The Hashemite University
Department of Physics
Fall Semester 2016/2017

First Exam
Graduate QM
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1. Consider a hydrogen atom with the potential $V(r) = -\frac{Ze^2}{r}$. The radial equation for an electron in the atom is given by

$$\frac{d^2u}{dr^2} + k_l^2(r)\, u = 0, \quad where\ \ k_l^2(r) = \frac{2m}{\hbar^2}\left[E - V(r) - \frac{\hbar^2 l(l+1)}{2mr^2}\right]$$, m is the reduced mass

For bound states solution, define $E = -\mathcal{E}$, with $\mathcal{E} > 0$

a) Using the change of variables technique, with $\rho = \chi r$ where $\chi^2 = 2m\mathcal{E}/\hbar^2$, show that the radial equation can be written as

$$\frac{d^2u}{d\rho^2} + \left[-1 + \frac{\alpha}{\rho} - \frac{l(l+1)}{\rho^2}\right]u = 0\,, \quad where\ \alpha = \frac{2mZe^2}{\hbar^2\chi}$$ (4 points)

b) Write down the last equation and it is general solution for the case $\rho \to 0$ (2 points)

c) Write down the last equation and it is general solution for the case $\rho \to \infty$ (2 points)

d) Write down the general solution of the radial equation that is valid everywhere (2 points)

2. The radial wave function of an electron in the Hydrogen atom is given by

$$R_{21}(r) = A\left(\frac{r}{2a_0}\right) e^{-r/2a_0}$$

a) Find the normalization constant A (3 points)

b) Calculate $\langle r^3 \rangle$ (3 points)

c) Calculate $\langle L_+ L_- \rangle$ for $l = 1, m = 1$ (2 points)

Hint: you need to express $L_+ L_-$ in terms of $L^2, L_z^2, and\ L_z$

3. A beam was prepared to be polarized in the $\vec{n}(\theta,\phi)$ direction. The beam is then directed into an analyzer that measures the spin along the x-direction. Now the general definition of an arbitrary spin state is defined by

$$\chi_{n+} = \begin{pmatrix} e^{-i\frac{\phi}{2}}\cos(\theta/2) \\ e^{+i\frac{\phi}{2}}\sin(\theta/2) \end{pmatrix} \qquad \text{and} \quad \chi_{n-} = \begin{pmatrix} -e^{-i\frac{\phi}{2}}\sin(\theta/2) \\ e^{+i\frac{\phi}{2}}\cos(\theta/2) \end{pmatrix}$$

Where χ_{n+} is the spin state that points up along $\vec{n}$, and χ_{n-} is the spin state that points down along $\vec{n}$

a) Find the probability of measuring $-\frac{\hbar}{2}$ (4 points)

b) Find $<S_x>$, where $S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ (3 points)