Phys 761
Quantum Mechanics
Problem Set # 7

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1. Consider a particle of mass m moving in a 1D harmonic oscillator potential $V(x) = \frac{1}{2}m\omega_0^2 x^2$. The particle is initially in the ground state $|0\rangle$. At $t = 0$, a perturbation, $H'(t) = 2\alpha x\ cos(\omega t)$ where α is a real constant, is turned on, calculate the transition probability $P_{n0}(t)$ from the ground state $|0\rangle$ to the nth excited state $|n\rangle$ after a sufficiently long time (i.e. $t \to \infty$)

2. Repeat the last problem, but with a perturbation of the form $H'(t) = \frac{\alpha x}{\sqrt{\pi}\tau} e^{-t^2/\tau^2}$, where both α and τ are real constants. Consider all states.

   Hint: $\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{\alpha}}$

3. Consider a particle of mass m moving in an infinite symmetric potential well given by

$$V(x) = \begin{cases} 0, & |x| \leq a/2; \\ \infty, & \text{elsewhere.} \end{cases}$$

   The eigenstates and eigenenergies are given by $u_n(x) = \sqrt{\frac{2}{a}}\ sin(\frac{n\pi}{a}(x - \frac{a}{2}))$ where $n = 1, 2, 3, ...$and $E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$. Notice that the potential is symmetric, hence there will be even and odd solutions.The particle is initially in the ground state of the well ($n = 1$). At $t = 0$, a small perturbation $H'(t) = x^2 e^{-t/\tau}$ is applied. Calculate the transition probabilities from the ground state to the first excited state ($n = 2$) and from the ground state to the second excited state ($n = 3$).

   Hint: $\int_0^{\infty} x^n e^{-\alpha x} dx = \frac{n!}{\alpha^{n+1}}$ and $cosA\ cosB = 1/2[cos(A - B) + cos(A + B)]$

4. Consider a hydrogen atom initially in its ground state. At $t = 0$, the atom is placed in a time-dependent electric field pointing along the z-direction $E = E_0\ e^{-t/\tau}$, where τ is a constant having the dimension of time. Calculate the probability that the atom will be found in the 2p state after a sufficiently long time (i.e. $t \to \infty$).

   Hint: take $\psi_i = \psi_{100}$ and $\psi_f = \psi_{210}$, where ψ_{nlm} refers to the eigenstates of the hydrogen atom

Good Luck