# Condensed Matter Physics (Phy 771)

## HW # 8 - solution

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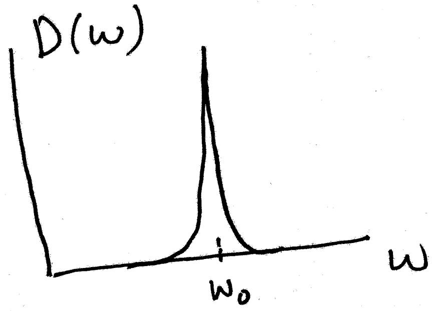
① for 1D solid of $N$ atoms, the density of states is $D(\omega) = N\,\delta(\omega-\omega_0)$. there are $N$ atoms in the 1D solid, and hence there are $N$ harmonic oscillators having the same frequency $\omega_0$. Each atom has 1 degree of freedom for vibration (one polarization state: longitudinal).




the total energy is

$$\mathcal{E} = \int_0^\infty \langle n(\omega)\rangle \hbar\omega_0\, D(\omega)\, d\omega$$

when $\langle n(\omega)\rangle = \dfrac{1}{e^{\beta\hbar\omega_0}-1}$, $\Rightarrow \beta = \dfrac{1}{k_B T}$

$$\mathcal{E} = \int_0^\infty \frac{\hbar\omega_0}{e^{\beta\hbar\omega_0}-1}\, N\,\delta(\omega-\omega_0)\, d\omega = \frac{\hbar\omega_0 N}{e^{\beta\hbar\omega_0}-1}\underbrace{\int_0^\infty d\omega\, \delta(\omega-\omega_0)}_{1}$$

$$= \frac{N\hbar\omega_0}{e^{\beta\hbar\omega_0}-1}$$, now the specif heat $C_V = \left(\dfrac{\partial \mathcal{E}}{\partial T}\right)_V$

$$C_V = N\hbar\omega_0\, e^{\frac{\hbar\omega_0}{k_B T}} \left(\frac{\hbar\omega_0}{k_B T^2}\right) \Big/ \left(e^{\frac{\hbar\omega_0}{k_B T}}-1\right)^2$$

$$C_V = N k_B \left(\frac{\hbar\omega_0}{k_B T}\right)^2 \frac{e^{\left(\frac{\hbar\omega_0}{k_B T}\right)}}{\left(e^{\frac{\hbar\omega_0}{k_B T}} - 1\right)^2} \quad ---- \quad (1)$$

- high $T$ limit ($k_B T \gg \hbar\omega_0$) : using $e^x = 1 + x$ for $x \ll 1$

$$C_V = N k_B \left(\frac{\hbar\omega_0}{k_B T}\right)^2 \frac{e^{\frac{\hbar\omega_0}{k_B T}}}{\left(1 + \frac{\hbar\omega_0}{k_B T} - 1\right)^2} = N k_B \underbrace{e^{\frac{\hbar\omega_0}{k_B T}}}_{\approx 1}$$

$= N k_B$ as expected for classical particles (ideal gas)

- low $T$ limit ($k_B T \ll \hbar\omega_0$) ; $\left(e^{\frac{\hbar\omega_0}{k_B T}} - 1\right)^2 \rightarrow \left(e^{\frac{\hbar\omega_0}{k_B T}}\right)^2$, so

$$C_V \approx N k_B \left(\frac{\hbar\omega_0}{k_B T}\right)^2 \frac{e^{\frac{\hbar\omega_0}{k_B T}}}{e^{2\frac{\hbar\omega_0}{k_B T}}} \qquad \Big\} = e^{\frac{2\hbar\omega_0}{k_B T}}$$

$= N k_B \left(\frac{\hbar\omega_0}{k_B T}\right)^2 e^{-\frac{\hbar\omega_0}{k_B T}}$. This result does not fit the observed low $T$ data which was observed to scale with $T^3$ at very low temperatures. So Einstein model works at high $T$, but fails at very low $T$. Now define $T_E = \frac{\hbar\omega_0}{k_B}$ as Einstein temperature, then we have

$$\frac{C_V}{N k_B} = \left(\frac{\hbar\omega_0}{k_B T}\right)^2 \frac{e^{\frac{\hbar\omega_0}{k_B T}}}{\left(e^{\frac{\hbar\omega_0}{k_B T}} - 1\right)^2} ;$$

$$\frac{C_v}{Nk_B} = \left(\frac{T_E}{T}\right)^2 \frac{e^{\frac{T_E}{T}}}{\left(e^{\frac{T_E}{T}}-1\right)^2}$$ ; to plot $\frac{C_v}{Nk_B}$ vs $T$, set

$T_E = 200K$ for example, where $T_E$ is a characteristic temperature that separates low and high $T$ limits.

