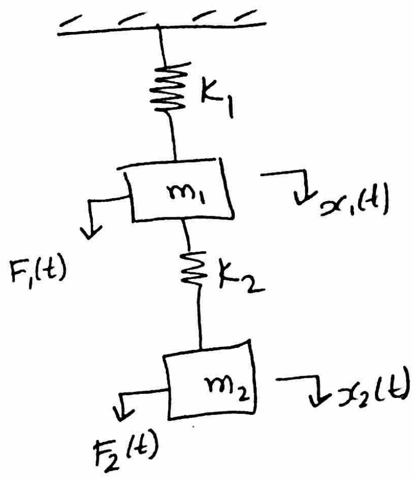
Natural Frequencies and modeshapes of Forced and damped MDOF systems

In this class we will consider obtaining natural Frequencies and modeshapes of Forced and/or damped Multidegree-of-freedom (MDOF) systems. Cases are:
- undamped and Forced MDOF
- Damped and unforced MDOF
- Damped and Forced MDOF

① Undamped and Forced MDOF

Equation of motion



$$[m]\{\ddot{x}\} + [K]\{x\} = \{F(t)\}$$

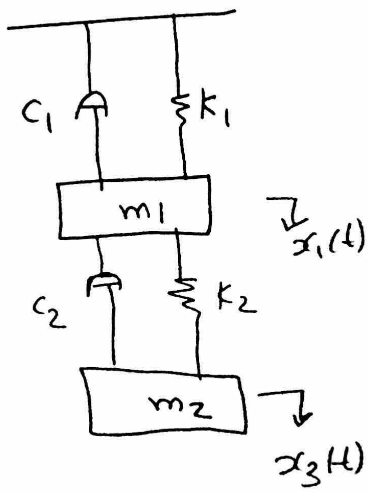
where $[m] = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$, $[K] = \begin{bmatrix} K_1+K_2 & -K_2 \\ -K_2 & K_2 \end{bmatrix}$

$$\{\ddot{x}(t)\} = \begin{Bmatrix} \ddot{x}_1(t) \\ \ddot{x}_2(t) \end{Bmatrix}, \quad \{x(t)\} = \begin{Bmatrix} x_1(t) \\ x_2(t) \end{Bmatrix}$$

$$\{F(t)\} = \begin{Bmatrix} F_1(t) \\ F_2(t) \end{Bmatrix} \qquad \begin{aligned} F_1(t) &= F_1 \cos\omega t \\ F_2(t) &= F_2 \cos\omega t \end{aligned}$$

② Damped and unforced MDOF

Equation of motion



$$[m]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{0\}$$

where $[m] = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$, $[C] = \begin{bmatrix} C_1+C_2 & -C_2 \\ -C_2 & +C_2 \end{bmatrix}$

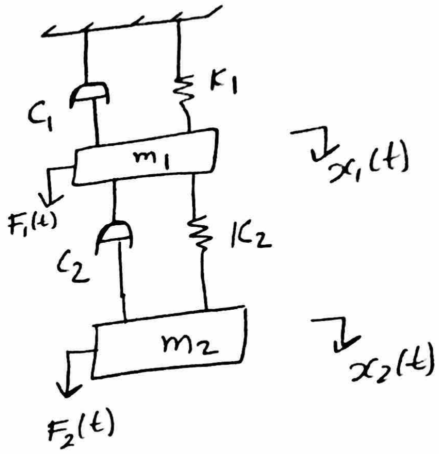
$$[K] = \begin{bmatrix} K_1+K_2 & -K_2 \\ -K_2 & +K_2 \end{bmatrix}$$

$[m] \to$ mass
$[C] \to$ Damping
$[K] \to$ Stiffness

$$\{\ddot{x}(t)\} = \begin{Bmatrix} \ddot{x}_1(t) \\ \ddot{x}_2(t) \end{Bmatrix}, \quad \{\dot{x}(t)\} = \begin{Bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{Bmatrix}, \quad \{x(t)\} = \begin{Bmatrix} x_1(t) \\ x_2(t) \end{Bmatrix}$$

Damped and Forced MDOF

Equation of motion



$$[m]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F(t)\}$$

where: $[m] = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$, $[C] = \begin{bmatrix} C_1+C_2 & -C_2 \\ -C_2 & C_2 \end{bmatrix}$

$$[K] = \begin{bmatrix} K_1+K_2 & -K_2 \\ -K_2 & K_2 \end{bmatrix}$$

$$\{\ddot{x}(t)\} = \begin{Bmatrix} \ddot{x}_1(t) \\ \ddot{x}_2(t) \end{Bmatrix}, \quad \{\dot{x}(t)\} = \begin{Bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{Bmatrix}, \quad \{x(t)\} = \begin{Bmatrix} x_1(t) \\ x_2(t) \end{Bmatrix}$$

$$\{F(t)\} = \begin{Bmatrix} F_1(t) \\ F_2(t) \end{Bmatrix}, \quad \begin{aligned} F_1(t) &= F_1 \cos\omega t \\ F_2(t) &= F_2 \cos\omega t \end{aligned}$$

