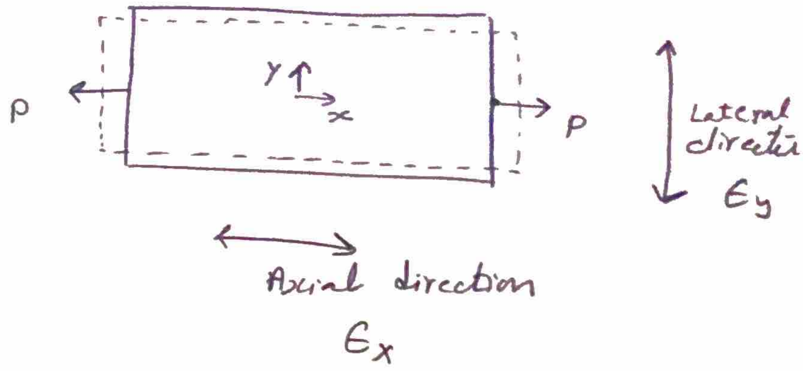
2.11 Poisson's ratio

Poisson's ratio

$$\nu = \frac{-\text{Lateral strain}}{\text{axial strain}}$$

nu



$$\nu = -\frac{\epsilon_y}{\epsilon_x} \quad [-]$$

$$0 < \nu < 0.5$$

also

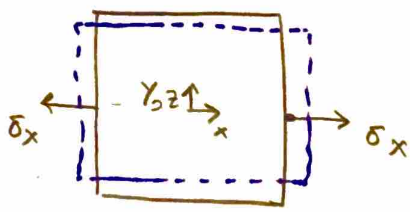
$$\nu = -\frac{\epsilon_z}{\epsilon_x} \quad [-]$$

Hooke's Law

Uniaxial Loading

$$\sigma_x = E\,\epsilon_x$$

$$\Rightarrow \epsilon_x = \frac{\sigma_x}{E}$$



but $\epsilon_y = -\nu\,\epsilon_x$

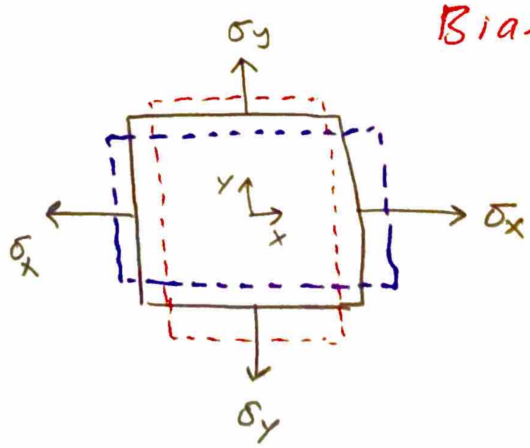
$$\Rightarrow \epsilon_y = -\frac{\nu}{E}\sigma_x$$