You can take $T_E$ any other number

```
(*Define the dimensionless specific heat function*)
CvOverNkB[TE_, T_] := (TE / T) ^ 2 * Exp[TE / T] / (Exp[TE / T] - 1) ^ 2

(*Set the Einstein temperature (you can change this value)*)
TE = 200; (*Example:Einstein temperature in Kelvin*)
(*Plot the dimensionless specific heat as a function of temperature*)
Plot[CvOverNkB[TE, T], {T, 0.1, 500}, (*Temperature range from 0.1 K to 500 K*)
 PlotRange → All, AxesLabel → {"Temperature (K)", "C_V / (N k_B)"},
 PlotLabel → "Einstein Model: Specific Heat of a 1D Solid",
 LabelStyle → Directive[Black, Bold, 12], PlotStyle → Thick, GridLines → Automatic]
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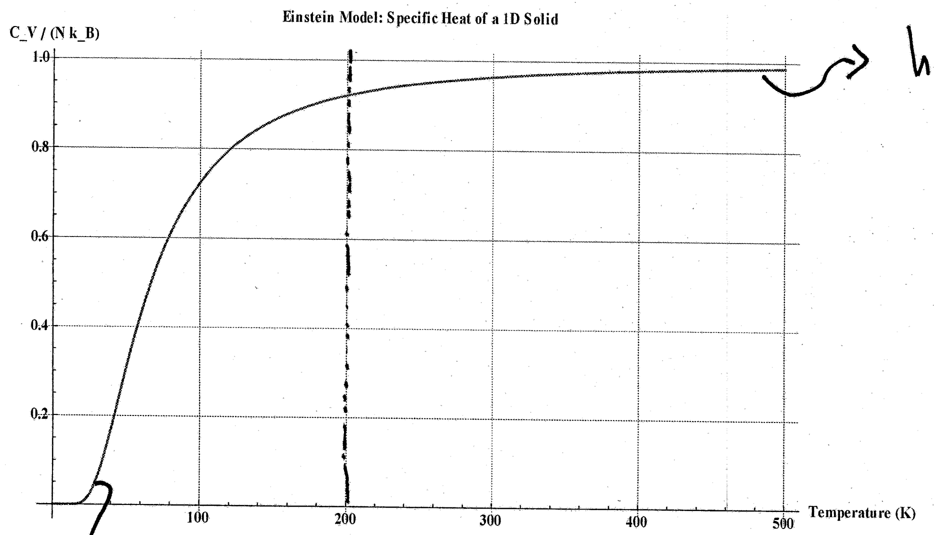
Out[11]=




high $T$ limit $\frac{C_v}{Nk_B} \approx 1$

$T_E$

low $T$ limit

$$C_v \propto \left(\frac{1}{T}\right)^2 e^{-\frac{\hbar\omega_0}{k_B T}} = \left(\frac{1}{T}\right)^2 e^{-\frac{T_E}{T}}$$

(2) Debye model in 2D:

$$D(\omega) = \sum_{\vec{k}\nu} \delta(\omega - \omega_{k\nu}) = \frac{1}{(2\pi)^d}\int d\Gamma \sum_{\nu} \delta(\omega-\omega_{k\nu}) \;;$$

where $d\Gamma = d^2q\, d^2k$ integral over phase space. Now in 2D, each phonon has two degrees of freedom (two polarization states: $\nu = 2$) $\Rightarrow$

$$D(\omega) = \frac{1}{(2\pi)^2}\cdot 2 \cdot \underbrace{\int d^2q}_{A = \text{area}} \int d^2k\, \delta(\omega - \omega_k) \;; \quad \omega_k = c\,k$$

$d\omega_k = c\, dk$ ; $c$: speed of sound

$$= \frac{2A}{4\pi^2}\int k\,dk\, d\phi\, \delta(\omega - \omega_k)$$

$$= \frac{A}{2\pi^2}\int_0^{2\pi} d\phi \int_0^{\infty} k\,dk\, \delta(\omega-\omega_k) \;; \quad \text{in polar coord } d^2k = k\,dk\,d\phi, \; 0<k<\infty, \; 0<\phi<2\pi$$

$$= \frac{2\pi A}{2\pi^2}\cdot\frac{1}{c^2}\int_0^{\infty} \omega_k\, d\omega_k\, \delta(\omega - \omega_k)$$

where I assumed that both polarization states have the same speed $c$

$$\Rightarrow D(\omega) = \frac{A}{\pi c^2}\,\omega$$

$$\Rightarrow D(\omega) = \begin{cases} \dfrac{A\omega}{\pi c^2}, & \omega < \omega_D \\ 0, & \omega > \omega_D \end{cases}$$

; where $\omega_D$ is Debye frequency

Now the total # of phonons in 2D is 2N, so $w_D$ can be found from

