We can also use complex numbers method in solving MDOF systems.

Consider 2DOF system we discussed

$$[m]\{\ddot{x}\} + [k]\{x\} = \{0\} \quad \text{— EOM}$$

$$m = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}, \quad [k] = \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}$$

$$\{\ddot{x}\} = \begin{Bmatrix} \ddot{x}_1(t) \\ \ddot{x}_2(t) \end{Bmatrix}, \quad \{x\} = \begin{Bmatrix} x_1(t) \\ x_2(t) \end{Bmatrix}$$

Using complex numbers method,

$$\{x\} = \{X\} e^{\hat{i}\omega t} \qquad \{X\} = \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}$$

$$\{\dot{x}\} = \hat{i}\omega \{X\} e^{\hat{i}\omega t}$$

$$\{\ddot{x}\} = -\omega^2 \{X\} e^{\hat{i}\omega t}$$

Substitute in EOM

$$-\omega^2 [m]\{X\} \cancel{e^{\hat{i}\omega t}} + [k]\{X\} \cancel{e^{\hat{i}\omega t}} = \{0\}$$

$$\Rightarrow \underbrace{([k] - \omega^2 [m])\{X\} = 0}$$

From this equation we can find

① Natural Frequencies (ω_1 and ω_2)

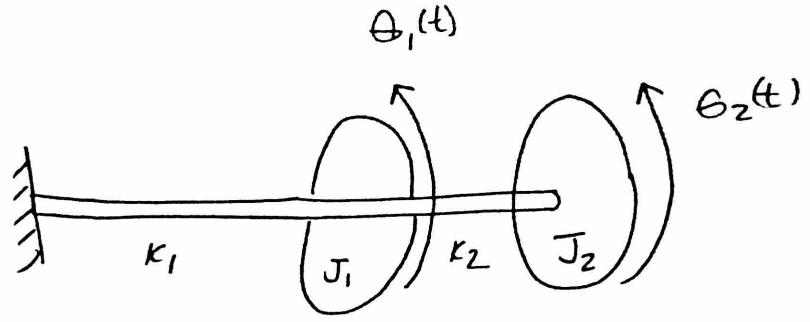
② Modeshape vectors ($\{X^{(1)}\}$ and $\{X^{(2)}\}$)

Problem 4.12

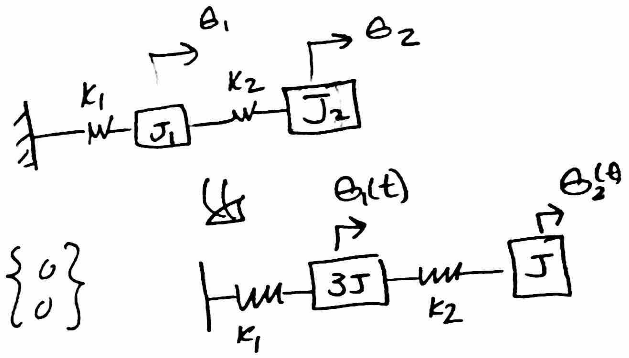
If $k_1 = k_2$
and $J_1 = 3J_2$

1- Find equation of motion (EOM)

2- Calculate natural Frequencies and modeshapes



Solution



① EOM let $k_1 = k_2 = k$ and $J_2 = J$

$$\Rightarrow \begin{bmatrix} 3J & 0 \\ 0 & J \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} 2k & -k \\ -k & +k \end{bmatrix} \begin{Bmatrix} \theta_1(t) \\ \theta_2(t) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

② Natural Frequencies and modeshapes

$$\left([k] - \omega^2 [m]\right)\{X\} = 0$$

$$\det\left[[k] - \omega^2 [m]\right] = 0 \Rightarrow \det \begin{bmatrix} 2k - \omega^2 3J & -k \\ -k & k - \omega^2 J \end{bmatrix} = 0$$

$$\Rightarrow 3J^2\omega^4 - 5Jk\omega^2 + k^2 = 0$$

$$\omega^2 = \lambda$$

$$\Rightarrow 3J^2\lambda^2 - 5Jk\lambda + k^2 = 0$$

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{+5Jk \pm \sqrt{(-5kJ)^2 - (4)(3J^2)(k^2)}}{6J^2}$$

cont'd

$$\Rightarrow \lambda_{1,2} = \frac{5JK \pm \sqrt{J^2K^2(25-12)}}{6J^2} = \frac{5JK \pm KJ\sqrt{13}}{6J^2}$$

$$\lambda_{1,2} = \left(\frac{5 \pm \sqrt{13}}{6}\right)\frac{K}{J} \Rightarrow \lambda_1 = 0.2324\frac{K}{J}\ , \ \lambda_2 = 1.434\frac{K}{J}$$

$$\Rightarrow \boxed{\omega_1 = 0.482\sqrt{\frac{K}{J}}\ , \ \omega_2 = 1.1976\sqrt{\frac{K}{J}}} \Rightarrow \text{Natural Frequecies.}$$

Mode Shapes

For $\omega_1^2 = 0.2324\frac{K}{J}$

$$\begin{bmatrix} 2K - 3J\left(0.2324\frac{K}{J}\right) & -K \\ -K & K - \left(0.2324\frac{K}{J}\right)J \end{bmatrix} \begin{Bmatrix} X_1^{(1)} \\ X_2^{(1)} \end{Bmatrix} = \{0\}$$

$$\Rightarrow X_1^{(1)} = 0.7676\, X_2^{(1)}\ , \quad \text{let } X_2^{(1)} = 0 \Rightarrow X_1^{(1)} = 0.7676$$

$$\boxed{\Rightarrow \{X_1\} = \begin{Bmatrix} 0.7676 \\ 1 \end{Bmatrix}} \longrightarrow \text{Modeshape 1}$$

For $\omega_2^2 = 1.434\frac{K}{J}$

$$\begin{bmatrix} 2K - 3J\left(1.434\frac{K}{J}\right) & -K \\ -K & K - \left(1.434\frac{K}{J}\right)J \end{bmatrix} \begin{Bmatrix} X_1^{(2)} \\ X_2^{(2)} \end{Bmatrix} = \{0\}$$

$$\Rightarrow X_1^{(2)} = -0.434\, X_2^{(2)}\ , \quad \text{let } X_2^{(2)} = 1 \Rightarrow X_1^{(2)} = -0.434$$

$$\boxed{\Rightarrow \{X_2\} = \begin{Bmatrix} -0.434 \\ 1 \end{Bmatrix}} \longrightarrow \text{Mode Shape 2}$$