2.5 Rotating unbalance

* A common source of troublesome vibration is rotating machinery. Many machines have rotating devices. Small irregularities (عدم انتظام) in the mass distribution can cause vibration which is called "rotating unbalance". shown In the Figure.

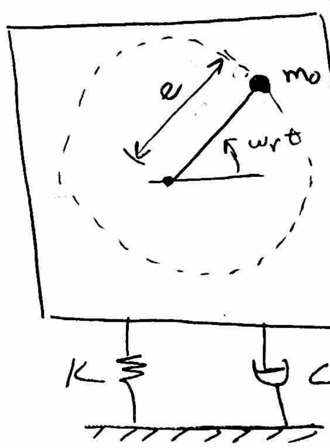
Total mass $= m$

rotating un balance mass $= m_o$

distance from center of rotation $= e$ → Eccentricity

Frequency of rotation $= \omega_r$

mass (m)



Equation of motion

$$m\ddot{x} + c\dot{x} + kx = m_o e \omega_r^2 \sin\omega_r t$$

If we let $F(t) = m_o e \omega_r^2 \sin\omega_r t$ and solve as we did before, the steady-state solution (non-Homog.) can be expressed as:-

$$x_p(t) = \underbrace{X}_{\text{Amplitude}} \sin(\omega_r t - \underbrace{\theta}_{\text{phase}})$$

$$X = \frac{m_o e}{m} \cdot \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

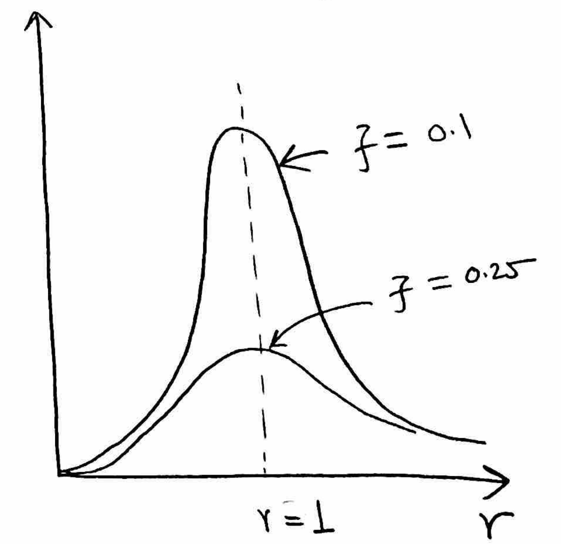
$$r = \frac{\omega_r}{\omega_n}$$

$$\frac{Xm}{m_o e} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

$$\theta = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right), \quad r = \frac{\omega_r}{\omega_n}$$

$$\omega_n = \sqrt{\frac{k}{m}}$$

same as before



please see the attached vedio for more details

Figure 2.21 text book page 162

Example For a rotating unbalance problem. At resonance ($r = 1$) maximum deflection is 0.1 m. The damping ratio of this system $\zeta = 0.05$. The unbalance mass (m_0) is 10% from total mass (m). Find (a) eccentricity (e)

(b) How much mass should we add to reduce deflection at resonance to 0.01 m.

Solution

(a)

$$\boxed{m_0 = \frac{1}{10} m} \quad , \quad \boxed{X = 0.1 m}$$

$$\frac{m X}{m_0 e} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \quad , \quad \text{If at resonance } r = 1$$

so

$$\frac{m X}{m_0 e} = \frac{1}{2\zeta} \Rightarrow m_0 e = 2\zeta m X$$

$$\Rightarrow e = \frac{2\zeta m X}{\underset{= \frac{1}{10} m}{m_0}} = \frac{(2)(0.05)(\cancel{m})(0.1)}{\frac{1}{10} \cancel{m}}$$

$$\Rightarrow \boxed{e = 0.1 \text{ m}}$$

(b) We need to add Δm to make $X = 0.01$ at $r = 1$

$$\frac{(m + \Delta m)\overset{X}{(0.01)}}{m_0 \underset{e}{(0.1)}} = \frac{1}{\underset{0.05}{2\zeta}} = \frac{1}{0.1} = 10$$

remember $m_0 = 0.1 m = \frac{1}{10} m$

$$\Rightarrow \frac{m + \Delta m}{0.1 m} = 100 \quad \Rightarrow \Delta m = 9 m$$

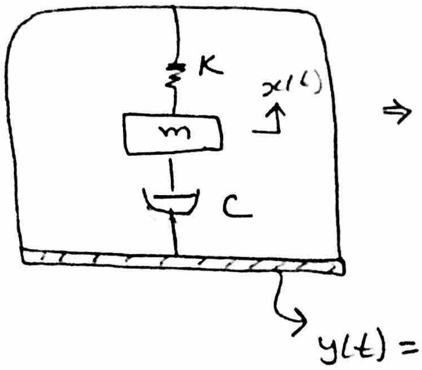
which means that the added mass should be 9 times mass (m)

2.6 Measurement devices

* An important application of the forced vibration and base excitation problems we discussed is in the design and analysis used to measure vibration usually by converting the mechanical motion into electrical voltage signal.