$$\int_0^{w_D} D(w)\,dw = 2N \Rightarrow \int_0^{w_D} \frac{Aw}{\pi c^2}\,dw = 2N\ ;$$

$$\Rightarrow \frac{A}{\pi c^2} \cdot \frac{w_D^2}{2} = 2N \Rightarrow w_D^2 = 4\left(\frac{N}{A}\right)\pi c^2 = 4n\pi c^2$$

$$\Rightarrow w_D = \left(4n\pi c^2\right)^{1/2}$$ , Now

$$\mathcal{E}(T) = \int_0^{\infty} \hbar w \langle n(w)\rangle D(w)\,dw = \int_0^{w_D} \frac{\hbar w}{e^{\frac{\hbar w}{k_B T}}-1} \frac{Aw}{\pi c^2}\,dw$$

$$= \frac{\hbar A}{\pi c^2} \int_0^{w_D} \frac{w^2\,dw}{e^{\frac{\hbar w}{k_B T}}-1}\ ;\quad \text{let } x = \frac{\hbar w}{k_B T}$$

$$\Rightarrow w = \frac{k_B T}{\hbar} x$$

$$w = 0 \Rightarrow x = 0$$

$$w = w_D \Rightarrow x = \frac{\hbar w_D}{k_B T} = \frac{T_D}{T}\ ;$$ where $T_D$ is $T_D = \frac{\hbar w_D}{k_B}$ Debye temper

$$\therefore \mathcal{E}(T) = \frac{\hbar A}{\pi c^2}\left(\frac{k_B T}{\hbar}\right)^3 \int_0^{T_D/T} \frac{x^2\,dx}{e^x-1}$$

i) high T limit ($k_B T \gg \hbar w$) ; $e^x \simeq 1+x$ for $x \ll 1$

$$\mathcal{E} = \frac{\hbar A}{\pi c^2}\left(\frac{k_B T}{\hbar}\right)^3 \int_0^{T_D/T} \frac{x^2\,dx}{(1+x-1)}\,dx$$

$$\Rightarrow \mathcal{E} = \frac{\hbar A}{\pi c^2}\left(\frac{k_B T}{\hbar}\right)^3 \int_0^{T_D/T} x\,dx = \frac{\hbar A}{\pi c^2}\left(\frac{k_B T}{\hbar}\right)^3 \cdot \frac{1}{2}\frac{T_D^2}{T^2}$$

$$= \frac{\hbar A}{\pi c^2}\frac{k_B^3 T^3}{\hbar^3}\cdot\frac{1}{2T^2}\left(\frac{\hbar^2 \omega_D^2}{k_B^2}\right) = \frac{A}{2\pi c^2} k_B T \omega_D^2$$

$$= \frac{\cancel{A}}{\cancel{2\pi c^2}}\cdot k_B T \cdot \frac{4N\cancel{\pi c^2}}{\cancel{A}} = 2Nk_B T \text{ as expected}$$

for classical ideal gas (classical particles).

(ii) low $T$ limit ($k_B T \ll \hbar\omega$) $\Rightarrow \frac{T_D}{T} \to \infty$

$$\Rightarrow \mathcal{E}(T) = \frac{\hbar A}{\pi c^2}\left(\frac{k_B T}{\hbar}\right)^3 \int_0^\infty \frac{x^2\,dx}{e^x - 1}$$

now using $\displaystyle\int_0^\infty \frac{x^{n-1}\,dx}{e^x-1} = \zeta(n)\,\Gamma(n)$ ;

$\zeta(n)$ → Rieman-Zeta function

$\Gamma(n)$ → Gamma function

$$\mathcal{E}(T) = \frac{\hbar A}{\pi c^2}\frac{k_B^3 T^3}{\hbar^3}\int_0^\infty \frac{x^{3-1}\,dx}{e^x-1} = \frac{\hbar A}{\pi c^2}\frac{k_B^3 T^3}{\hbar^3}\underbrace{\zeta(3)}_{1.2}\underbrace{\Gamma(3)}_{2!=2}$$

