Phys 761
Quantum Mechanics
Problem Set # 2

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1. (a) Show that each component of the orbital angular momentum commutes with the kinetic energy operator

$$[\hat{l}_i, \hat{\mathbf{p}}^2/2m] = 0$$

   (b) Prove that

$$\frac{d}{dt}\langle \mathbf{L} \rangle = -\langle \mathbf{r} \times \nabla V \rangle = \langle \mathbf{r} \times \mathbf{F} \rangle$$

   as expected from correspondence with classical physics

   (c) Show that the orbital angular momentum operator $\hat{\mathbf{L}}$ commutes with any scalar function of $\hat{\mathbf{p}}^2$ and $\hat{\mathbf{r}}^2$

2. Consider the translation operator that is defined by $\hat{\mathbf{D}}\psi(\mathbf{r}) = \psi(\mathbf{r} - \boldsymbol{a})$

   (a) Show that for infinitesimal translation $\delta \boldsymbol{a}$, $\hat{\mathbf{D}} \approx 1 - \frac{i}{\hbar}\delta \boldsymbol{a}.\hat{\mathbf{p}}$

   (b) Show that for finite translation $\boldsymbol{a}$, $\hat{\mathbf{D}} \approx e^{-\frac{i}{\hbar}\boldsymbol{a}.\hat{\mathbf{p}}}$

   (c) Consider a particle described by the Hamiltonian $H = p^2/2m + V(x)$, Show that H has translational symmetry only if $V(x) = V_0$.

3. Show that

$$e^{-i\frac{\alpha}{2}(\sigma.\mathbf{n})} = \mathbf{I}cos\frac{\alpha}{2} - i(\sigma.\mathbf{n})sin(\frac{\alpha}{2})$$

   where $\mathbf{n}$ is a unit vector and $\mathbf{I}$ is the 2×2 identity matrix

4. Abeam of atoms were initially prepared to have a spin in the direction $\mathbf{n}(\theta, \phi)$. The beam is then directed into an analyzer that measures the spin along the z-axis.

   (a) What is the probability that the component of the spin along the z-axis will be measured to be $+\hbar/2$ and $-\hbar/2$

   (b) Find the expectation value of S_z

   (c) Check your answers for the case where the original beam is polarized along the z-axis (i.e $\theta = 0$)

5. Find the normalized eigenfunctions, energy eigenvalues and their degeneracy of the plane rotor. The plane rotor is a system with one degree of freedom (a polar angle ϕ) and described by the Hamiltonian

$$\hat{H} = \frac{\hbar^2 \hat{l}_z^2}{2I}$$

   where I is the moment of inertia

6. Show that $(\sigma.\boldsymbol{a})(\sigma.\boldsymbol{b}) = \sigma_0(\boldsymbol{a}.\boldsymbol{b}) + i\sigma.(\boldsymbol{a} \times \boldsymbol{b})$

Good Luck