

* Applications to ODE's: Boundary Value Problems (BVP)

(1)

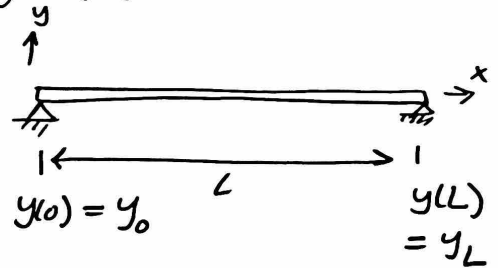
So far, we have been dealing with ODE's with initial value Problems (IVP). In IVP, the conditions are always given at "initial" value like $(t \text{ or } x=0)$, as: $y(0) = y_0$ and $y'(0) = v_0$

However, for BVP, the conditions are given over a certain domain or space, for like beams with length (L) :

$$y(0) = y_0 \text{ and } y(L) = y_L$$

"Boundary conditions"

Boundary value Problems are widely used in Mechanical Engineering applications!



For BVP, we use exactly the same methods we have discussed so far.

Example = Solve $y'' - y' - 6y = 0$ $0 \leq x \leq L$, $y(0) = y_1$, $y(L) = y_2$

Sol'n \Rightarrow 2nd order ODE with constant coefficients

$$\Rightarrow \text{Charact Eq'n} \Rightarrow \lambda^2 - \lambda - 6 = 0$$

roots $\lambda_1 = -2$ and $\lambda_2 = 3$ "Distinct roots"

$$y(x) = A_1 e^{-2x} + B_1 e^{3x} \quad , \quad A_1 \text{ and } B_1 \text{ from BC's}$$

If $y_1 = 0$ and $y_2 = 0$

$$y(0) = 0 = A_1 + B_1$$

$$y(L) = 0 = A_1 e^{-2L} + B_1 e^{3L}$$

} If L is known, we have two eq'ns and 2 unknowns \Rightarrow Find A_1 and B_1