$$= 2.4\,\frac{A k_B^3 T^3}{\pi c^2 \hbar^2}$$ ; let us write this in terms

of Debye temperature, $T_D$.

using $T_D = \frac{\hbar \omega_D}{k_B} \Rightarrow T_D^2 = \frac{\hbar^2 \omega_D^2}{k_B^2} = \frac{\hbar^2}{k_B^2} \cdot \frac{4N\pi c^2}{A}$

$$\Rightarrow \pi c^2 \hbar^2 = \frac{T_D^2 \cdot A \cdot k_B^2}{4N}$$

$$\Rightarrow \mathcal{E} = 2.4\, A k_B^3 T^3 \cdot \frac{4N}{A \cdot k_B^2 \cdot T_D^2} = 4.8\,(2Nk_B)\frac{T^3}{T_D^2}$$

Now $C_V = \left(\frac{\partial \mathcal{E}}{\partial T}\right)_V = \begin{cases} 2Nk_B, & \text{high } T \\ 14.4\,(2Nk_B)\frac{T^2}{T_D^2}, & \text{low } T \end{cases}$

$$\frac{C_V}{2Nk_B} = \begin{cases} 1, & \text{high } T \\ 14.4\,\frac{T^2}{T_D^2}, & \text{low } T \end{cases}$$

to find exact expression for $\frac{C_V}{2Nk_B}$ Vs $T$, we start from

$$\mathcal{E} = \frac{\hbar A}{\pi c^2} \int_0^{\omega_D} \frac{\omega^2\, d\omega}{e^{\frac{\hbar\omega}{k_B T}} - 1},$$

$$C_V = \left(\frac{\partial \mathcal{E}}{\partial T}\right)_V = \frac{\hbar A}{\pi c^2} \int_0^{\omega_D} \frac{-\omega^2\, d\omega \left(e^{\frac{\hbar\omega}{k_B T}} \cdot \left(-\frac{\hbar\omega}{k_B T^2}\right)\right)}{\left(e^{\frac{\hbar\omega}{k_B T}} - 1\right)^2}$$

let $x = \frac{\hbar \omega}{k_B T} \Rightarrow \omega = \frac{k_B T}{\hbar} x \Rightarrow$

when $\omega = 0 \Rightarrow x = 0$

$\omega = \omega_D \Rightarrow x = \frac{\hbar \omega_D}{k_B T} = \frac{T_D}{T}$ ; $T_D = \frac{\hbar \omega_D}{k_B}$

$\downarrow$

Debye Temperature

$$\Rightarrow C_V = \frac{\hbar^2 A}{\pi c^2} \frac{1}{k_B T^2} \left(\frac{k_B T}{\hbar}\right)^4 \int_0^{T_D/T} \frac{x^3 e^x}{(e^x - 1)^2} dx$$

$$= \frac{A k_B^3 T^2}{\pi c^2 \hbar^2} \int_0^{T_D/T} \frac{x^3 e^x}{(e^x-1)^2} dx \ ; \text{ using } \pi c^2 \hbar^2 = \frac{T_D^2 . A . k_B^2}{4N}$$

$$= \cancel{A} k_B^{\cancel{3}} T^2 . \frac{4N}{T_D^2 . \cancel{A} . \cancel{k_B^2}} \int_0^{T_D/T} \frac{x^3 e^x}{(e^x-1)^2} dx =$$

$$= 4N k_B \frac{T^2}{T_D^2} \int_0^{T_D/T} \frac{x^3 \ e^x}{(e^x-1)^2} dx$$

$$\Rightarrow \frac{C_V}{2N k_B} = 2 \left(\frac{T}{T_D}\right)^2 \int_0^{T_D/T} \frac{x^3 e^x}{(e^x - 1)^2} dx$$

plot this relation, with $T_D = 200K$ for example. you can change it to any number

# Plotting $C_V$ for 2D Debye model

