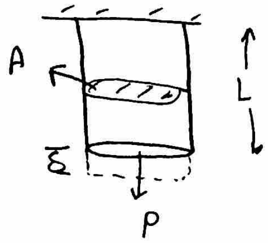
2.9 statically indeterminate problems

Last video/class

$$\delta = \frac{PL}{AE}$$

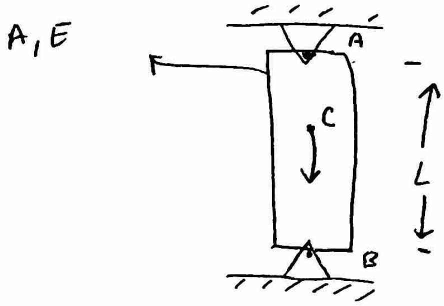


For multisections

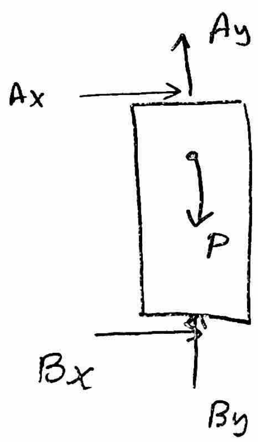
$$\delta = \sum_{i=1}^{n} \frac{P_i L_i}{A_i E_i}$$, n # of sections. { Change in → material, area, Force }

2.9

* Find reactions at A and B



F.B.D



$\overset{+}{\rightarrow} \sum F_x = 0$

$\boxed{A_x + B_x = 0}$ — (1)

$+\uparrow \sum F_y = 0$

$\boxed{A_y + B_y = P}$ — (2)

$+\circlearrowleft \sum M_A = 0 \Rightarrow B_x L = 0 \Rightarrow \boxed{B_x = 0} \Rightarrow \boxed{A_x = 0}$

we still need A_y and B_y ? # Equations < # unknowns

- statics is not enough here (statically indeterminate)
- we need one more equation from mechanics of materials ($\delta = 0$)

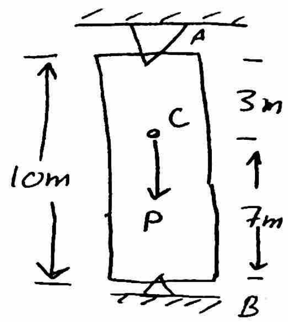
Example

Steel member $E = 200\,GPa$

cross-sectional area $A = 0.5\ m^2$

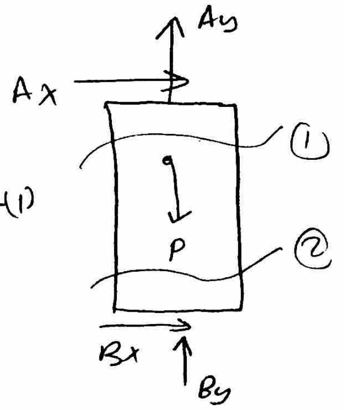
$P = 30\ kN$

Find stresses in AC and BC



Solution

F.B.D



$\Rightarrow A_x = B_x = 0$

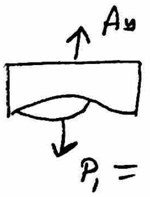
$A_y + B_y = 30\ kN \quad -(1)$

we have one more equation ($\delta = 0$)

$$\delta = \sum_{i=1}^{2} \frac{P_i L_i}{A_i E_i} = \frac{P_1 L_1}{A_1 E_1} + \frac{P_2 L_2}{A_2 E_2} = \frac{1}{AE}\left(P_1 L_1 + P_2 L_2\right) = 0$$

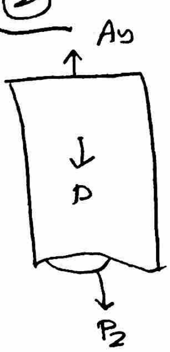
$$\Rightarrow \boxed{P_1 L_1 + P_2 L_2 = 0} \quad -(2)$$

Section 1



$P_1 = A_y \quad , L_1 = 3\,m$

Section 2



$+\uparrow \sum F_y = 0 \Rightarrow A_y - P - P_2 = 0$

$P_2 = A_y - P$

$L_2 = 7\,m$

$\Rightarrow$ Back to equation (2) $\Rightarrow A_y(3) + (A_y - P)(7) = 0 \quad , P = 30{,}000$

$\Rightarrow A_y = 21\ kN \quad \Rightarrow B_y = 9\ kN$

cont'd

Stress in AC

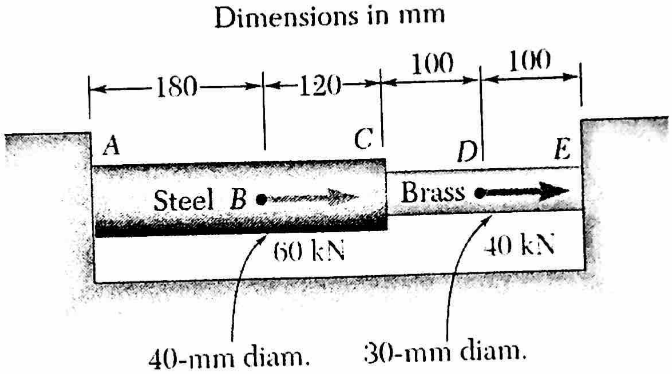
$$\sigma_{AC} = \frac{P_1}{A_{AC}} = \frac{21 \times 10^3}{0.5} = 42 \text{ kPa (tension)}$$

