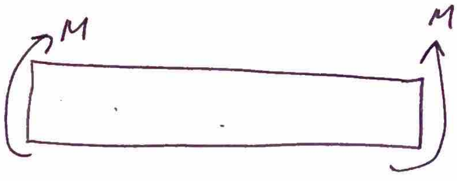
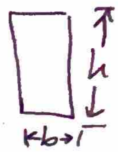
4.6 Bending of members made of several materials.

Last class/video

$$\sigma = \frac{-My}{I}$$





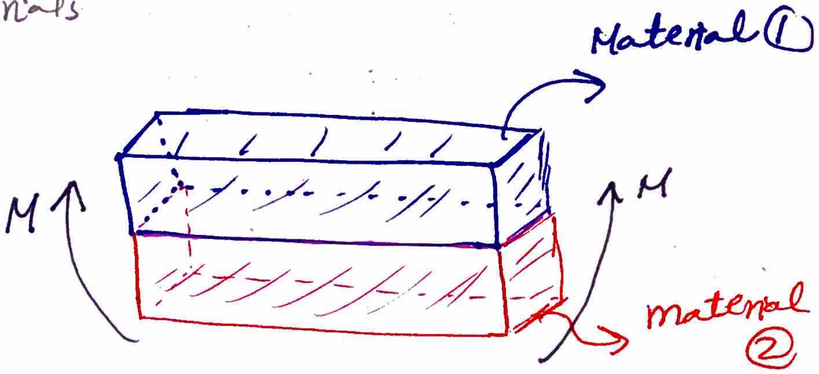
$$\epsilon = \frac{-My}{EI} \quad , \quad \frac{1}{\rho} = \frac{My}{EI}$$

one material

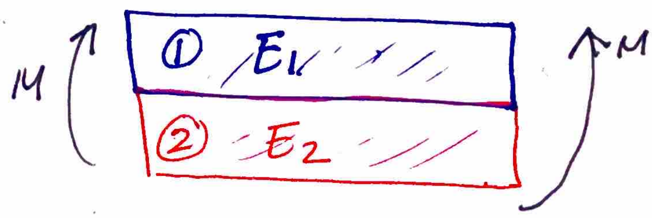
Beams made of several materials

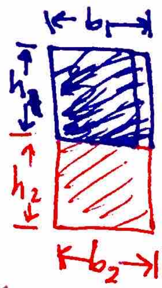
Solution procedure

- we need to transform one material to the other material, mathematically.



- we either transform material ① to material ② or, transform material ② to material ①.





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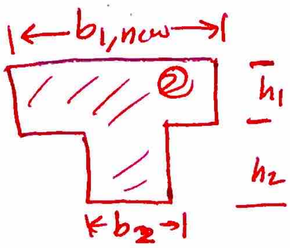
* Transform ① to ②

$$n = \frac{E_1}{E_2} \Rightarrow b_{1,new} = n b_1$$

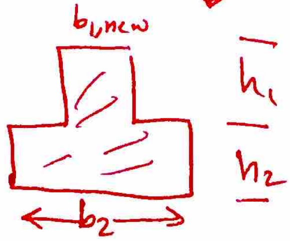
$E_1 > E_2$ $(n > 1) \Rightarrow b_{1,new} > b_1$



All is material ②

$$\sigma_2 = \frac{-My}{\underset{\hookrightarrow new}{I}} \Rightarrow \sigma_1 = n\sigma_2$$

$E_1 < E_2$ $(n < 1) \Rightarrow b_{1,new} < b_1$

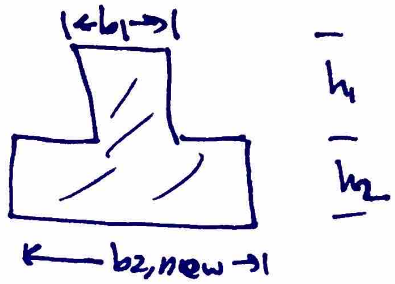


$$\sigma_2 = \frac{-My}{\underset{\hookrightarrow new}{I}} \quad , \quad \sigma_1 = n\sigma_2$$

or

* Transform ② to ①

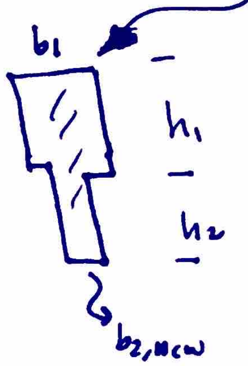
$$n = \frac{E_2}{E_1} \Rightarrow b_{2,new} = n\, b_2$$