```
In[77]:= (*Define the dimensionless specific heat function for the Debye model in 2D*)
CvOver2NkB[TD_, T_] := 2 * (T / TD) ^2 * NIntegrate[(x^3 * Exp[x]) / (Exp[x] - 1) ^2, {x, 0, TD / T}]

(*Set the Debye temperature (adjust as needed)*)
TD = 200; (*Example:Debye temperature in Kelvin*)
(*Plot the specific heat for a wider temperature range to see the high-T limit*)
Plot[CvOver2NkB[TD, T], {T, 0.1, 500}, (*Extended temperature range to T>>Θ_D*)
 PlotRange → {0, 1.1}, AxesLabel → {"Temperature (K)", "C_V / (2N k_B)"},
 PlotLabel → "Debye Model: Specific Heat of a 2D Solid",
 LabelStyle → Directive[Black, Bold, 12], PlotStyle → Thick, GridLines → Automatic]

Out[78]=
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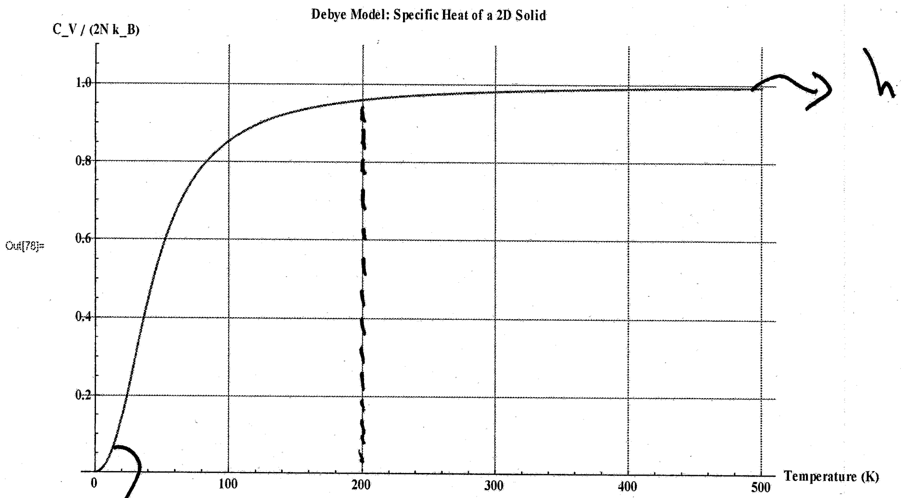


high T limit

$\frac{C_V}{2Nk_B} \approx 1$

$T_D$

low T limit

$\frac{C_V}{2Nk_B} \propto T^2$

③ 1D solid of N atoms

$$D(\omega) = \sum_{\vec{k}\nu} \delta(\omega - \omega_{k\nu}) = \frac{1}{(2\pi)^d} \int d\Gamma \sum_{\nu} \delta(\omega - \omega_{k\nu})$$

in 1D $d=1$ and $\nu = 1$ (one state of polarization)
and $d\Gamma = dq\, dk$

$$\Rightarrow D(\omega) = \frac{1}{2\pi} \int dq\, dk\, \delta(\omega - \omega_k) = \frac{1}{2\pi} \underbrace{\int dq}_{L} \int dk\, \delta(\omega - \omega_k)$$

$$= \frac{L}{2\pi} \int \frac{1}{c} d\omega_k\, \delta(\omega - \omega_k) \quad ; \text{ when } \begin{array}{l} \omega_k = ck \\ d\omega_k = c\, dk \end{array}$$

in 1D, we have to multiply by factor 2 to account for degeneracy as $\omega(k) = \omega(-k)$; i.e. $k$ can take positive and negative values

$$\Rightarrow D(\omega) = \frac{2L}{2\pi c} \underbrace{\int_0^\infty d\omega_k\, \delta(\omega - \omega_k)}_{1} = \frac{L}{\pi c}$$

in 1D, the # of phonons is fixed to N

$$\Rightarrow \int_0^{\omega_D} D(\omega)\, d\omega = N \Rightarrow \frac{L}{\pi c} \omega_D = N \Rightarrow \omega_D = \frac{N}{L} \pi c$$

$$= n \pi c,$$

where $n = \frac{N}{L}$ phonon density

$$\Rightarrow D(\omega) = \begin{cases} \frac{L}{\pi c}, & \omega < \omega_D \\ 0, & \omega > \omega_D \end{cases}$$

the total energy is

$$\mathcal{E}(T) = \int_0^\infty \hbar\omega \langle n(\omega)\rangle D(\omega)\, d\omega = \frac{\hbar L}{\pi c}\int_0^{\omega_D} \frac{\omega\, d\omega}{e^{\frac{\hbar\omega}{k_B T}}-1}$$

let $x = \frac{\hbar\omega}{k_B T}$ $\Rightarrow$ $\omega = 0 \Rightarrow x = 0$

$\omega = \omega_D \Rightarrow x = \frac{\hbar\omega_D}{k_B T} = \frac{T_D}{T}$ ; $T_D = \frac{\hbar\omega_D}{k_B}$

$$\Rightarrow \mathcal{E} = \frac{\hbar L}{\pi c}\left(\frac{k_B T}{\hbar}\right)^2 \int_0^{T_D/T} \frac{x\, dx}{e^x - 1}$$

