Chapter one : Introduction

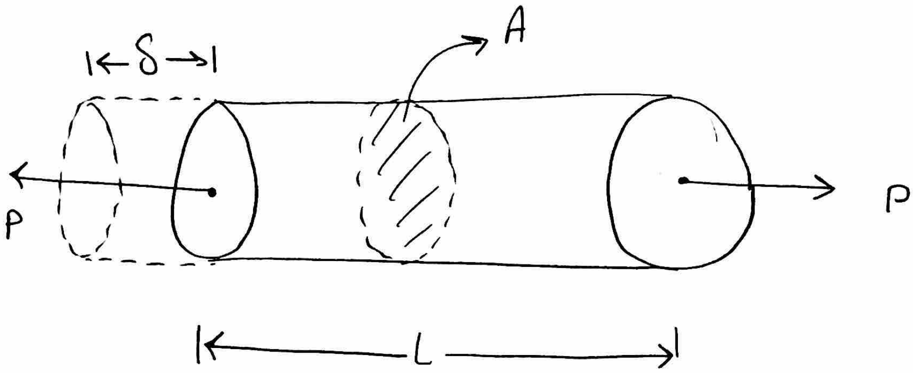
1.1 Review on Elementary mechanics of materials

1.1.1 Axial Loading

P: Force (N)

A: Cross-sectional area (m^2)

L: Length (m)



Stress (σ)

$$\sigma = \frac{P}{A} \qquad [Pa]$$

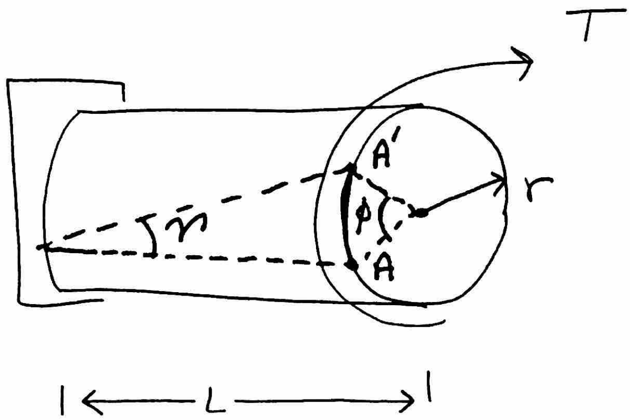
Deflection (δ)

$$\delta = \frac{PL}{AE}$$, E : Elastic modulus (Pa)

Strain (ϵ)

$$\epsilon = \frac{\delta}{L} \qquad [-]$$ Dimensionless

1.1.2 Torsional Loads



T: Torque (N/m)

r: Radius (m)

L: Length (m)

* Shear stress (τ)

$$\tau = \frac{Tr}{J}$$

, J: Polar moment of inertia (m^4)

$$J = \frac{1}{2}\pi r^4$$

[Pa]

* Angle of twist ϕ

$$\phi = \frac{TL}{GJ}$$

, G: Shear modulus (Pa)

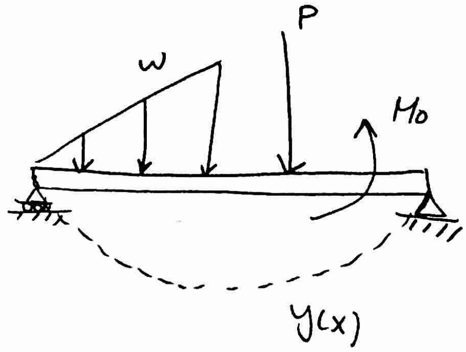
[rad]

* Shear strain (γ)

$$\gamma = \frac{r\phi}{L}$$

[rad]

1.1.3 Bending Loads



* Stress (σ)

$$\sigma = \frac{M c}{I}$$

M: Bending moment ($N \cdot m$)

C: Distance from Centroid (m)

I: moment of intertia (m^4)

* Deflection ($y(x)$)

- Elastic Curve formula

$$\frac{d^2 y}{dx^2} = \frac{M(x)}{E I}$$, E: Elastic modulus

Integrate twice to find $y(x)$

* Shear stress (τ)

$$\tau = \frac{V Q}{I t}$$

V: Shear force (N)

Q: 1st moment of area (m^3)

t: Smallest thickness (m)

1.2 Methods of analysis

* We generally aim to find relation between

1- Load and Stress

or

2- Load and deflection

If we have deflections, then it is easy to find strain (δ/L) and Stress.

there are three common methods for that

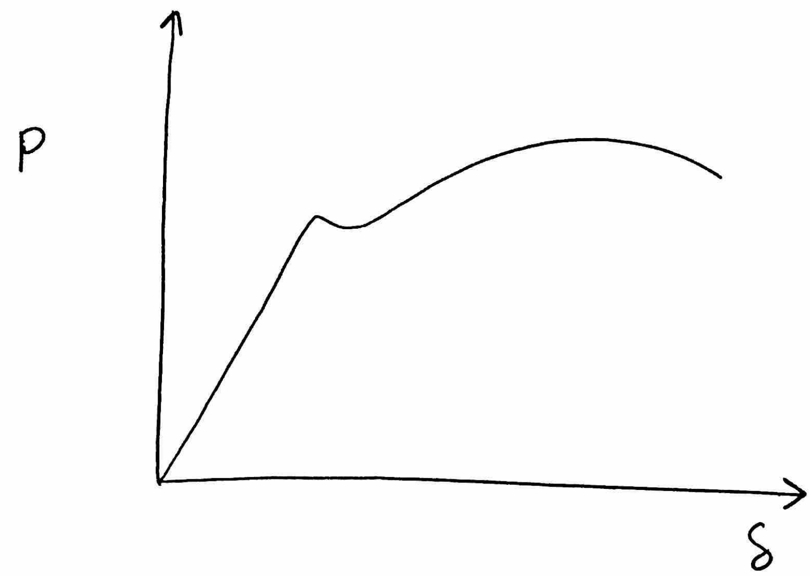
① Method of mechanics of materials
"Free-body diagram"

