2.10 Problems with temperature changes

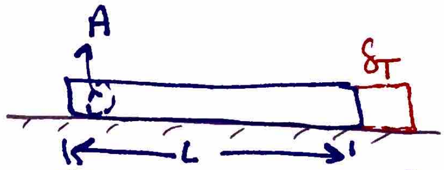
$\delta_T = \alpha (\Delta T) L$

→ thermal deflection (m)

α: Coefficient of thermal expansion (CTE) [1/°C]

L: Length (m)

$\Delta T = T_2 - T_1$ (°C)



Temperature (T_1)

→ Heating (T_2)

$\Delta T = T_2 - T_1$

For multisections
$$\delta_T = \sum_{i=1}^{n} \alpha_i (\Delta T) L_i$$

$$\epsilon_T = \frac{\delta_T}{L} = \frac{\alpha (\Delta T) L}{L} \Rightarrow \epsilon_T = \alpha (\Delta T)$$

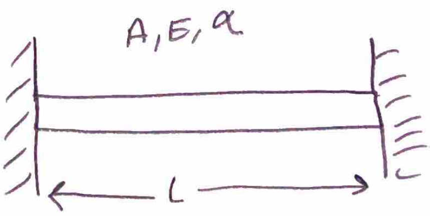
↑ thermal strain

$\sigma_T = 0$ // No internal forces (reactions)

thermal stress

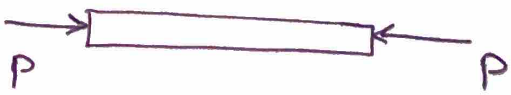
Free expansion

$\delta_T = 0$



Heating $T_1 \rightarrow T_2$

⇒ we need to find reaction forces



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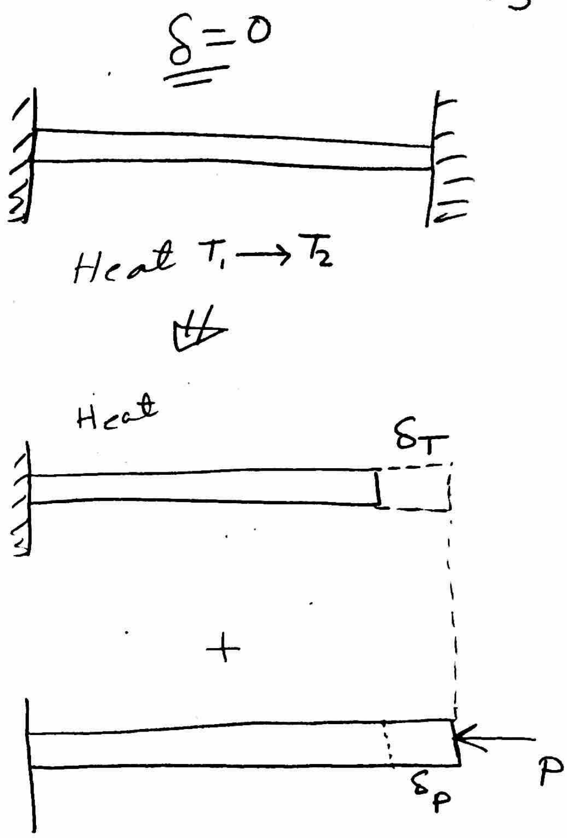


$$\delta_T + \delta_P = 0$$

$$\delta_T = \alpha(\Delta T)L$$

$$\delta_P = \frac{PL}{AE}$$

$$\Rightarrow \alpha(\Delta T)\cancel{L} + \frac{P\cancel{L}}{AE} = 0$$

$$\Rightarrow P = -AE\,\alpha(\Delta T)$$

$$\sigma_T = \frac{P}{A} = -E\alpha(\Delta T)$$

(−: compression)

