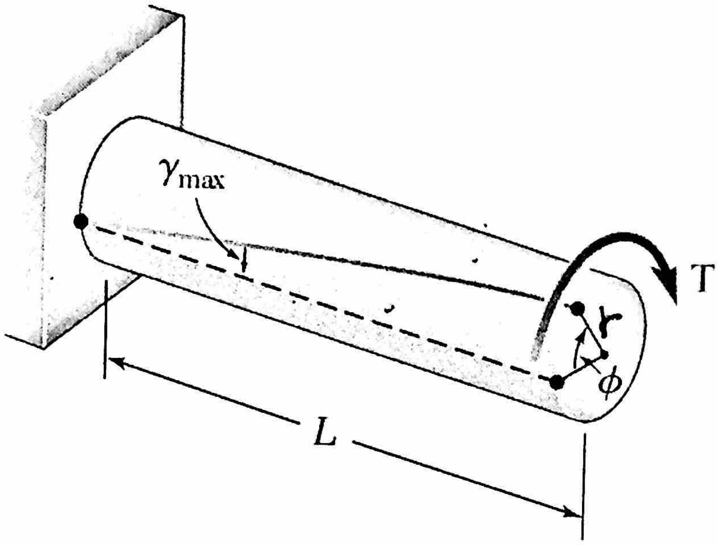
3.5 Angle of twist

$\tau = \frac{Tr}{J}$ (shear stress)

$\gamma = \frac{r\phi}{J}$ (shear strain)

$\phi = \frac{TL}{GJ}$ (Angle of twist)

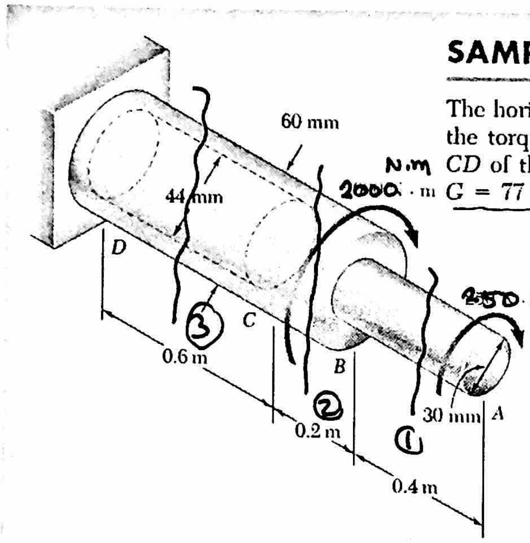


For multisections

$$\phi = \sum_{i=1}^{n} \frac{T_i L_i}{G_i J_i}$$ → n # of sections

Sections → Change in → area (J)
→ Material (G)
→ Load (T)

SAMPLE PROBLEM 3.3



The horizontal shaft AD is attached to a fixed base at D and is subjected to the torques shown. A 44-mm-diameter hole has been drilled into portion CD of the shaft. Knowing that the entire shaft is made of steel for which $G = 77$ GPa, determine the angle of twist at end A.

ϕ_A

3 sections

Solution

$$\phi_A = \sum_{i=1}^{n=3} \frac{T_i L_i}{G_i J_i} = \frac{T_1 L_1}{G_1 J_1} + \frac{T_2 L_2}{G_2 J_2} + \frac{T_3 L_3}{G_3 J_3}$$

$$\phi_A = \frac{1}{G}\left(\frac{T_1 L_1}{J_1} + \frac{T_2 L_2}{J_2} + \frac{T_3 L_3}{J_3}\right)$$

$$= \frac{1}{(77)(10^9)}\left[\frac{(250)(0.4)}{0.0795\times10^{-6}} + \frac{(2250)(0.2)}{1.272\times10^{-6}} + \frac{(2250)(0.6)}{0.904\times10^{-6}}\right]$$