$E_2 > E_1 \quad (n>1) \Rightarrow b_{2,new} > b_2$

$$\sigma_1 = \frac{-My}{\underbrace{I}_{new}}, \quad \sigma_2 = n\sigma_1$$

All is material ①



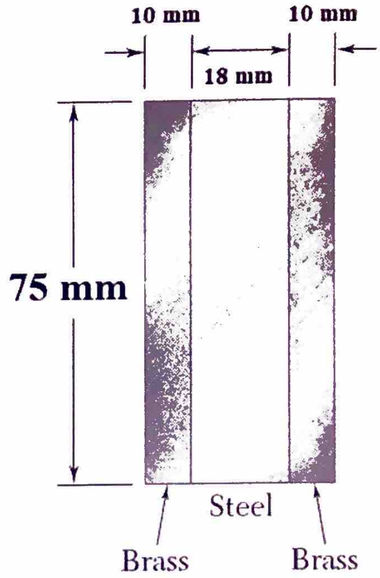
$E_2 < E_1 \quad (n<1) \Rightarrow b_{2,new} < b_2$

$$\sigma_1 = -\frac{My}{\underbrace{I}_{new}}, \quad \sigma_2 = n\sigma_1$$

Examples, next page!

Example:

A bar obtained by bonding together pieces of steel ($Es = 200\ GPa$) and brass ($Eb = 100\ GPa$) has the cross section shown. Determine the maximum stress in the steel and in the brass when the bar is in pure bending with a bending moment $M = 4.5\ kN.m$.



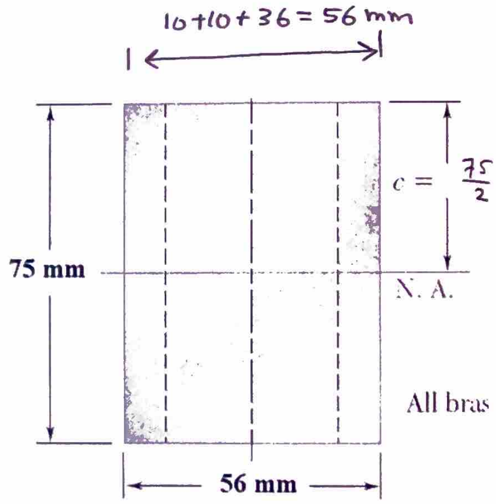
Solution

Transform steel to brass

$$n = \frac{E_s}{E_b} = \frac{200}{100} \Rightarrow \boxed{n = 2}$$

$$b_{steel,new} = 18 \times 2 = 36$$

$$\sigma_{b,max} = \left| -\frac{My}{I} \right| = \frac{+(4.5)(10^3)(75/2)(10^{-3})}{\frac{1}{12}(0.056)(0.075)^3}$$



$$\boxed{\sigma_{b,max} = 85.7\ MPa}$$

brass

$$\sigma_{s,max} = n\,\sigma_{b,max}$$

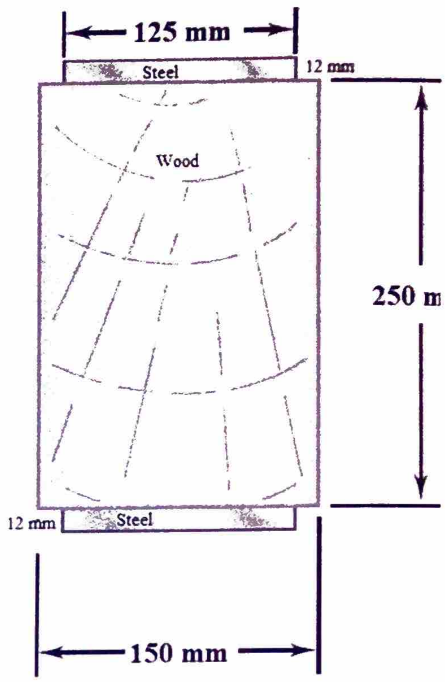
steel

$$= (2)(85.7)$$

$$\boxed{\sigma_{s,max} = 171.4\ MPa}$$

Example:

A bar obtained by bonding together pieces of steel ($E_s = 200\ GPa$) and wood ($E_w = 20\ GPa$) has the cross section shown. Determine the maximum tensile stress in the steel and in the wood when the bar is in pure bending with a bending moment $M = 100\ kN.m$.