- high $T$ limit ($k_B T \gg \hbar\omega$) ; $e^x \approx 1 + x$ , $x \ll 1$

$$\Rightarrow \mathcal{E} = \frac{L k_B^2}{\pi c \hbar} T^2 \int_0^{T_D/T} \frac{x\, dx}{1 + x - 1} = \frac{L k_B^2}{\pi c \hbar} T^2 \left(\frac{T_D}{T}\right) = \frac{L k_B^2}{\pi c \hbar} T_D T$$

but $\omega_D = \frac{N}{L}\pi c \Rightarrow \pi c = \frac{L}{N}\omega_D \Rightarrow \pi c \hbar = \frac{L}{N}\hbar\omega_D = \frac{L}{N} k_B T_D$

where $\hbar\omega_D = k_B T_D$

$$\Rightarrow \mathcal{E} = L k_B^2 \cdot \frac{N}{L k_B T_D} T_D T = N k_B T$$

- low $T$ limit ($k_B T \ll \hbar\omega$) ; $\frac{T_D}{T} \to \infty$

$$\mathcal{E} = \frac{\hbar L}{\pi c}\left(\frac{k_B T}{\hbar}\right)^2 \int_0^\infty \frac{x\, dx}{e^x - 1} = \frac{\hbar L}{\pi c}\left(\frac{k_B T}{\hbar}\right)^2 \underbrace{\int_0^\infty \frac{x^{2-1}\, dx}{e^x - 1}}_{\underbrace{\zeta(2)}_{\frac{\pi^2}{6}}\ \underbrace{\Gamma(2)}_{1}}$$

$$= \frac{L k_B^2}{\pi c \hbar} T^2 \cdot \frac{\pi^2}{6} = \frac{\pi^2}{6} \cdot L k_B^2 \cdot T^2 \cdot \frac{N}{L k_B T_D}$$

$$= \frac{\pi^2}{6} \frac{N k_B}{T_D} T^2$$

$$\Rightarrow C_V = \left(\frac{\partial \mathcal{E}}{\partial T}\right)_V = \begin{cases} N k_B \,; & \text{high } T \\ \frac{\pi^2}{3} \frac{N k_B}{T_D} T ; & \text{low } T \end{cases}$$

$$\Rightarrow \frac{C_V}{N k_B} = \begin{cases} 1, & \text{high } T \\ \frac{\pi^2}{3 T_D} T, & \text{low } T \end{cases}$$

to find exact expression for $C_V/N k_B$, we start from $\mathcal{E} = \frac{\hbar L}{\pi c} \int_0^{\omega_D} \frac{\omega \, d\omega}{e^{\frac{\hbar\omega}{k_B T}} - 1}$ , $C_V = \left(\frac{\partial \mathcal{E}}{\partial T}\right)_V$

$$\Rightarrow C_V = \frac{\hbar L}{\pi c} \frac{\partial}{\partial T} \int_0^{\omega_D} \frac{\omega \, d\omega}{e^{\frac{\hbar\omega}{k_B T}} - 1} = \frac{\hbar L}{\pi c} \frac{\hbar}{k_B T^2} \int_0^{\omega_D} \frac{\omega^2 e^{\hbar\omega/k_B T} d\omega}{\left(e^{\hbar\omega/k_B T} - 1\right)^2}$$

let $x = \hbar\omega/k_B T \Rightarrow \omega = 0 \Rightarrow x = 0$

$\omega = \omega_D \Rightarrow x = \frac{\hbar \omega_D}{k_B T} = \frac{T_D}{T}$ ; $T_D = \frac{\hbar \omega_D}{k_B}$

$$\Rightarrow C_V = \frac{\hbar^2 L}{\pi c k_B T^2} \left(\frac{k_B T}{\hbar}\right)^3 \int_0^{T_D/T} \frac{x^2 e^x \, dx}{(e^x - 1)^2}$$

