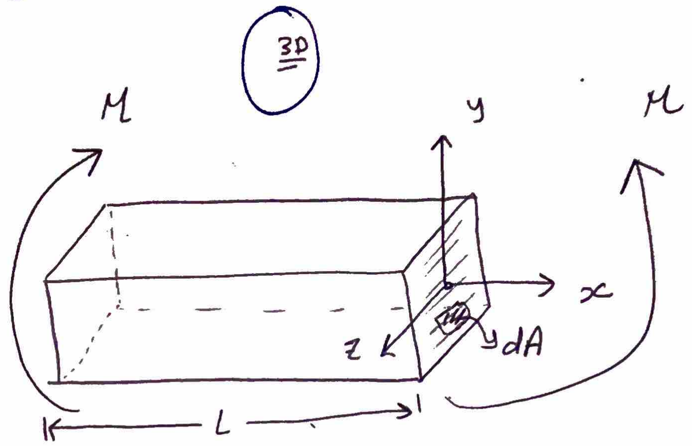
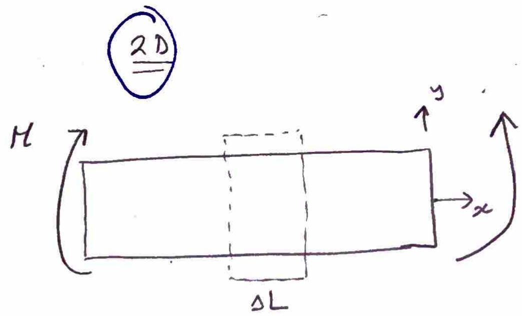
Chapter Four : Pure Bending

$$\text{Strain} = \frac{\text{change of length}}{\text{original length}}$$



$$\epsilon = \frac{\Delta L' - \Delta L}{\Delta L}$$

$$\epsilon = \frac{(\rho - y)\theta - \rho\theta}{\rho\theta} = \frac{-y}{\rho}$$

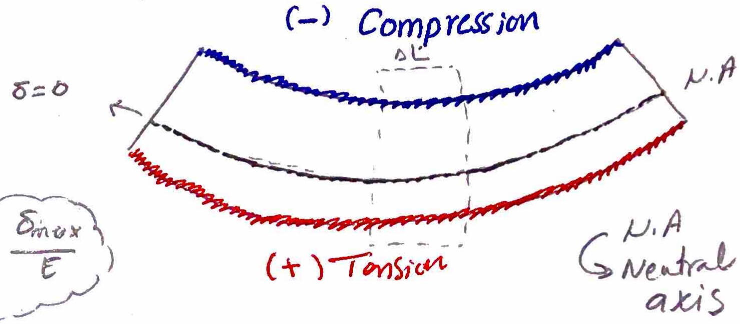


$$\epsilon = \frac{-y}{\rho}$$

$$\epsilon_{max} = \frac{-y_{max}}{\rho} = \frac{+C}{\rho} \leftarrow \text{Absolute}$$

$$\frac{\epsilon}{\epsilon_{max}} = \frac{-y/\rho}{+c/\rho} = \frac{y}{c}$$

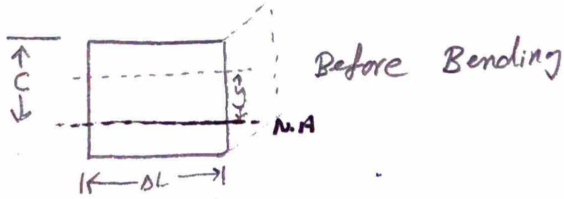
$$\epsilon = \frac{-y}{c}\,\epsilon_{max}$$



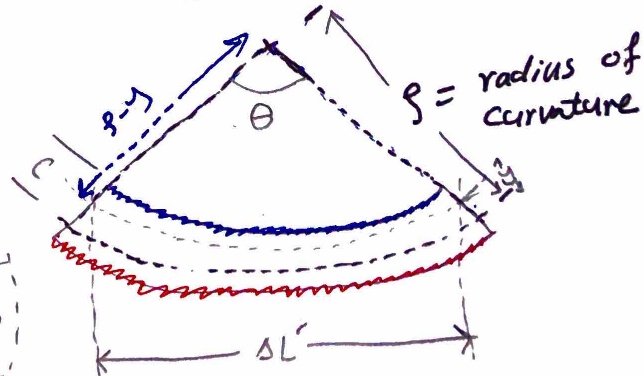
Hooke's Law

$$\sigma = E\epsilon \Rightarrow \epsilon = \frac{\sigma}{E}, \quad \epsilon_{max} = \frac{\sigma_{max}}{E}$$

$$\frac{\sigma}{E} = \frac{-y}{C} \cdot \frac{\sigma_{max}}{E}$$



$$\sigma = \frac{-y}{C} \cdot \sigma_{max}$$



$$\Sigma F = 0 \Rightarrow \int_0^A \sigma \, dA = 0$$

$$\int \frac{-y}{c}\,\sigma_{max}\, dA = 0 \Rightarrow \frac{-\sigma_{max}}{c}\int y\, dA = 0$$