* One important measurement devices is the accelerometer which is commonly used to measure acceleration. and it is used for undamped systems only.

Equation of motion (base Excitation)



$$m\ddot{x} + c(\dot{x}-\dot{y}) + K(x-y) = 0$$

using relative displacement

$$z(t) = x(t) - y(t)$$

the equation of motion becomes

$$m\ddot{z} + c\dot{z} + Kz = -m\ddot{y}$$

Remember the Solution of this equations $\Rightarrow z(t) = Z\cos(w_b t - \theta)$

$$z(t) = \frac{w_b^2 Y}{\sqrt{(w_n^2 - w_b^2)^2 + (2\zeta w_b w_n)^2}} \cos(w_b t - \theta) \; , \quad \theta = \tan^{-1}\left(\frac{2\zeta w_n w_b}{w_n^2 - w_b^2}\right)$$

↳ Eq(1)

Also, remember

$$\frac{Z}{Y} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \; , \quad \theta = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right)$$

We can write eq(1) as:- (take $\frac{1}{w_n^2}$ and mutiply Left hand side by w_n^2)

$$w_n^2 z(t) = \frac{1}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} \boxed{w_b^2 Y(\cos w_b - \theta)} - \ddot{y}(t)$$

$$\Rightarrow \quad w_n^2 z(t) = \frac{-1}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} \ddot{y}(t)$$

For small (r) or $r \to 0$

$$\Rightarrow \lim_{r\to 0} \frac{+1}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} = 1$$

So,

$$z(t) = \frac{-\ddot{y}(t)}{w_n^2} \quad \text{for small } r \quad (r \leq 0.5)$$

which means that we can easily estimate the displacement z(t) only by dividing the base accaleration ($\ddot{y}$) by natural frequency squared (w_n^2)

Example 2.6.1 (book)

This is the last topic in chapter 2

Example 2.6.1

This example illustrates how an independent measurement of acceleration can provide a measurement of a transducer's mechanical properties. An accelerometer is used to measure the oscillation of an airplane wing caused by the plane's engine operating at 6000 rpm (628 rad/s). At this engine speed the wing is known, from other measurements, to experience 1.0-g acceleration. The accelerometer measures an acceleration of 10 m/s^2. If the accelerometer has a 0.01-kg moving mass and a damped natural frequency of 100 Hz (628 rad/s), the difference between the measured and the known

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acceleration is used to calculate the damping and stiffness parameters associated with the accelerometer.

Solution From equation (2.93), the amplitude of the measured values of acceleration $|\omega_n^2 z(t)|$ is related to the actual values of acceleration $|\ddot{y}(t)|$ by

$$\frac{|\omega_n^2 z(t)|}{|\ddot{y}(t)|} = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = \frac{10\ \text{m/s}^2}{9.8\ \text{m/s}^2} = 1.02$$

Rewriting this expression yields one equation in ζ and r:

$$(1-r^2)^2 + (2\zeta r)^2 = 0.96$$

A second expression in ζ and r can be obtained from the definition of the damped natural frequency:

$\omega_d = \omega_n \sqrt{1-\zeta^2}$

$$\frac{\omega_b}{\omega_d} = \frac{\omega_b}{\omega_n}\frac{1}{\sqrt{1-\zeta^2}} = r\frac{1}{\sqrt{1-\zeta^2}} = \frac{628\ \text{rad/s}}{628\ \text{rad/s}} = 1 \Rightarrow r = \sqrt{1-\zeta^2}$$

$r^2 = 1-\zeta^2$

Thus $r = \sqrt{1-\zeta^2}$, providing a second equation in ζ and r. This can be manipulated to yield $\zeta^2 = (1-r^2)$, which when substituted with the preceding expression for r and ζ yields $\Rightarrow \zeta^2 = 1-r^2$

$$\zeta^4 + 4\zeta^2(1-\zeta^2) = 0.96$$

This is a quadratic equation in ζ^2:

$$3\zeta^4 - 4\zeta^2 + 0.96 = 0$$

$\Rightarrow$ Underdamped

This quadratic expression yields the two roots $\zeta = 0.56,\ 1.01$. Using $\boxed{\zeta = 0.56,}$ the damping constant is $(\sqrt{1-\zeta^2} = 0.83,\ \omega_n = \omega_d/\sqrt{1-\zeta^2} = 758.0\,\text{rad/s})$

$$c = 2m\omega_n\zeta = 2(0.01)(758.0)(0.56) = 8.49\,\text{N}\cdot\text{s/m}$$

Similarly, the stiffness in the accelerometer is

$$k = m\omega_n^2 = (0.01)(758.0)^2 = 5745.6\ \text{N/m}$$

□