Physics I

Mechanics

classical mechanics, concerning the motion of objects that are large relative to atoms and move at speeds much slower than the speed of light.

Chapter 1 Physics and measurement

1.3 Dimensional analysis

* The symbols we use to specify the dimensions of length, mass, and time are L, M, and T.

length L

~~M~~mass M
time T

* dimensions of speed (v) are $[v] = \frac{L}{T}$

* dimensional analysis can be used to check specific equations because dimensions can be treated as algebraic quantites. for example quantities can be added or subtracted only if they have the same dimensions.

* Any relationship can be correct only if the dimensions on both sides of the eqhation are the same.

example. equation

$$x = \frac{1}{2} a t^2$$

$[x] = L$, $[a] = \frac{L}{T^2}$, $[t] = T$

The dimensional form of the equation:

$$\Rightarrow L = \frac{L}{T^2} \cdot T^2 = L$$

* dimentional analysis is used to set up an expression of the form

$$x \propto a^n t^m$$

$$\Rightarrow [a^n t^m] = L = L^1 T^0$$

$$\left(\frac{L}{T^2}\right)^n \cdot T^m = L^1 \cdot T^0$$

$$\Rightarrow \left(L^n T^{m-2n}\right) = L^1 T^0$$

$$\Rightarrow \boxed{n = 1} \quad m - 2n = 0 \Rightarrow \boxed{m = 2n = 2}$$

$$\Rightarrow \boxed{x \propto a t^2}$$

example ① Show that the expression $v = at$, where v is speed ~~and~~, a is acceleration and t is time is dimensionally correct.

$[v] = \frac{L}{T}$, $[at] = \frac{L}{T^2} \cdot T = \frac{L}{T}$

$\Rightarrow v = at$ is dimensionally correct.

example ② accelaration(a) of particle is ~~given~~ ~~by~~ propotional to some power of r (say r^n) and some power of v, (say v^m), find n, and m values.

$$a = K r^n v^m$$

(K → constant)

$$\Rightarrow \frac{L}{T^2} = L^n \left(\frac{L}{T}\right)^m = \frac{L^{n+m}}{T^m}$$

$\Rightarrow 1 = n+m$ and $m = 2$

$\Rightarrow n = 1 - m = -1$

$$\Rightarrow a = K r^{-1} v^2 = K \frac{v^2}{r}$$

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problem

Newtons law of universal gravitation is represent by. $F = G \frac{Mm}{r^2}$, where M, and m masses of the objects and r is a distance. F is the gravitational force ~~F~~ and it has the SI units $kg \cdot m/s^2$. What are the SI units of the proportionality constant G?

$$\Rightarrow \frac{F \cdot r^2}{Mm} = G \frac{Mm}{Mm} \Rightarrow G = \frac{F \cdot r^2}{Mm}$$

$$[G] = \frac{kg \cdot m/s^2 \cdot m^2}{kg \cdot kg} = \frac{m^3}{kg \cdot s^2}$$