$$\Rightarrow \phi_A = 0.0403 \text{ rad}$$

$$= 2.31^\circ \text{ (degree)}$$

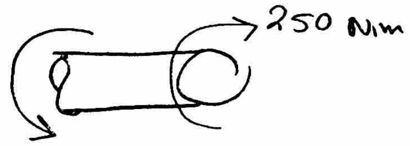
Section ①

$L_1 = 0.4$ m

$G_1 = 77\times10^9$ Pa

$J = \frac{\pi}{2}(0.015)^4$

$J = 0.0795\times10^{-6}$ m^4



$T_1 = 250$ N.m

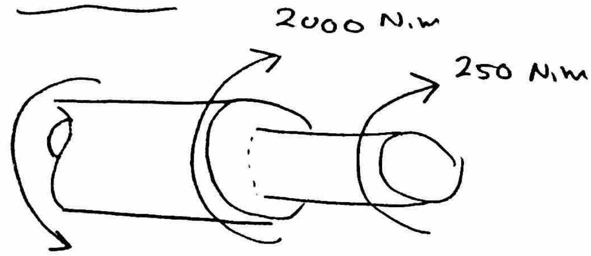
Section ②

$L_2 = 0.2$ m

$G_2 = 77\times10^9$ Pa

$J = \frac{\pi}{2}(0.03)^4$

$= 1.272\times10^{-6}$ m^4



$T_2 = 2250$ N.m $(\Sigma T = 0)$

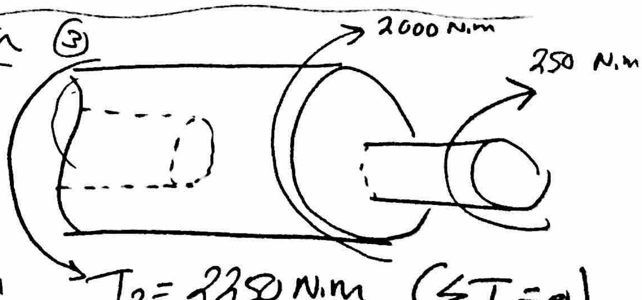
Section ③

$L_3 = 0.6$ m

$G_3 = 77\times10^9$ Pa

$J = \frac{\pi}{2}\left[(0.03)^4 - (0.022)^4\right]$

$J = 0.904\times10^{-6}$ m^4



$T_3 = 2250$ N.m $(\Sigma T = 0)$

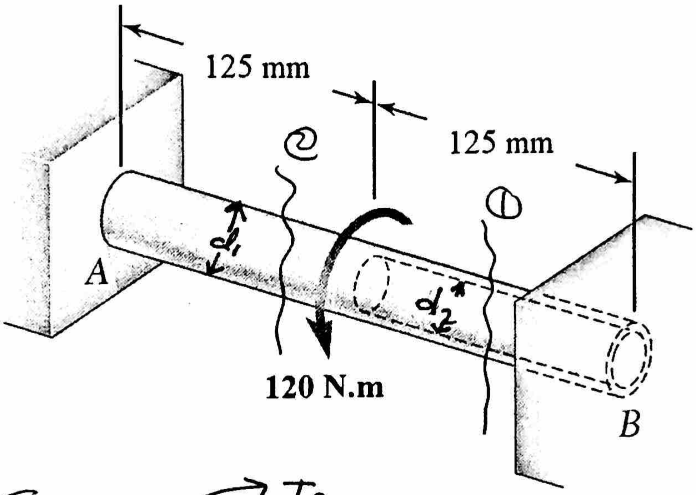
3.6 Statically indeterminate shafts

$d_1 = 22$ mm

$d_2 = 16$ mm

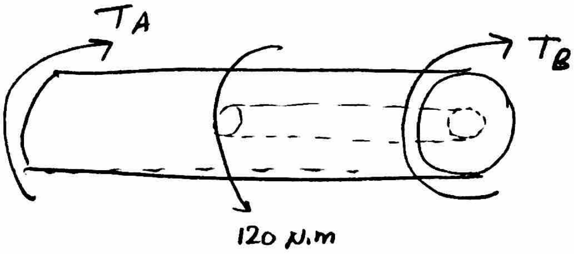
$G = 12$ GPa

Find reactions at A and B



Solution

F.B.D



$\Sigma T = 0$

$\Rightarrow T_A + T_B = 120$ —(1)

statically indeterminate

we need one more equation from mechanics of materials ($\phi = 0$)

$$\phi_B = 0 \Rightarrow \phi_B = \sum_{i=1}^{n=2} \frac{T_i L_i}{J_i G_i} = \frac{T_1 L_1}{J_1 G_1} + \frac{T_2 L_2}{J_2 G_2} = 0$$

$$0 = \frac{T_B (125)(10^{-3})}{(1.66 \times 10^{-8})(12)(10^9)} + \frac{(T_B - 120)(125)(10^{-3})}{(12)(10^9)(2.3 \times 10^{-8})}$$

