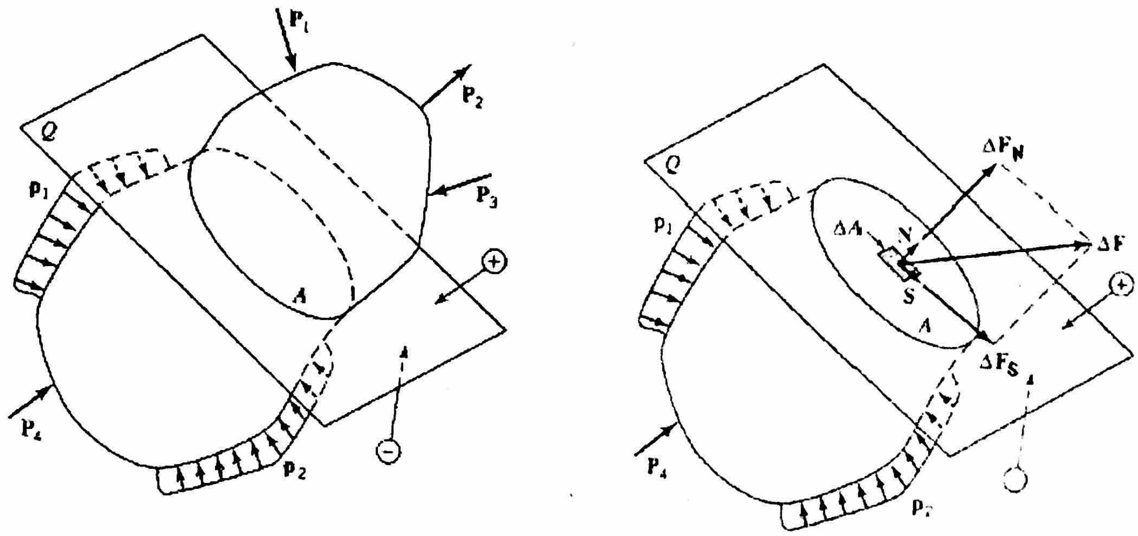
Chapter Two : Theories of Stress and strain

2.1 Definition of stress

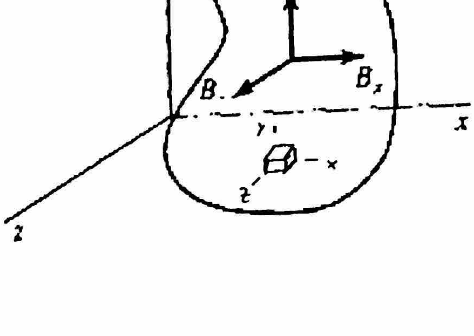


* Normal force ΔF_N : will produce a normal stress

$$\sigma_N = \frac{\Delta F_N}{\Delta A}$$

* Shear force ΔF_S : Will produce shear stress

$$\sigma_S = \frac{\Delta F_S}{\Delta A}$$



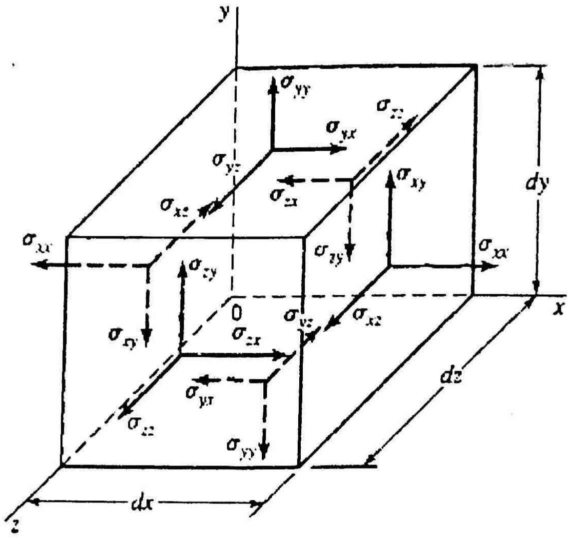
B_x, B_y, B_z are Body forces in x, y, z directions

2.2 Stress Notation

2.3 Stress ~~Symmetry~~ Symmetry

σ_{ij} → stress direction

↳ i: plane direction (Normal to the plane)