Solution

Transform steel to wood

$$E = \frac{E_s}{E_w} = \frac{200}{20} \Rightarrow n = 10$$

$$b_{steel,new} = n\, b_{steel} = (10)(125)$$

$$= 1250\ mm$$

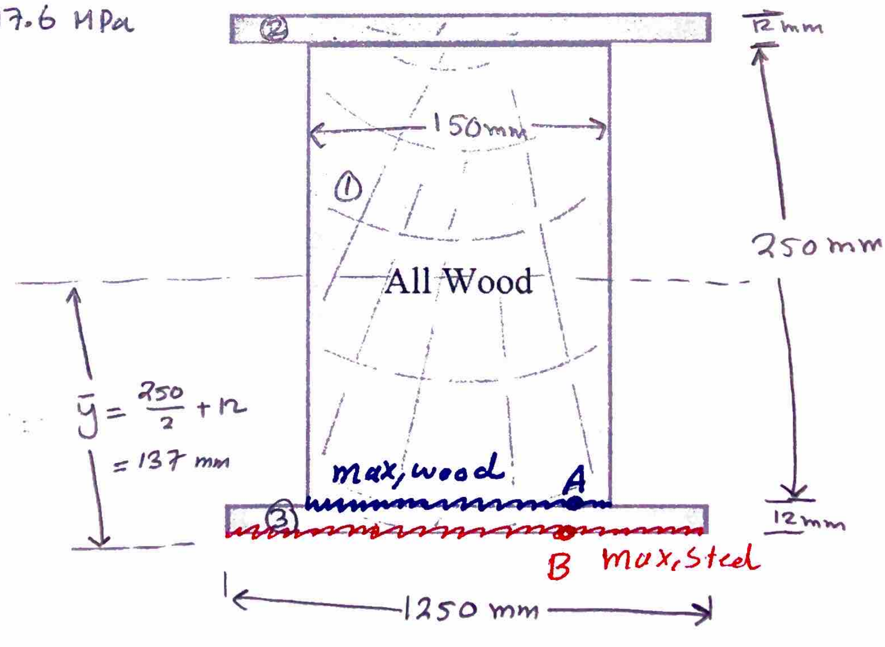
$$\sigma_{w,max} = \frac{-My_A}{I} = \frac{-(100)(10^3)\left(\frac{-250}{2}\right)(10^{-3})}{710.5 \times 10^{-6}}$$

$$\Rightarrow \sigma_{w,max} = 17.6\ MPa$$

$$\sigma_{s,max} = n\left(\frac{-My_B}{I}\right)$$

$$= 10\left(\frac{-(100)(10^3)\,(-137 \times 10^{-3})}{(710.5 \times 10^{-6})}\right)$$

$$\sigma_{s,max} = 192.8\ MPa$$



$$I = I_1 + I_2 + I_3$$

$$I = I_1 + 2I_2$$

$$= 710.5 \times 10^{-6}\ m^4$$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2$$