Stress in BC

$$\sigma_{BC} = \frac{P_2}{A_{BC}} = \frac{-9 \times 10^3}{0.5} = -18 \text{ kPa (compression)}$$

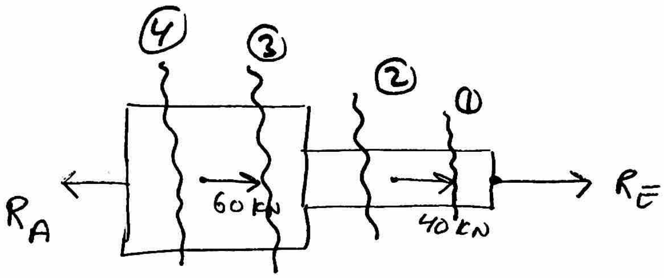
Example:

Two cylindrical rods, one of steel and the other of brass, are joined at ***C*** and restrained by rigid supports at ***A*** and ***E***. For the loading shown and knowing that $E_s = 200\ GPa$ and $E_b = 105\ GPa$.
Determine:
(*a*) Reactions at ***A*** and ***E***.
(*b*) Stress in potion **BC**.



Solution

F.B.D



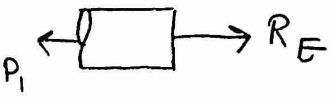
$\overset{+}{\rightarrow} \Sigma F_x = 0$

$$\Rightarrow \quad -R_A + 60 + 40 + R_E = 0 \quad \Rightarrow \quad \boxed{R_A - R_E = 100\ kN} \quad — (1)$$

$$\delta = 0$$

$$\delta = \sum_{i=1}^{n=4} \frac{P_i L_i}{A_i E_i} = 0 \Rightarrow \frac{P_1 L_1}{A_1 E_1} + \frac{P_2 L_2}{A_2 E_2} + \frac{P_3 L_3}{A_3 E_3} + \frac{P_4 L_4}{A_4 E_4} = 0$$

Section 1



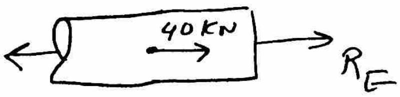
$P_1 = R_E$

$L_1 = 100\ mm$

$A_1 = \pi (15 \times 10^{-3})^2 = 706.4 \times 10^{-3}\ m^2$

$E_1 = 105 \times 10^9\ Pa$

Section 2



$L_2 = 100\ mm$

$A_2 = 706.4 \times 10^{-3}\ m^2$

$E_2 = 105 \times 10^9\ Pa$

$\overset{+}{\rightarrow} \Sigma F_x = 0$

$P_2 = 40 + R_E$

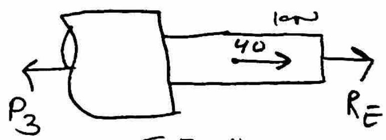
Section 3

$L_3 = 120\ mm$

$A_3 = \pi (20 \times 10^{-3})^2$

$= 1.257 \times 10^{-3}\ m^2$

$E_3 = 200 \times 10^9\ Pa$



$\Sigma F = 0$

$P_3 = 40 + R_E$

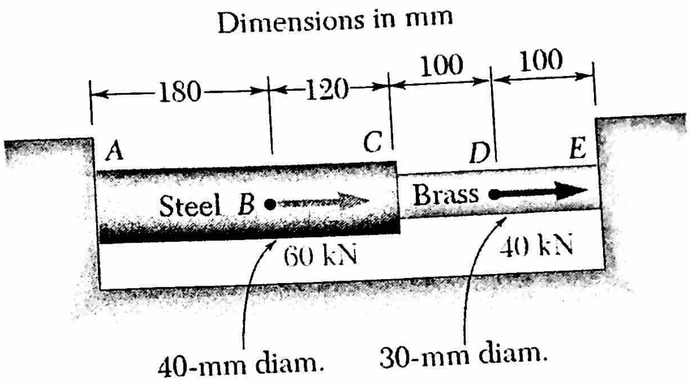
Section 4

$L_4 = 180\ mm$

$A_4 = 1.257 \times 10^{-3}\ m^2$

$E_4 = 200 \times 10^9\ Pa$

$P_4 = 100 + R_E$



$$\delta = \frac{P_1 L_1}{A_1 E_1} + \frac{P_2 L_2}{A_2 E_2} + \frac{P_3 L_3}{A_3 E_3} + \frac{P_4 L_4}{A_4 E_4} = 0$$

$$0 = \frac{(R_E)(100)(10^{-3})}{(706.4 \times 10^{-3})(105)(10^9)} + \frac{(40 + R_E)(100)(10^{-3})}{(706.4 \times 10^{-3})(105)(10^9)}$$

$$+ \frac{(40 + R_E)(120)(10^{-3})}{(1.257 \times 10^{-3})(200)(10^9)} + \frac{(100 + R_E)(180)(10^{-3})}{(1.257 \times 10^{-3})(200)(10^9)}$$

$$\Rightarrow R_E = -54.520 \text{ kN}$$

Eq(1) $\quad R_A = 45.48$ kN

(b) Stress in BC

$$\sigma_{BC} = \frac{P_3}{A_{BC}} = \frac{40 + R_E}{A_{BC}} = \frac{-14.52 \times 10^3}{1.257 \times 10^{-3}} = -11.55 \text{ MPa}$$

A_{BC} " A_3

#