$\circlearrowleft \sum M = 0$

$$\Rightarrow \quad \sigma_{xy} = \sigma_{yx}$$

$$\sigma_{xz} = \sigma_{zx}$$

$$\sigma_{yz} = \sigma_{zx}$$

$\sigma \Rightarrow$ is only defined by 6 components.

$$\sigma = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

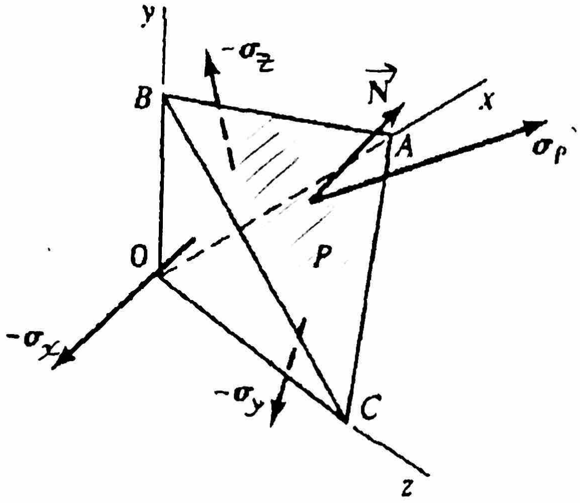
* Define stress vectors $\vec{\sigma_x}$, $\vec{\sigma_y}$, $\vec{\sigma_z}$ on x, y, z planes

so,

$$\left.\begin{aligned} \vec{\sigma_x} &= \sigma_{xx}\hat{i} + \sigma_{xy}\hat{j} + \sigma_{xz}\hat{k} \\ \vec{\sigma_y} &= \sigma_{yx}\hat{i} + \sigma_{yy}\hat{j} + \sigma_{yz}\hat{k} \\ \vec{\sigma_z} &= \sigma_{zx}\hat{i} + \sigma_{zy}\hat{j} + \sigma_{zz}\hat{k} \end{aligned}\right\} \quad [\sigma] = \begin{bmatrix} \vec{\sigma_x} \\ \vec{\sigma_y} \\ \vec{\sigma_z} \end{bmatrix}$$

Stress on arbitrary plane

- a plane (P) with a normal vector $\vec{N}$



- A stress on plane (P) is $\vec{\sigma_P}$

$$\vec{N} = l\hat{i} + m\hat{j} + n\hat{k}$$

$l, m, n \Rightarrow$ components in x, y, z.

$$\vec{\sigma_p} = l\,\vec{\sigma_x} + m\,\vec{\sigma_y} + n\,\vec{\sigma_z}$$

Substitute $\vec{\sigma_x}$, $\vec{\sigma_y}$ and $\vec{\sigma_z}$ above

$$\Rightarrow \vec{\sigma_p} = \sigma_{px}\,\hat{i} + \sigma_{py}\,\hat{j} + \sigma_{pz}\,\hat{k}$$

where $\sigma_{px} = l\,\sigma_{xx} + m\,\sigma_{yx} + n\,\sigma_{zx}$

$$\sigma_{py} = l\,\sigma_{xy} + m\,\sigma_{yy} + n\,\sigma_{zy}$$

$$\sigma_{pz} = l\,\sigma_{xz} + m\,\sigma_{yz} + n\,\sigma_{zz}$$

Normal & shear stresses on an oblique plane

* The normal stress σ_{PN} on plane (P) is the projection of $\vec{\sigma_P}$ in the direction of $\vec{N}$

$$\Rightarrow \sigma_{PN} = \vec{\sigma_P} \cdot \vec{N}$$

$$\sigma_{PN} = l^2 \sigma_{xx} + m^2 \sigma_{yy} + n^2 \sigma_{zz} + 2mn\,\sigma_{yz} + 2ln\,\sigma_{xz} + 2lm\,\sigma_{xy}$$

* The shear stress (σ_{PS}) on plane (P),

$$\sigma_{PS} = \sqrt{\sigma_P^2 - \sigma_{PN}^2} = \sqrt{\sigma_{Px}^2 + \sigma_{Py}^2 + \sigma_{Pz}^2 - \sigma_{PN}^2}$$