Chapter 11 Fourier Analysis

we only need Sections 11.1, 11.2 & 11.5

11.1 : Fourier Series

- Fourier series are infinite series that represents periodic functions in terms of cosines and sines. Commonly used in signal analysis.

- For a periodic function $f(x)$: $f(x+p) = f(x)$ or a function repeats itself over a period p. For "n" periods: $f(x+np) = f(x)$.

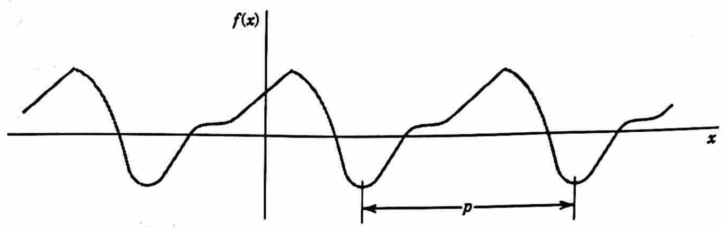

Fig. 258. Periodic function of period *p*

- Common examples of periodic functions are cosine, sine, tangent and cotangent. Periodic over period 2π.

- Common examples of non-periodic functions: $e^{(x)}$, x, x^2, $\cosh x$, $\ln(x)$

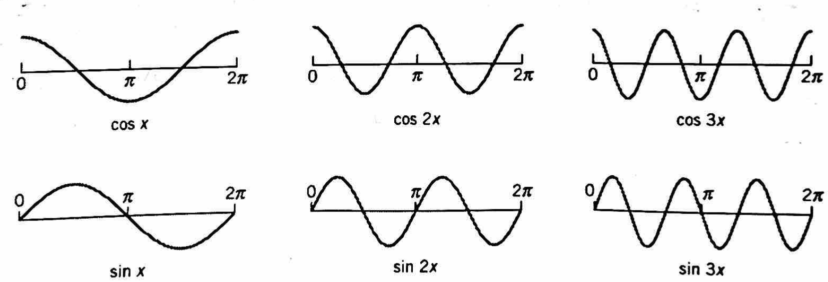

Fig. 259. Cosine and sine functions having the period 2π (the first few members of the trigonometric system (3), except for the constant 1)

- For a function $f(x)$, that is periodic over period of 2π, can be expressed in terms of Fourier series:

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(nx) + b_n \sin(nx))$$

where: $n = 1, 2, \ldots$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx)\,dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx)\,dx$$

Euler Formulas!

11.2 Arbitrary Period. Even & odd functions. Half-range expansions. ②

- For a function $f(x)$ periodic over arbitrary period $2L$ "instead of 2π"

The Fourier series:

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi}{L}x + b_n \sin\frac{n\pi}{L}x\right)$$

where:

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi}{L}x\,dx \qquad n = 1, 2, \ldots$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$

- If $f(x)$ is an even function $(f(-x) = f(x))$, the Fourier Series above becomes:

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi}{L}x \qquad n = 1, 2, \ldots$$

where: $a_0 = \frac{1}{L}\int_0^L f(x)\,dx$, $a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi}{L}x\,dx$

- If $f(x)$ is an odd function $(f(-x) = -f(x))$, the Fourier Series above becomes:-

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi}{L}x$$

where $b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi}{L}x$

For the period $2L = 2\pi$ "$L = \pi$"

Even $f(x)$

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx$$

$$a_0 = \frac{1}{\pi}\int_0^{\pi} f(x)\,dx$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos(nx)\,dx$$

$n = 1, 2, \ldots$

Odd $f(x)$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$$

$n = 1, 2, \ldots$