$$= \frac{1}{12}(150)(250)^3 + (150)(250)(0) = 195.3 \times 10^{-6}\ m^4$$

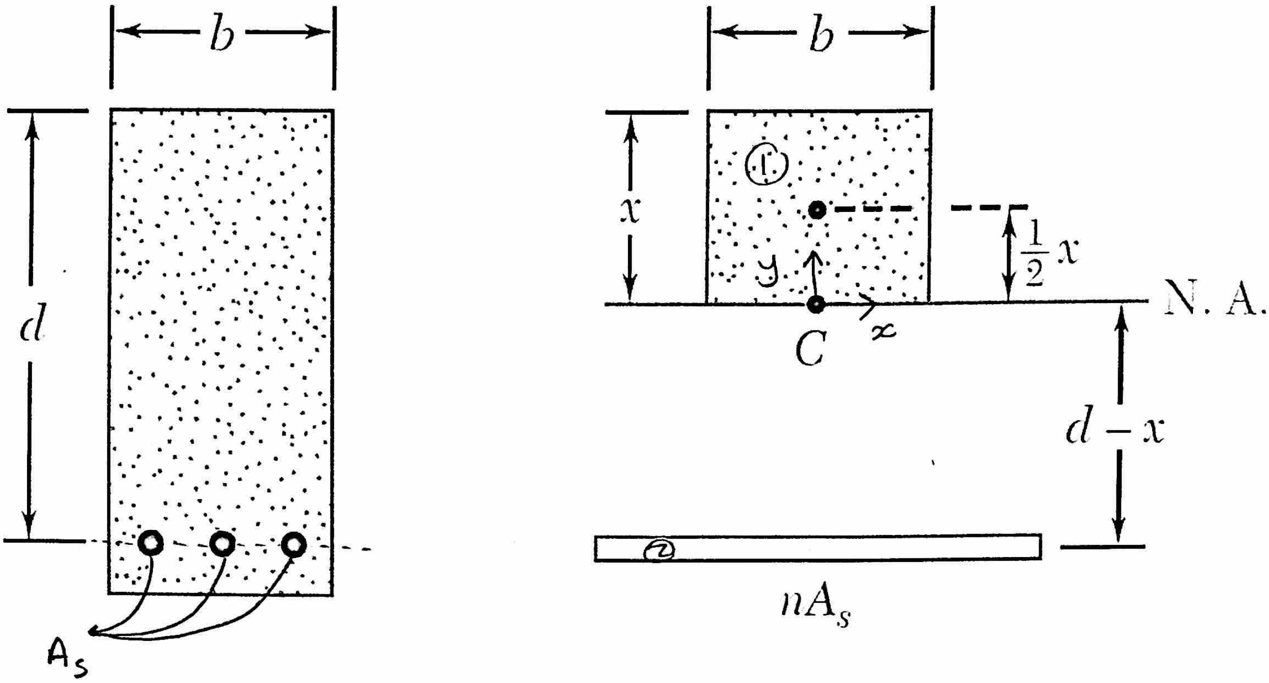
$$I_2 = I_3 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2$$

$$= \frac{1}{12}(1250)(12)^3 + (1250)(12)(137-6)^2$$

$$= 257.6 \times 10^{-6}\ m^4$$

* 4.6: Members of several materials

Concrete and Steel Beams



* Transform steel to concrete

$$n = \frac{E_s}{E_c}$$

$$\sigma_{c,max} = \frac{-Mx}{I}$$

$$\sigma_{s,max} = (n)\left(-\frac{M(d-x)}{I}\right)$$

* Find centroid

$$0 = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{(b)(x)(\frac{x}{2}) + nA_s(d-x)}{(bx) + nA_s} \Rightarrow$$

$$\frac{bx^2}{2} + nA_s(d-x) = 0 \Rightarrow \text{two root } (+, -)$$

take +ve

* Moment of Inertia

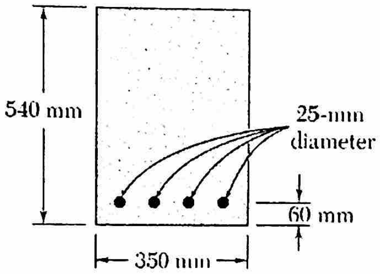
$$I = I_1 + I_2$$

$$I_1 = \frac{1}{12} b h^3 + Ad^2$$
$$= \frac{1}{12} bx^3 + (bx)(x/2)^2 \Rightarrow I_1 = \frac{1}{3} bx^3$$

$$I_2 = \cancel{\frac{1}{12} b_2 h_2^3}^{\,0} + A_2 d_2^2 \Rightarrow I_2 = nA_s(d-x)^2$$

$h \approx 0$

$$I = \frac{1}{3} bx^3 + nA_s(d-x)^2$$



PROBLEM 4.47 The reinforced concrete beam shown is subjected to a positive bending moment of 175 kN · m. Knowing that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine (*a*) the stress in the steel, (*b*) the maximum stress in the concrete.