↳ only true if N.A is on Controid (1st moment of area)

Cont'd

$$\Sigma M = 0 \Rightarrow M - \int \sigma y \, dA = 0$$

$$M - \int \frac{-y}{c} \sigma_{max} y \, dA = 0$$

$$\Rightarrow M = \frac{-\sigma_{max}}{c} \boxed{\int y^2 dA} \longrightarrow$$ 2nd moment of area / Moment of Inertia (I)

$$\Rightarrow M = \frac{-\sigma_{max} I}{c}$$

$$\Rightarrow \sigma_{max} = \frac{-Mc}{I}$$

In general

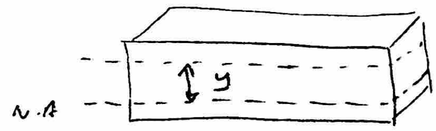
$$\sigma = \frac{-My}{I}$$

M: Bending moment (N.m)

y: Distance from Centroid (Neutral axis) (m)

I: Moment of Inertia (m^4) About Centroidal axis

Hooke's Law $\sigma = E\epsilon$, $\epsilon = \frac{\sigma}{E}$



$$\epsilon = \frac{-My}{EI}$$

also, $\epsilon = \frac{-y}{\rho} = \frac{-My}{EI}$

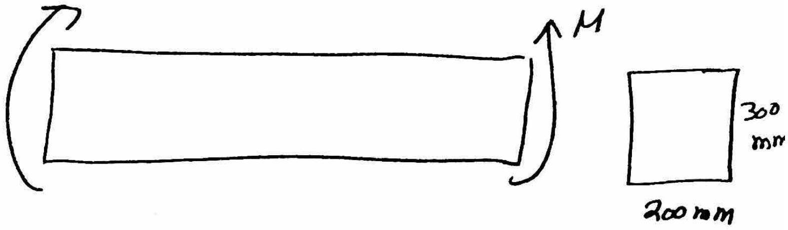
$$\Rightarrow \frac{1}{\rho} = \frac{M}{EI}$$

radius of Curvature

Example

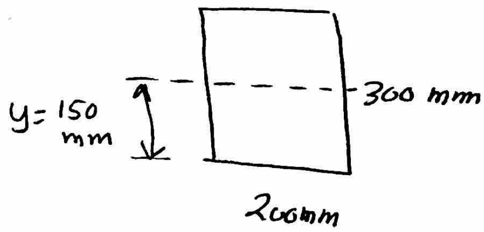
$M = 1000$ N.m

- Find stress at the top surface of the Beam



Solution

$$\sigma = \frac{-My}{I}$$



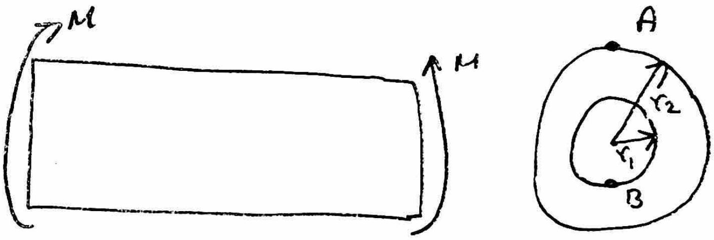
$$= \frac{-(1000)(150)(10^{-3})}{\frac{1}{12}(200\times10^{-3})(300\times10^{-3})^3}$$

$$I = \frac{1}{12} bh^3$$

$$= -0.33 \text{ MPa} \quad (\text{compression})$$

Example

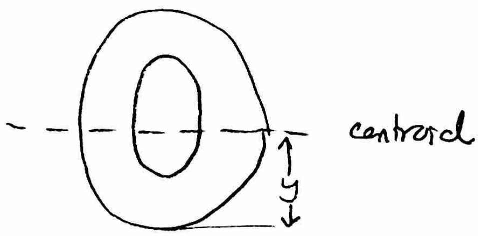
$M = 5$ KN.m , $r_1 = 120$ mm
$r_2 = 200$ mm



