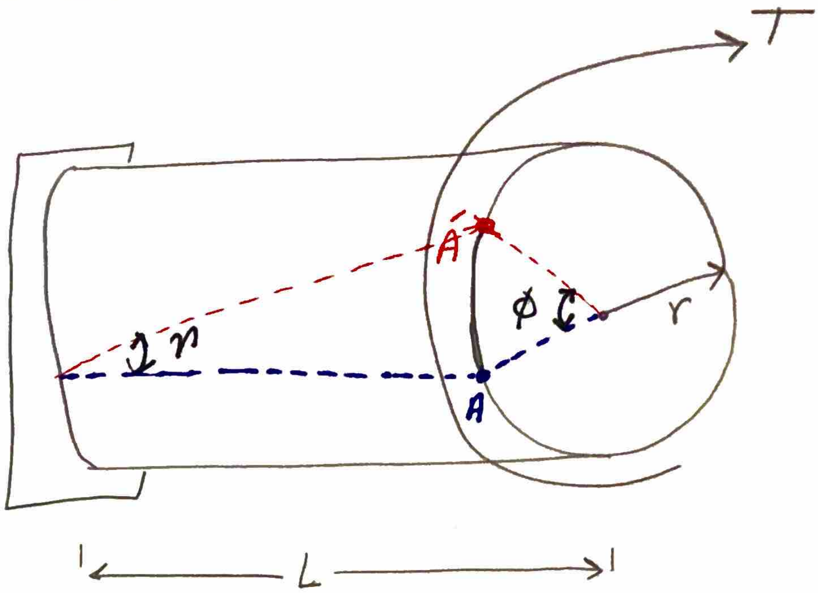
* Chapter Three : Torsion

T: Torque

ϕ : Angle of twist

γ : shear strain



Torsional Load will induce shear stress (τ)

$$\tau = \frac{Tr}{J}$$

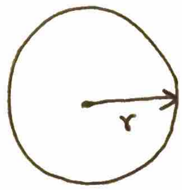
τ: shear stress (Pa)

T: Torque (N.m)

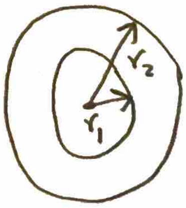
r: radius (r)

J : Polar moment of Inertia (m^4)

$$J = \frac{1}{2}\pi r^4$$



$$J = \frac{\pi}{2}(r_2^4 - r_1^4)$$



Shear strain γ

$$\gamma = \tan\gamma = \frac{r\phi}{L} \Rightarrow \boxed{\gamma = \frac{r\phi}{L}}$$

rad

* Remember $\tau = G\gamma$

G: shear modulus (Pa)

L: Length (m)

$$\frac{T\cancel{r}}{J} = G\frac{\cancel{r}\phi}{L} \Rightarrow \boxed{\phi = \frac{TL}{GJ}}$$

rad

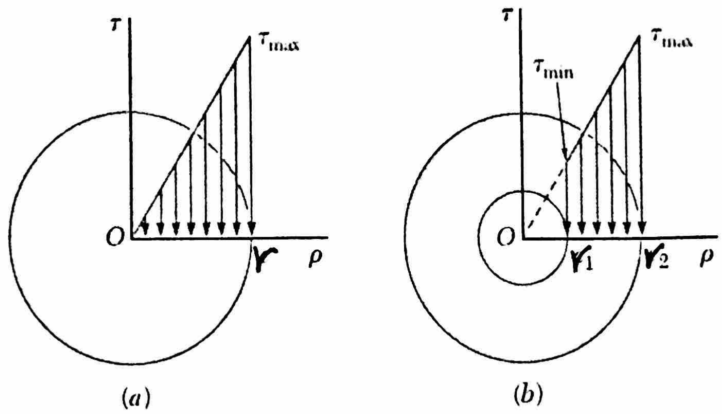


Fig. 3.14 Distribution of shearing stresses.