$$= \frac{L k_B^2 T}{\pi c \hbar} \int_0^{T_D/T} \frac{x^2 e^x \, dx}{(e^x - 1)^2} = L k_B^2 T \cdot \frac{N}{L k_B T_D} \int_0^{T_D/T} \frac{x^2 e^x \, dx}{(e^x - 1)^2}$$

$$= N k_B \frac{T}{T_D} \int_0^{T_D/T} \frac{x^2 e^x \, dx}{(e^x - 1)^2}$$

$$\Rightarrow \frac{C_V}{Nk_B} = \frac{T}{T_D}\int_0^{T_D/T} \frac{x^2 e^x \, dx}{(e^x-1)^2}, \text{ let us plot } C_V$$

Take $T_D = 200\,K$, for example,

# Plot of $C_V/Nk_B$ (Debye model for 1D solid)

In[28]:=

```
(*Define the dimensionless specific heat function for the Debye model*)
CvOverNkB[TD_, T_] := (T / TD) * NIntegrate[(x^2 * Exp[x]) / (Exp[x] - 1)^2, {x, 0, TD / T}]

(*Set the Debye temperature (you can change this value)*)
TD = 200; (*Example:Debye temperature in Kelvin*)
(*Plot the dimensionless specific heat as a function of temperature*)
Plot[CvOverNkB[TD, T], {T, 0.1, 500}, (*Temperature range from 0.1 K to 500 K*)
 PlotRange → All, AxesLabel → {"Temperature (K)", "C_V / (N k_B)"},
 PlotLabel → "Debye Model: Specific Heat of a 1D Solid",
