2.6 Existence and Uniqueness of Solution: Wronskain.

* General linear homogeneous 2nd order ODE:

$$y'' + p(x)y' + q(x)y = 0 \qquad \text{— Eq(1)}$$

with contineous variable coefficients $p(x)$ and $q(x)$

The general solution "Using Principle of Superposition"

$$y(x) = y_1(x) + y_2(x) \qquad \text{— Eq(2)}$$

- Where the initial conditions are:

$$y(x_0) = K_0 \quad \text{and} \quad y'(x_0) = K_1 \qquad \text{— Eq(3)}$$

Theorem # 1: Existence and Uniqueness theorem for initial Value Problems. (IVP)

⇒ If $p(x)$ and $q(x)$ are contineous functions on Some open interval (I), and the initial value (x_0) is in (I), then the IVP of Eq(1) and Eq(3) has a unique solution on the interval (I).

* Linear independence of Solutions

For ODE of Eq(1) and Solution of Eq(2), we call $y_1(x)$ and $y_2(x)$ "Linearly independent" on (I), if:

$$C_1 y_1 + C_2 y_2 = 0 \quad \text{implying} \quad C_1 = C_2 = 0$$

or, they are "linearly dependent" on (I), if:

$$C_1 y_1 + C_2 y_2 = 0 \quad \text{if} \quad C_1 \neq 0 \text{ or } C_2 \neq 0$$

In this case, and only this case, y_1 and y_2 are proportional to each other, so:

$$y_1 = C\, y_2 \quad \text{or} \quad y_2 = C\, y_1$$

Definition : Wronskain (W)

determinnate

$$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1'$$

Theorem #2 : Linear dependence and Independence of solutions

$\Rightarrow$ For eq(1) and two solutions y_1 and y_2, are linearly dependent, if and only if, their Wronskain is zero $(W=0)$. for $x = x_0$. Hence, if there is $x = x_0$ at which $W \neq 0$, then y_1 and y_2 are linearly independent.

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