Find stress at A and B

Solution

$$\sigma_A = \frac{-My_A}{I} = \frac{-(5)(10^3)(200)(10^{-3})}{\frac{\pi}{4}\left[0.20^4 - 0.12^4\right]}$$



$$\sigma_A = -0.914 \text{ MPa} \quad (\text{compression})$$

$$I_{circle} = \frac{\pi}{4} r^4$$

$$\sigma_B = \frac{-My_B}{I} = \frac{-(5)(10^{+3})(-120)(10^{-3})}{\frac{\pi}{4}\left[0.2^4 - 0.12^4\right]}$$

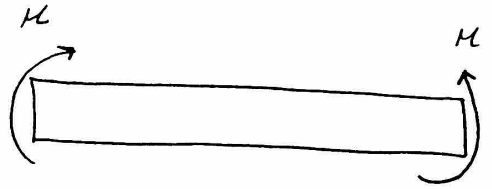
$$I = \frac{\pi}{4}(r_2^4 - r_1^4)$$

$$= 0.548 \text{ MPa} \quad (\text{Tension})$$

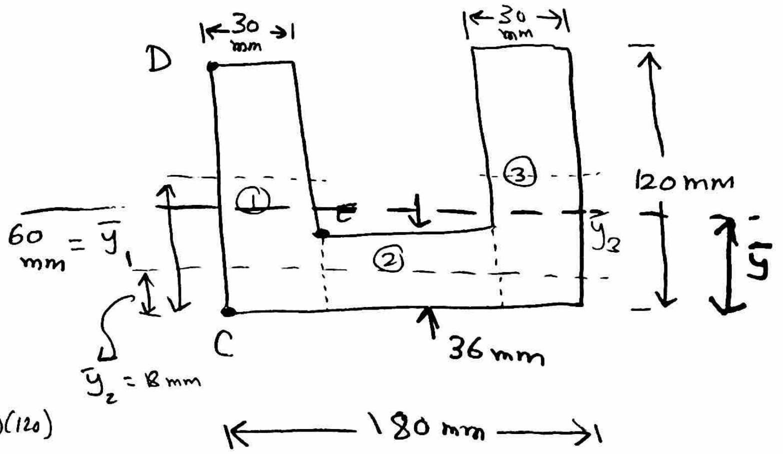
Example

$M = 25$ kNm

Find stress at C, D and E



Solution



⊛ Centroid

$$\bar{y} = \frac{\sum_{i=1}^{3} \bar{y}_i A_i}{\sum_{i=1}^{3} A_i} = \frac{\bar{y}_1 A_1 + \bar{y}_2 A_2 + \bar{y}_3 A_3}{A_1 + A_2 + A_3}$$

$$= \frac{(60)(30)(120) + (18)(120)(36) + (60)(30)(120)}{(30)(120) + (120)(36) + (30)(120)}$$

$$= 44.25 \text{ mm}$$

$$I = I_1 + I_2 + I_3$$

$$I = 13.87 \times 10^{-6} \text{ mm}^4 = 13.87 \times 10^{-6} \text{ m}^4$$

$$\sigma_C = \frac{-M y_C}{I} = \frac{-(25)(10^3)(\overline{44.25})(10^{-3})}{13.87 \times 10^{-6}}$$

$$\Rightarrow \sigma_C = 79.8 \text{ MPa (tension)}$$

$$\sigma_D = \frac{-M y_D}{I} = \frac{-(25)(10^3)(120 - 44.25)(10^{-3})}{13.87 \times 10^{-6}}$$

$$\sigma_D = -136.5 \text{ MPa (compression)}$$

$$\sigma_E = \frac{-M y_E}{I} = \frac{-(25)(10^{+3})(36 - 44.25)(10^{-3})}{13.87 \times 10^{-6}}$$

$$\sigma_E = 14.87 \text{ MPa (tension)}$$

Parallel axis theorem

$$I_1 = \frac{1}{12} b_1 h_1^3 + A d_1^2$$

$$= \frac{1}{12}(30)(120)^3 + (120)(30)(60 - 44.25)^2$$

$$= 5.213 \times 10^6 \text{ mm}^4$$

$$I_3 = I_1$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2$$

$$= \frac{1}{12}(120)(36)^3 + (120)(36)(44.25 - 18)^2$$

$$I_2 = 3.4433 \times 10^6 \text{ mm}^4$$