In All cases above, If we want to find natural Frequencies and modeshapes, we drop (or remove) any damping and forcing terms so the equation of motion would become:-

$$[m]\{\ddot{x}\} + [K]\{x\} = \{0\}$$

then we can find $\omega_i \longleftarrow$ natural Frequencies

$\{X\} \longleftarrow$ Modeshapes

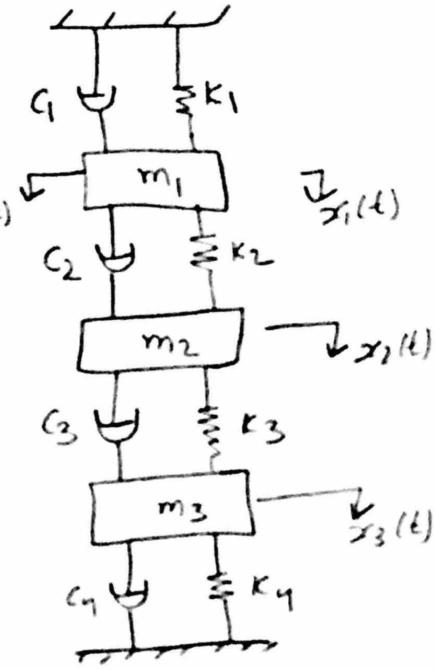
as we did previously

* Note: We will not consider finding $x_1(t)$ and $x_2(t)$ for damped/Forced MDOF systems.

ple (3DOF)

the equation of motion of this system is

$$[m]\{\ddot{x}\} + [c]\{\dot{x}\} + [k]\{x\} = \{F(t)\}$$



where $[m] = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}$, $[C] = \begin{bmatrix} c_1+c_2 & -c_2 & 0 \\ -c_2 & c_2+c_3 & -c_3 \\ 0 & -c_3 & c_3+c_4 \end{bmatrix}$

$[K] = \begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3+k_4 \end{bmatrix}$, $\{F(t)\} = \begin{Bmatrix} F_1(t) \\ 0 \\ 0 \end{Bmatrix}$, $F_1(t) = \cos 3t$

and $m_1 = 10\,kg$, $m_2 = 5\,kg$, $m_3 = 1\,kg$

$c_1 = 1000\,kg/s$, $c_2 = 600\,kg/s$, $c_3 = c_4 = 500\,kg/s$

$k_1 = 500\,N/m$, $k_2 = 300\,N/m$, $k_3 = k_4 = 250\,N/m$

* Find natural Frequencies and modeshapes of this system.

Solution

* To Find Natural Frequencies and modeshapes $\Longrightarrow$ Drop $[C]$ and $\{F\}$

so,

$$[m]\{\ddot{x}\} + [k]\{x\} = \{0\}$$

substitute

As we did previously

$$\{x(t)\} = \{X\}\sin(\omega t + \phi)$$

$$\Rightarrow \underbrace{\left([K] - \omega^2 [m]\right)}_{\det(\;) = 0}\{X\} = 0$$

$$\Rightarrow \det \begin{bmatrix} 800 - \omega^2 10 & -300 & 0 \\ -300 & 550 - \omega^2 5 & -250 \\ 0 & -250 & 500 - \omega^2 \end{bmatrix} = 0$$

$\omega_1^2 = 39$, $\omega_2^2 = 121$, $\omega_3^2 = 530$

$\Rightarrow$ $\omega_1 = 6.25$ rad/s , $\omega_2 = 11$ rad/s , $\omega_3 = 23$ rad/s

Find mode shapes we apply ω_1^2, ω_2^2 and ω_3^2 in

$$([K] - \omega^2[m])\{X\} = 0$$

To find

these are normalized modes shapes

$$\{X^{(1)}\} = \begin{Bmatrix} -0.2242 \\ -0.3062 \\ -0.1662 \end{Bmatrix}, \quad \{X^{(2)}\} = \begin{Bmatrix} -0.2229 \\ 0.3043 \\ 0.2007 \end{Bmatrix}, \quad \{X^{(3)}\} = \begin{Bmatrix} 0.0077 \\ -0.1160 \\ 0.9654 \end{Bmatrix}$$

$$\Rightarrow [U_n] = \begin{bmatrix} -0.2242 & -0.2229 & 0.0077 \\ -0.3062 & 0.3043 & -0.1160 \\ -0.1662 & 0.2007 & 0.9654 \end{bmatrix}$$

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the end of chapter 4