# Phys 741
# Statistical Mechanics
# Problem Set # 5

Dr. Gassem Alzoubi
*The Hashemite University Department of Physics, Zarqa, Jordan*

1. The Hamiltonian of a 1D harmonic oscillator is given by

$$H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 q^2$$

   (a) Calculate the classical partition function, taking the phase space element to be $dqdp/\tau$, where $\tau$ is a constant
   (b) Calculate the quantum partition function
   (c) Compare the quantum partition function in the limit ($\beta \to 0$) with the classical partition function and show that $\tau = h$, Planck's constant

2. Consider a system of $N$ noninteracting spins, whose energies in a magnetic field $B$ are given by $\pm\mu_0 B$. Ignoring translational motion $p^2/2m$ of the particles
   (a) Calculate the partition function $Q_N$
   (b) Calculate the average magnetic moment $\langle M \rangle$
   (c) Find the mean square fluctuations $\langle M^2 \rangle - \langle M \rangle^2$

3. Pathria 5.1

4. (a) Consider the density matrix $\rho = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ that represents a beam of electrons. Calculate $\langle S_x \rangle$, $\langle S_y \rangle$, and $\langle S_z \rangle$, where $S_i = \frac{\hbar}{2}\sigma_i$, $\sigma_i$ are Pauli matrices. What is the physical interpretation of your results.
   (b) Consider the density matrix $\rho = \frac{1}{2}\begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}$ that represents a beam of electrons. Calculate $\langle S_x \rangle$, $\langle S_y \rangle$, and $\langle S_z \rangle$. What is the physical interpretation of your results.

5. This problem is a generalization of problem 5. A beam of electrons was prepared to be polarized in the $\vec{n}(\theta, \phi)$ direction. Let $\chi_{n+}$ be the eigenstate of the spins that point up along $\vec{n}$, which is defined by

$$\chi_{n+} = \begin{pmatrix} e^{-i\phi/2} \cos(\frac{\theta}{2}) \\ \\ e^{i\phi/2} \sin(\frac{\theta}{2}) \end{pmatrix}$$

   (a) Construct the density matrix that represents the electron beam in terms of the angles $\theta$ and $\phi$
   (b) Find the density matrices $\rho_x$, $\rho_y$, and $\rho_z$ that represents a beam of electrons polarized along the x, y, and z axes respectively.
   (c) To check your findings in part b), consider $\rho_y$ and show that $\langle S_y \rangle = \hbar/2$, $\langle S_x \rangle = \langle S_z \rangle = 0$

6. Consider the density matrix of a harmonic oscillator that is given by $\rho = A\ e^{-H/k_B T}$, where $A$ is a constant and $H\ |n\rangle = E_n\ |n\rangle$, $n = 0, 1, 2, ....$ Clearly in these basis $|n\rangle$, $\rho$ is diagonal; i.e. $\rho_{nn} = \langle n|\rho|n\rangle$
   (a) Calculate the normalization constant $A$
   (b) Find an expression for the density matrix $\rho_{nn}$
   (c) Using the density matrix obtained in (b), find an expression for $\langle E \rangle$

*Good Luck*