$$\epsilon_z = -\frac{\nu}{E}\sigma_x$$

Biaxial Loading

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}\sigma_y$$

$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}\sigma_x$$



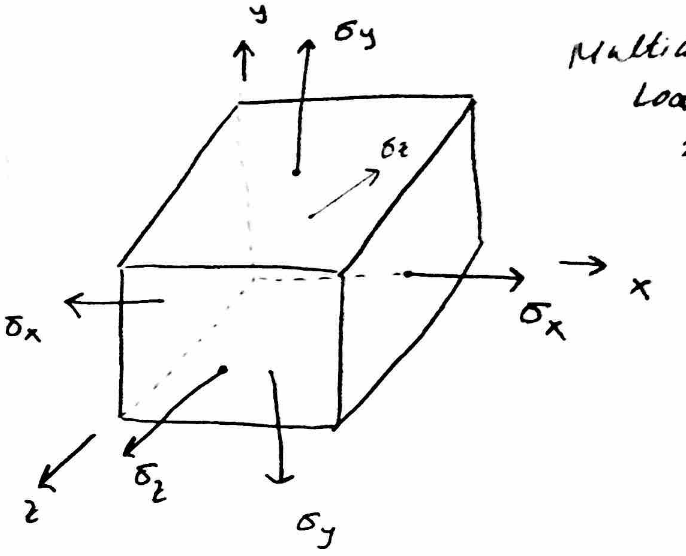
3 Dimensional

Multiaxial Loading 2.12

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z)$$

$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_x + \sigma_z)$$

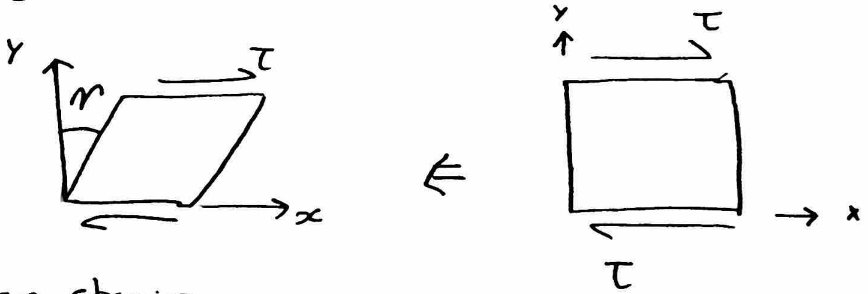
$$\epsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_x + \sigma_y)$$



↗ Generalized Hook's Law

2.14 Shear strain

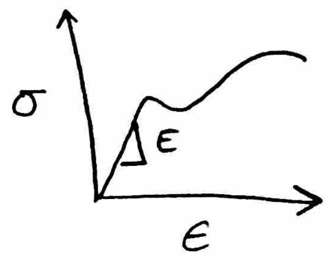
γ : Gamma



$\tau = G\gamma$

- γ → shear strain
- G → shear modulus

Normal

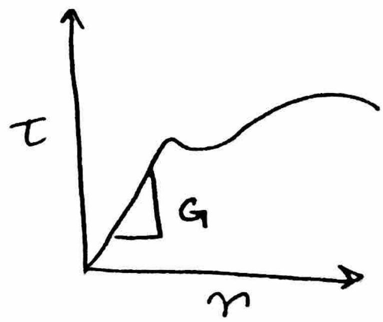


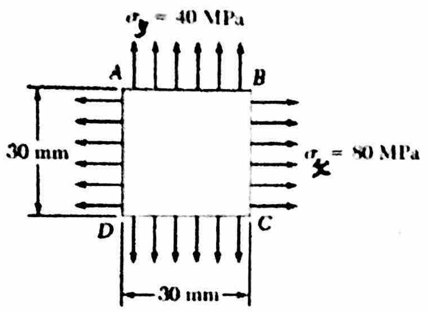
Now, we have 3 material properties

E, G, ν

$$G = \frac{E}{2(1+\nu)}$$

Shear





PROBLEM 2.68

A 30-mm square was scribed on the side of a large steel pressure vessel. After pressurization, the biaxial stress condition at the square is as shown. For $E = 200$ GPa and $\nu = 0.30$, determine the change in length of (*a*) side *AB*, (*b*) side *BC*, (*c*) diagnonal *AC*.

(a) Change length AB (x-direction)

$\delta_x = L_{AB}\,\epsilon_x \quad , \quad \epsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z)$ (σ_z = 0)

$\delta_x = (30)(10^{-3})(340)(10^{-6}) \qquad \epsilon_x = \frac{(80)(10^6)}{(200)(10^9)} - \frac{0.3}{(200)(10^9)}(40)(10^6) = 340\times10^{-6}$

$\delta_x = \delta_{AB} = 10.20\ \mu m$ (micro)

(b) Change length BC (y-direction)

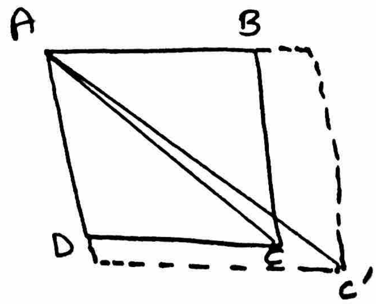
$\delta_y = L_{BC}\,\epsilon_y \quad , \quad \epsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_x + \sigma_z)$ (σ_z = 0)

$\epsilon_y = \frac{(40)(10^6)}{(200)(10^9)} - \frac{0.3}{(200)(10^9)}(80)(10^6) = 80\times10^{-6}$

$\delta_y = (30)(10^{-3})(80)(10^{-6})$

$\delta_y = \delta_{BC} = 2.40\ \mu m$

(c) Change in AC



$\delta_{AC} = L_{AC'} - L_{AC}$

$L_{AC'} = \sqrt{\left[30\times10^{-3} + 10.2\times10^{-6}\right]^2 + \left[30\times10^{-3} + 2.4\times10^{-6}\right]^2}$

$= 42.4353$ mm

$L_{AC} = \sqrt{(30\times10^{-3})^2 + (30\times10^{-3})^2}$

$= 42.426$ mm

$\Rightarrow \delta_{AC} = 42.4353 - 42.426$

$\delta_{AC} = 8.91\ \mu m$