Phys 761
Quantum Mechanics
Problem Set # 3

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1. Suppose that a 3D scattering potential has a translation symmetry $V(\mathbf{r}+\mathbf{R}) = V(\mathbf{r})$, where $\mathbf{R}$ is a constant vector. Starting from the scattering solutions defined in the class

$$\psi_{\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}.\mathbf{r}} - \tfrac{m}{2\pi\hbar^2}\int d^3\mathbf{r}'\; G(\mathbf{r},\mathbf{r}')\; V(\mathbf{r}')\; \psi_{\mathbf{k}}(\mathbf{r}')$$

and using the fourier representation of $G(\mathbf{r})$; $G(\mathbf{r}) = \frac{1}{2\pi^2}\int \frac{1}{k'^2-k^2} e^{i\mathbf{k}.\mathbf{r}} d^3k'$

   (a) Prove that the above scattering solutions are block wave functions, since they satisfy $\psi_{\mathbf{k}}(\mathbf{r}+\mathbf{R}) = e^{i\mathbf{k}.\mathbf{R}}\psi(\mathbf{r})_{\mathbf{k}}$

   (b) Show that the scattering amplitude vanishes unless $\mathbf{q} = \mathbf{k} - \mathbf{k}'$ is a reciprocal lattice vector which satisfies the Laue condition $\mathbf{q}.\mathbf{R} = (\mathbf{k}-\mathbf{k}').\mathbf{R} = 2\pi n$, where n is an integer

2. Consider the problem of scattering by a spherical barrier that is defined by

$$V(r) = \begin{cases} V_0\ , & r < R; \\ 0\ , & r > R. \end{cases}$$

   (a) Find the scattering amplitude at an arbitrary energy in the first Born approximation

   (b) Based on the result of part *a*), find the total cross section for the low energy limit.

   (c) Use the method of phase shift analysis to find the low energy phase shift, scattering amplitude, and the total cross section. Compare your findings with part *b*). Discuss the case of the very high potential (i.e. $V_0 \to \infty$)

3. Consider the problem of scattering by the Yukawa potential that is defined by $V(r) = \frac{g}{r}e^{-\mu r}$, where g and μ are constants.

   (a) Find the scattering amplitude and the total scattering cross section for an arbitrary energy in the Born approximation

   (b) Find the total scattering cross section for the low energy limit (long wavelength limit)

   (c) Find the total scattering cross section for the high energy limit (short wavelength limit)

4. Consider the problem of scattering by the potential $V(r) = -V_0\ e^{-r/a}$, where $V_0 > 0$ and a is constant.

   (a) Find the differential scattering cross section for an arbitrary energy in the Born approximation

   (b) Find the total scattering cross section for the low energy limit (long wavelength limit)

   (c) Find the total scattering cross section for the high energy limit (short wavelength limit)

5. Consider the problem of scattering by a spherical delta function shell $V(r) = V_0\ \delta(r-a)$, where $V_0 > 0$ and a is constant.

   (a) Find the differential scattering cross section for an arbitrary energy in the Born approximation

(b) Find the total scattering cross section for the low energy limit in the Born approximation

6. Consider the problem of scattering by the potential $V(r) = Ae^{-\mu r^2}$, where A and μ are constants.

   (a) Find the scattering amplitude and the total scattering cross section for an arbitrary energy in the Born approximation

   (b) Find the total scattering cross section for the low energy limit (long wavelength limit)

   (c) Find the total scattering cross section for the high energy limit (short wavelength limit)

7. Consider the problem of scattering by the potential $V(r) = \frac{A}{r^2}$, where A is constant. Calculate the total scattering cross section for this potential. Does your result converge?. Discuss the validity of Born approximation for this long range potential.

8. Consider the problem of scattering by the Coulomb potential $V(r) = \frac{A}{r}$, where A is constant. Calculate the scattering amplitude. Does your result converge?. Discuss the validity of Born approximation for this long range potential. Explain how to overcome the problem of divergence of the scattering amplitude of this potential by taking the limit of the screened Yukawa potential as $\mu \to 0$. Calculate the differential scattering cross section. Does your result converge?. Calculate the total scattering cross section. Does your result converge?.

Good Luck