Solution

$$A_s = (4)\left(\frac{\pi}{4}\right)(25)^2 = 1.96 \times 10^3 \text{ mm}^2$$

- $n = \frac{E_s}{E_c} = \frac{200}{25} \Rightarrow n = 8$

$$d = 540 - 60 = 480 \text{ mm}$$

- Centroid

$$\frac{b}{2}x^2 + nA_s(d-x) = 0 \Rightarrow \frac{350}{2}x^2 + (8)(1.96)(10)^2(480-x) = 0$$

$$\boxed{x_1 = 167.48 \text{ mm}} \qquad x_2 = -(\ ) \times$$

- moment of Inertia

$$I = \frac{1}{3}bx^3 + nA_s(d-x)^2$$

$$= \frac{1}{3}(350)(167.48)^3 + (8)(1.96)(10^{-3})\left[480 - 167.48\right]^2$$

$$I = 2.08 \times 10^{-3} \text{ m}^4$$

$$* \ \sigma_{c,max} = -\frac{Mx}{I} = \frac{-(175)(10^3)(480)(10^{-3})}{2.08 \times 10^{-3}} = -14.08 \text{ MPa}$$

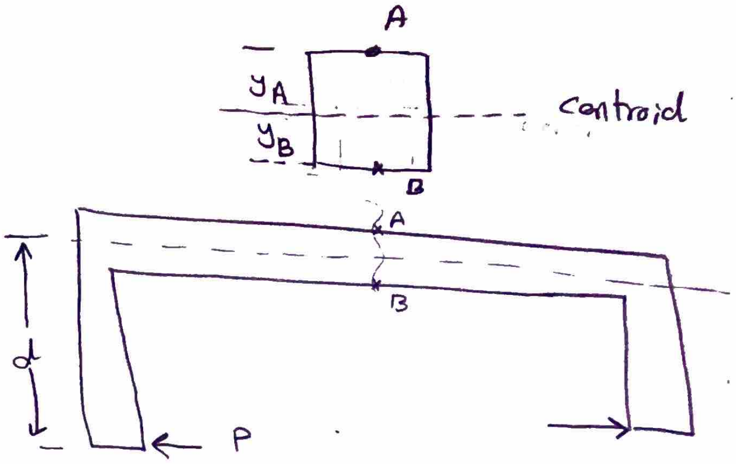
(Compression)

$$* \ \sigma_{s,max} = (n)\left(-\frac{M(d-x)}{I}\right) = (8)\left(\frac{-(175)(10^{-3})(480-167.48)(10^{-3})}{2.08 \times 10^{-3}}\right)$$