In Figure (a)

$$\tau_{max} = \frac{Tr}{J}, \quad \tau_{min} = 0, \quad J = \frac{\pi}{2} r^4$$

In figure (b)

$$\tau_{max} = \frac{Tr_2}{J}, \quad \tau_{min} = \frac{Tr_1}{J}, \quad J = \frac{\pi}{2}(r_2^4 - r_1^4)$$

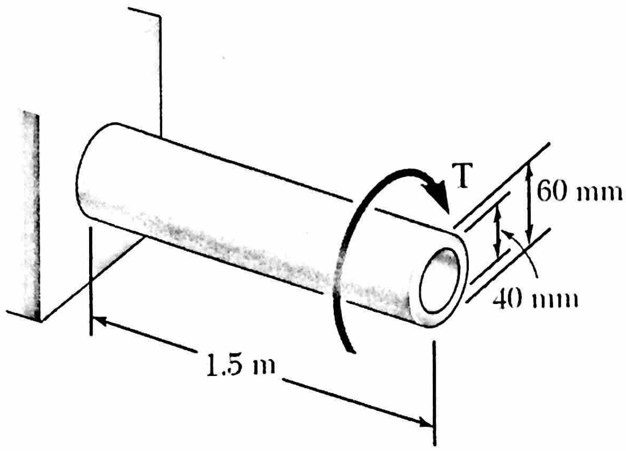
$$\frac{\tau_{min}}{\tau_{max}} = \frac{Tr_1/J}{Tr_2/J} \Rightarrow \frac{\tau_{min}}{\tau_{max}} = \frac{r_1}{r_2}$$

$$\frac{\tau_{min}}{\tau_{max}} = \frac{r_{min}}{r_{max}}$$

Example

$T = 400\ N.m$

$G = 12\ GPa$ (Aluminum)



Find

① Max and min shear stress

② Angle of twist

Solution

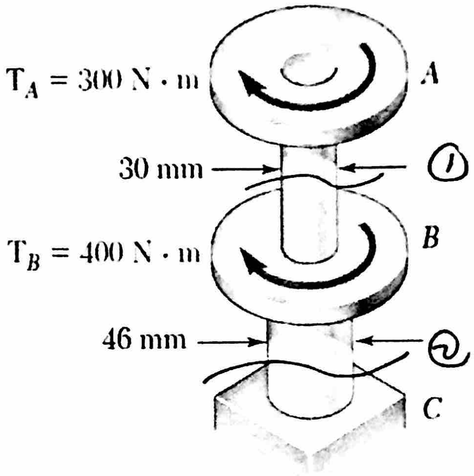
$$\tau_{max} = \frac{T r_{max}}{J} = \frac{(400)(30)(10^{-3})}{\frac{\pi}{2}\left[(30\times10^{-3})^4 - (20\times10^{-3})^4\right]} \Rightarrow \tau_{max} = 11.75\ MPa$$

$$\tau_{min} = \frac{T r_{min}}{J} = \frac{(400)(20)(10^{-3})}{\frac{\pi}{2}\left[(30\times10^{-3})^4 - (20\times10^{-3})^4\right]} \Rightarrow \tau_{min} = 7.83\ MPa$$

$$\phi = \frac{TL}{GJ} = \frac{(400)(1.5)}{(12)(10^9)\frac{\pi}{2}\left[(30\times10^{-3})^4 - (20\times10^{-3})^4\right]} = 0.0489\ rad$$

Example

The torques shown are exerted on pulleys *A* and *B*. Knowing that both shafts are solid, determine the maximum shearing stress in (*a*) in shaft *AB*, (*b*) in shaft *BC*.

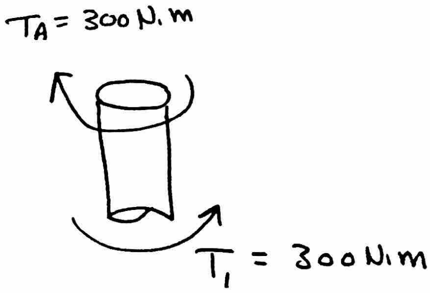


Solutin

① shear stress in AB

$$\tau_{AB} = \frac{T_1 \, r_{AB}}{J_{AB}} = \frac{(300)(15)(10^{3})}{\frac{\pi}{2}(15 \times 10^{-3})^4}$$

$$\tau_{AB} = 56.6 \text{ MPa}$$



② Shear stress in BC

$$\tau_{BC} = \frac{T_2 \, r_{BC}}{J_{BC}} = \frac{(700)(23)(10^{-3})}{\frac{\pi}{2}(23 \times 10^{-3})^4}$$

$$\tau_{BC} = 36.6 \text{ MPa}$$