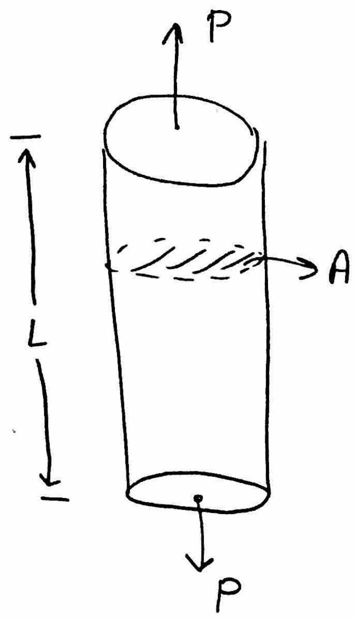
② methods of Continuum mechanics
Ch. 2 and 3

③ Energy methods
Ch. 5

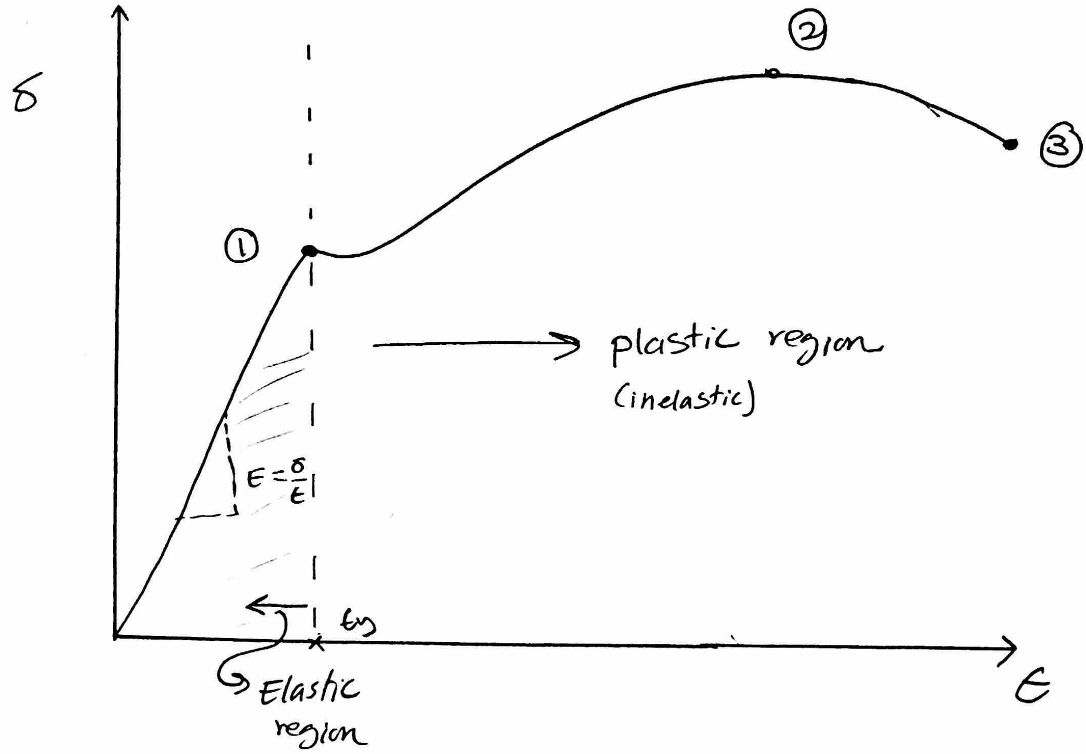
Section 1.3 Stress-strain Relations

Load-Deflection Curve





Divide P/A and δ/L $\Rightarrow$ Stress-Strain Curve



(1) Yield stress (σ_y)

(2) Ultimate tensile stress (UTS) — Necking

(3) Fracture (rupture) stress (σ_F)

* Slope of Linear part (E) — Elastic modulus

$$E = \frac{\sigma}{\epsilon} \quad (Pa) \quad \Rightarrow \quad \sigma = E\,\epsilon$$

* Poisson's ratio (ν)

$$\nu = \frac{\text{Lateral strain}}{\text{axial strain}}$$

* modulus of resilience (U_r)

area under curve till yield stress

$$U_r = \frac{1}{2}\sigma_y \epsilon_y = \frac{1}{2}\frac{\sigma_y^2}{E}, \quad \sigma_y = E\,\epsilon_y$$

* **Modulus of toughness (U_F)**

area under all curve

Energy required to cause fracture

* please re-read section 1.3 from text book

1.4 Failure and limits on Design

1.4.1 Failure modes

Reasons or types of failures are:

① Excessive Deformations

→ Elastic Deflections

→ Deformations causes by Creep (Load combined with temperature for Long Period of times)

② Failure by Yielding $\sigma > \sigma_y$

③ Fracture

→ Sudden fracture of Brittle materials (glass)

→ Fracture of Cracked structure (Stress concentration)

→ Progressive fracture (fatigue)

④ Instability

→ Buckling of Columns.