$$= 210 \text{ MPa}$$

4.12 Eccentric Bending

Find stresses at A and B



Stress at A

Tension from P

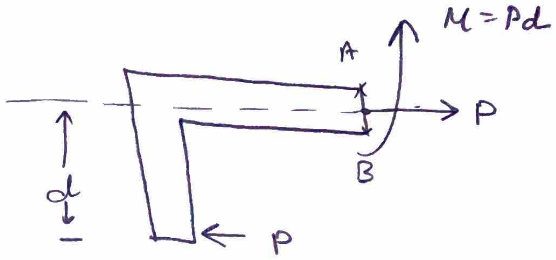
and compression from M

$$\sigma_A = \sigma_P + \sigma_M$$

$$\sigma_A = \frac{P}{A} - \frac{My_A}{I}$$

$$\sigma_P = \frac{P}{A}$$

$$\sigma_m = \frac{-My_A}{I}$$



Stress at B

Tension from P

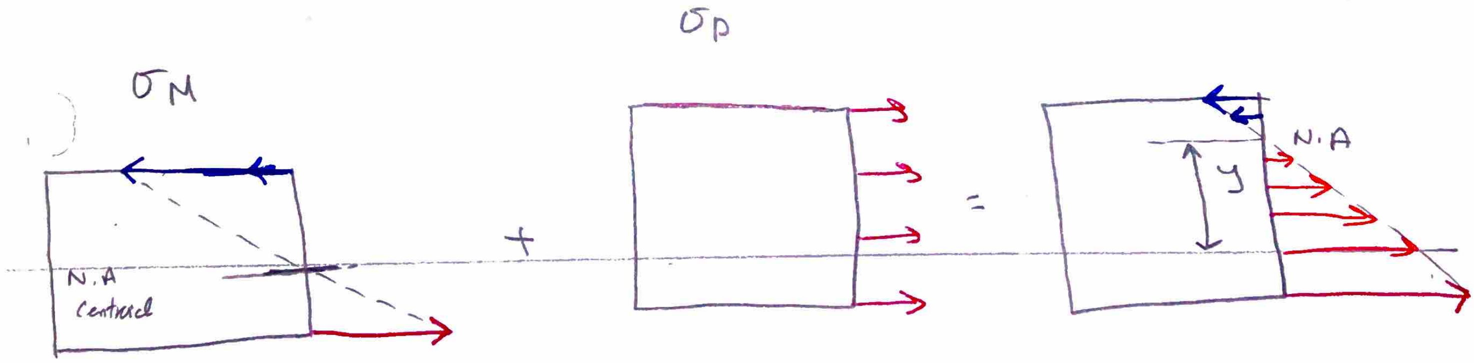
Tension from M

$$\sigma_B = \sigma_P + \sigma_M$$

$$= \frac{P}{A} + \frac{My_B}{I}$$

$$\sigma_P = \frac{P}{A}$$

$$\sigma_m = \frac{-M(-y_B)}{I} = \frac{My_B}{I}$$



$$\sigma = 0 \Rightarrow \frac{P}{A} + \frac{-My}{I} = 0$$

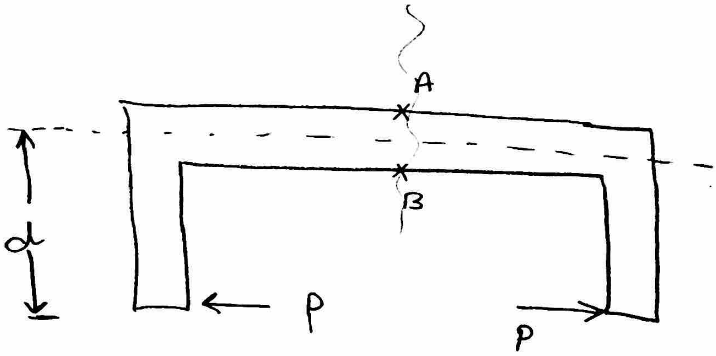
$$\Rightarrow \boxed{y = \frac{PI}{MA}}$$ distance from centroid to N.A

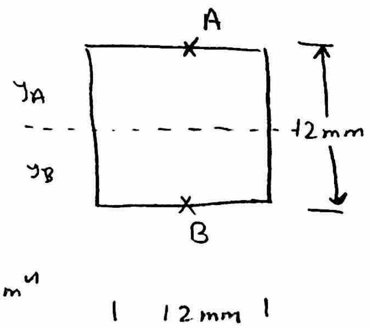
Example

$p = 800\ kN$

$d = 15\ mm$

Find ① stress at A
② Stress at B
③ Location of N.A From centroid



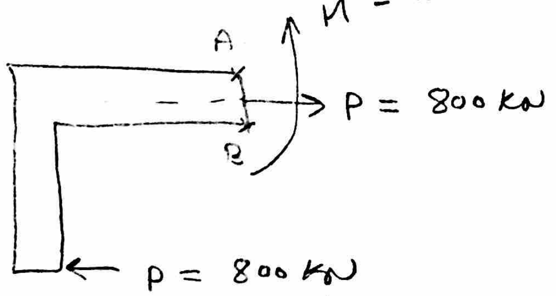


Solution

$$A = (12)(12) = 144 \times 10^{-6}\ m^2$$

$$I = \frac{1}{12}(12)(12)^3 = 1.728 \times 10^{-9}\ m^4$$

At A



$$\sigma_A = \frac{P}{A} - \frac{M y_A}{I}$$

$$= \frac{(800)(10^3)}{144 \times 10^{-6}} - \frac{(12)(10^3)(6)(10^{-3})}{1.728 \times 10^{-9}}$$

$$= -36.11\ MPa$$

At B

$$\sigma_B = \frac{P}{A} + \frac{M y_B}{I} = \frac{(800)(10)^3}{144 \times 10^{-6}} + \frac{(12)(10^3)(6)(10^{-3})}{1.728 \times 10^{-9}} \Rightarrow$$

$$= 47.22\ MPa$$

location N.A

$$y = \frac{PI}{MA} = \frac{(800)(10^3)(1.728)(10^{-9})}{(12)(10^3) \cdot (144)(10^{-6})} = 0.8\ mm$$