$\Rightarrow T_B = 50.2$ N.m

$T_A = 69.8$ N.m

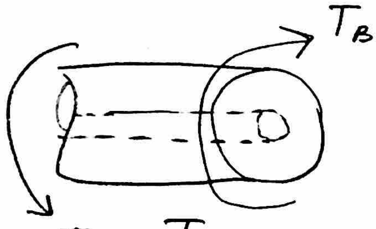
Section ①

$L_1 = 125$ mm

$G_1 = 12$ GPa

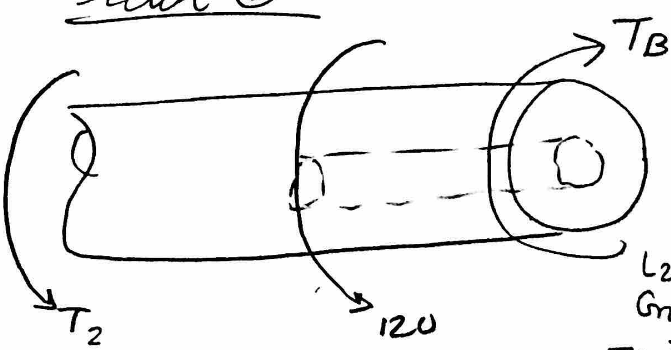
$J = \frac{\pi}{2}\left[0.011^4 - 0.008^4\right]$

$= 1.66 \times 10^{-8}$ m^4



$T_1 = T_B$

Section ②



$L_2 = 125$ mm

$G_2 = 12$ GPa

$J = \frac{\pi}{2}(0.011)^4$

$= 2.3 \times 10^{-8}$ m^4

$\Sigma T = 0 \Rightarrow T_2 + 120 - T_B = 0$

$\Rightarrow T_2 = T_B - 120$

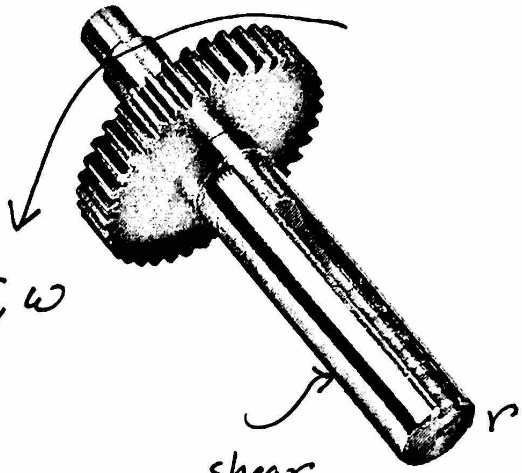
3.7 Design of transmission shafts

T: Torque (N.m)

ω: Angle velocity (rad/s)

Power (P) = $T\omega$, $\omega = 2\pi f$

P: Watt   f: frequency (Hz)



$$\tau = \frac{Tr}{J}$$

Example

$P = 7.5$ K Watt

$f = 60$ Hz

$\tau_{all} = 60$ MPa

Find shaft radius (r)

Solution

$$P = T\omega = T(2\pi f) \Rightarrow T = \frac{P}{2\pi f}$$

$$\tau = \frac{Tr}{J} = \frac{Pr}{2\pi f\ J} , \quad J = \frac{\pi}{2} r^4$$

$$\tau = \frac{Pr}{2\pi f \frac{\pi}{2} r^{4}} \Rightarrow \tau = \frac{P}{\pi^2 f r^3}$$

$$\Rightarrow r = \sqrt[3]{\frac{P}{\pi^2 \tau_{all} f}} \Rightarrow r = \sqrt{\frac{7.5 \times 10^3}{\pi^2 (60)(60)(10^6)}}$$

$r = 11.91$ mm or Larger