Phys 761
Quantum Mechanics
Problem Set # 5

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1. There are special spectroscopic notations used to label quantum states in atoms. The notation takes the form $^{2s+1}nL_j$, where n is the principle quantum number (which takes the values $1, 2, 3, ...$), s is the spin angular momentum quantum number, j is the total angular momentum quantum number, and L is the orbital angular momentum quantum number (which takes the values $0, 1, 2, 3, ...$). Using these notations, write down the possible states that can be obtained for an electron in the state $n = 3$ of the hydrogen atom. What are the corresponding Landue g_j factor of each state. What is the consequence of having different landue g_j factors for different states.

2. Calculate the energy spectrum of the $n = 1$ and $n = 2$ states in the real hydrogen atom, ignoring the hyperfine structure. How is the spectrum changed when the atom is placed in a magnetic fields of strength 0.1 T and 10 T.

3. The nucleus of a deuterium atom consists of a proton and a neutron that are both spin 1/2 particles.The total spin angular momentum is $\vec{S} = \vec{S_p} + \vec{S_n}$. Assuming the state $|s_p s_n s m\rangle$ is a simultaneous eigenstate of $\vec{S_p}, \vec{S_n}, S^2$, and S_z,

   (a) What are the allowed values of the total spin quantum number s. For each s-value, what are the allowed m quantum numbers.

   (b) Show that $|s_p s_n s m\rangle$ is an eigenstate of $\vec{S_p} \, . \, \vec{S_n}$ and find the corresponding eigenvalue.

4. Consider the vanadium ion V^{+2} that has electron configuration of (Ar$3d^3$).

   (a) Write down the possible quantum numbers for the total electron spin angular momentum in this configuration?

   (b) If two electrons reside on two different orbitals, what are the possible values for total spin and the multiplicity? What values are possible for three electrons on different orbitals?

5. (a) Write down all the possible spectroscopic notations that can be obtained from the electron configurations $2s^1 2p^1$, Ar$4s^2 3d^{10} 4p^5$ (Bromine atom)

   (b) What information that can be obtained about a given atom if it exists in the state $^3F_4$?

6. Consider the emission spectrum of potassium atom, which contains lines at $\lambda_1 = 766.70\ nm$ and $\lambda_2 = 770.11\ nm$. The two emission lines result from the two states $^2P_{1/2}$ and $^2P_{3/2}$, which are split by the spin-orbit coupling, and decay to the ground state $^2S_{1/2}$. Hence the energy difference between the two emission lines gives the energy difference between the spin-orbit split states $^2P_{1/2}$ and $^2P_{3/2}$.

   (a) Based on the two transition lines, calculate the energy difference between the two states $^2P_{1/2}$ and $^2P_{3/2}$ and compare it with the value that is obtained using the formula discussed in the class for spin orbit coupling correction for hydrogen like atom ($Z = 19$).

   (b) If the potassium atom is placed in a magnetic field of strength 10 T, calculate the Zeeman splitting for the state $^2P_{1/2}$. Compare your result with the strength of the spin orbit coupling of the same state. Does the spin orbit coupling strong or weak for the potassium atom.

Good Luck