Phys 761
Quantum Mechanics
Problem Set # 4

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1. A particle of mass m is confined in a 1D potential well of width a, which is defined by

$$V(x) = \begin{cases} 0\ , & 0 \leq x \leq a; \\ 0\ , & \text{elsewhere.} \end{cases}$$

Suppose we add a delta function bump in the middle of this potential , $H' = \alpha\delta(x - a/2)$, where α is a constant,

   (a) Find the 1^{st} order correction to the allowed energies $E_n^{(1)}$. Explain why energies are not perturbed for even n

   (b) Find the 1^{st} three nonzero terms in the corrections to the wave function of the ground state ($n = 1$)

   (c) Find the 2^{nd} order correction to the energies $E_n^{(2)}$

2. A particle of mass m is confined in a 2D potential well of width a that is defined by

$$V(x, y) = \begin{cases} 0\ , & 0 \leq x \leq a \ \ \textit{and} \ \ 0 \leq y \leq a; \\ 0\ , & \text{elsewhere.} \end{cases}$$

Let us introduce the following perturbation

$$H'(x, y) = \begin{cases} w_0\ , & 0 \leq x \leq a/2 \ \ \textit{and} \ \ 0 \leq y \leq a/2; \\ 0\ , & \text{elsewhere.} \end{cases}$$

   (a) Find the lowest order (in w_0) energy correction to the ground state

   (b) Find the lowest order (in w_0) energy correction to the first exited state

3. Consider a particle of mas m moving in a slightly perturbed 2D H.O potential given by

$$V(x, y) = \tfrac{1}{2}mw^2(x^2 + y^2) + \beta m\omega^2 xy\ ,$$

where β is a small constant.

   (a) Calculate to 1^{st} order in β the energy of the ground state

   (b) Calculate to 1^{st} order in β the energy of the first excited state

4. Consider the problem of 1D harmonic oscillator that is slightly perturbed by the potential $H' = \beta(a^\dagger a^\dagger + aa)$, where $a^\dagger$ and a are creation and annihilation operators, respectively, and β is a real constant. Calculate the energies to the 2^{nd} order in β

5. Consider a rigid rotator whose Hamiltonian is given by $\hat{H}_0 = \hat{L}^2/2I$, where $\hat{L}$ is the angular momentum vector operator, and I is the moment of inertia. The magnetic moment of this rotator can be written as $\vec{\mu} = \alpha\vec{L}$, where α is a constant. If the rotator is placed in a magnetic filed that points in the z-direction, calculate the energy correction.

Good Luck