σ_T: thermal stress

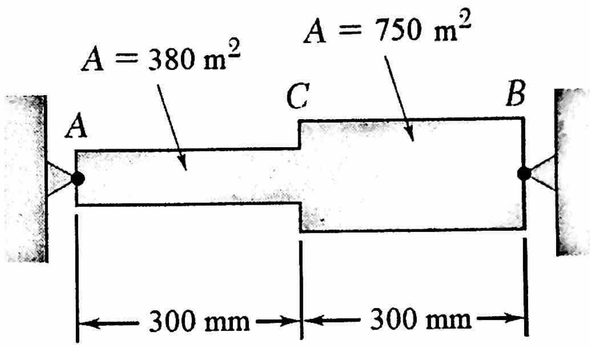
For Multisections

$$\sum_{i=1}^{n} \alpha_i(\Delta T)L_i + \sum_{j=1}^{m} \frac{P_j L_j}{A_j E_j} = 0$$

n **not** always equal m

Example:

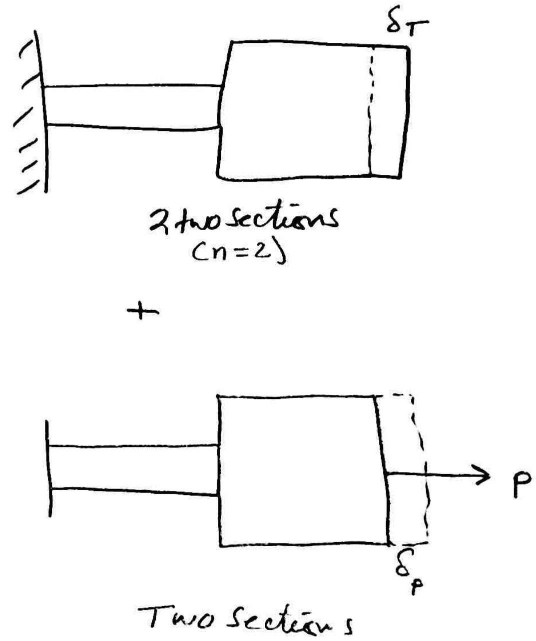
Determine the values of the stress in portions AC and CB of the steel bar shown when the temperature of the bar is 24 °C, knowing that a cooling has happened to the temperature of -45 °C. Use the values $E = 200\ GPa$ and $\alpha = 11.7 \times 10^{-6}/°C$ for steel.



Solution

$$\delta_T + \delta_P = 0 \qquad , T_2 = -45\ °C, \quad T_1 = 24\ °C$$

$$\sum_{i=1}^{n} \alpha_i (\Delta T) L_i + \sum_{j=1}^{m} \frac{P_j L_j}{A_j E_j} = 0$$



$$\alpha_1 (\Delta T) L_1 + \alpha_2 (\Delta T) L_2 + \frac{P_1 L_1}{A_1 E_1} + \frac{P_2 L_2}{A_2 E_2} = 0$$

$$\alpha (\Delta T) \left[L_1 + L_2 \right] + \frac{PL}{E} \left(\frac{1}{A_1} + \frac{1}{A_2} \right) = 0$$

$$11.7 \times 10^{-6} (-45 - 24) \left[300 \times 10^{-3} + 300 \times 10^{-3} \right]$$

$$+ \frac{(P)(300)(10^{-3})}{200 \times 10^{9}} \left(\frac{1}{380 \times 10^{-6}} + \frac{1}{750 \times 10^{-6}} \right) = 0$$

$$\Rightarrow P = 81.34\ kN$$

$$\sigma_{AC} = \frac{P}{A_{AC}} = \frac{81.34 \times 10^{3}}{380 \times 10^{-6}} = 214.1\ MPa$$

$$\sigma_{BC} = \frac{P}{A_{BC}} = \frac{81.34 \times 10^{3}}{750 \times 10^{-6}} = 108.5\ MPa$$

2.7 Fatigue

Fatigue is failure from repeated loading.

Section 2.7 is self-reading and <u>included</u> in ALL exams.