 LabelStyle → Directive[Black, Bold, 12], PlotStyle → Thick, GridLines → Automatic]
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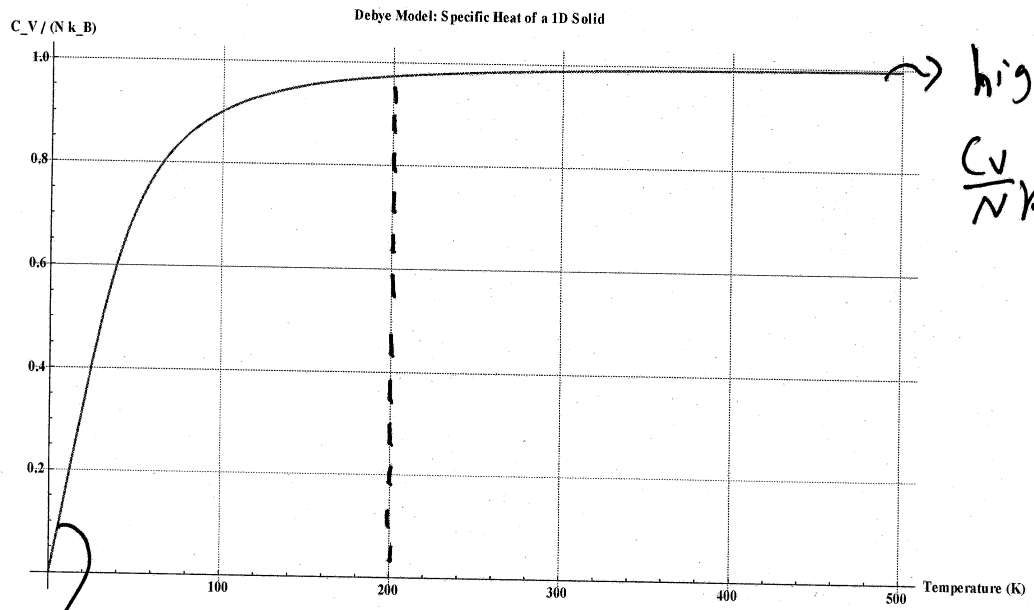
Out[29]=




high T limit $\frac{C_V}{Nk_B} \approx 1$

$T_D$

low T limit $C_V \propto T$

(4) we already found that for metals,

$$\frac{C_V}{T} = \frac{\pi^2}{2}\frac{Nk_B}{T_F} + \frac{234Nk_B}{T_D^3}T^2 \quad , \text{ for solids } C_V \approx C_P$$

$$\Rightarrow \frac{C_P}{T} = A + BT^2 \; ; \text{ where } A = \frac{\pi^2}{2}\frac{Nk_B}{T_F} \text{ and } B = \frac{234Nk_B}{T_D^3}$$

now $Nk_B = nR$ ; where $n=1$ and $R = 8.314 \frac{J}{mol.K}$ .

in the next figure, we use the high $T$ data to estimate $T_D$ and low $T$ data to estimate $T_F$

$\underline{\underline{T_D:}}$ $\text{slope} \approx B \approx 0.2\times10^{-3}\frac{J}{mol.K} = \frac{234\times R}{T_D^3}$

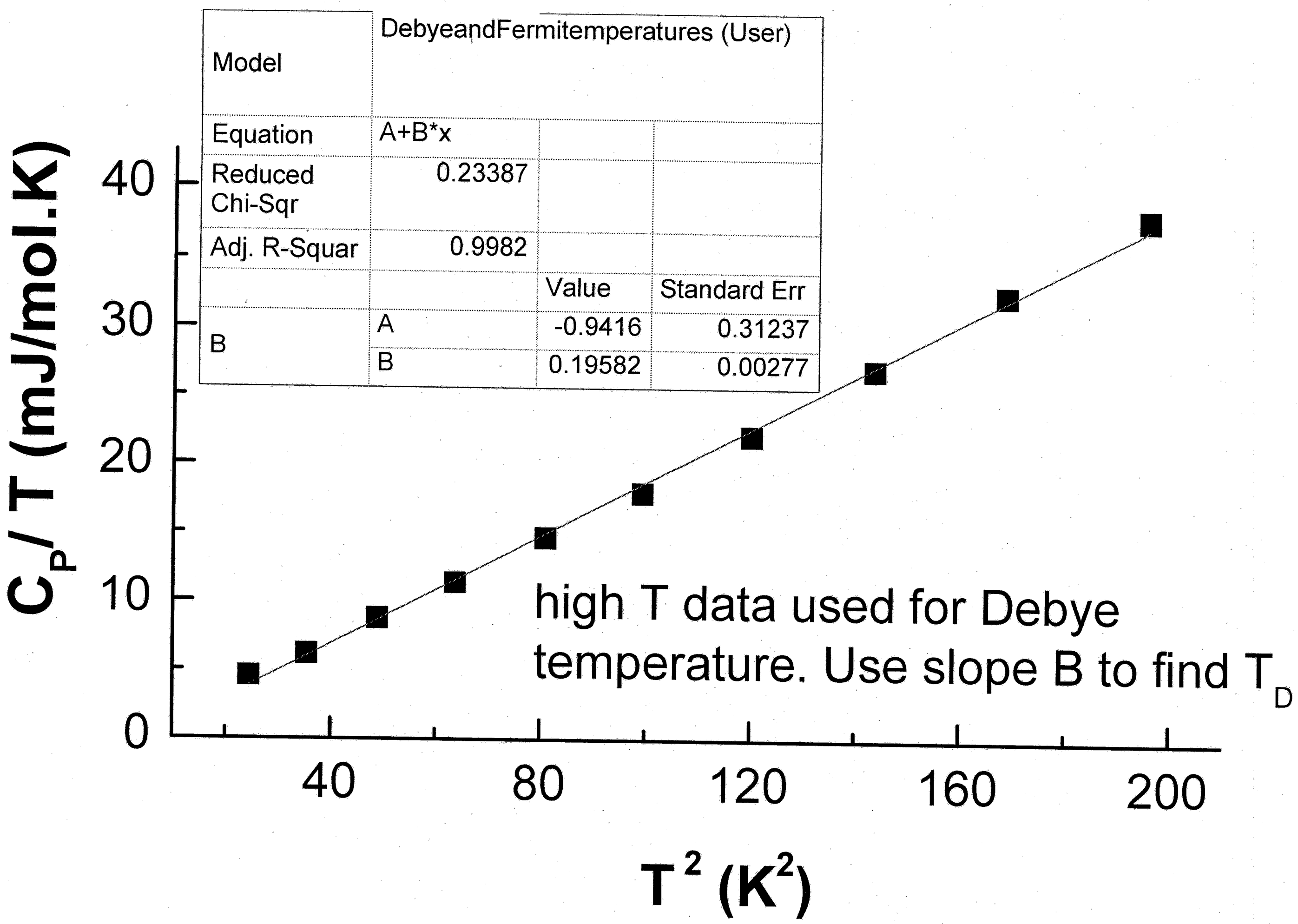
$$\Rightarrow T_D^3 = \frac{234\times 8.314}{0.2\times10^{-3}} \quad \Rightarrow T_D \approx 213\,K$$ which is

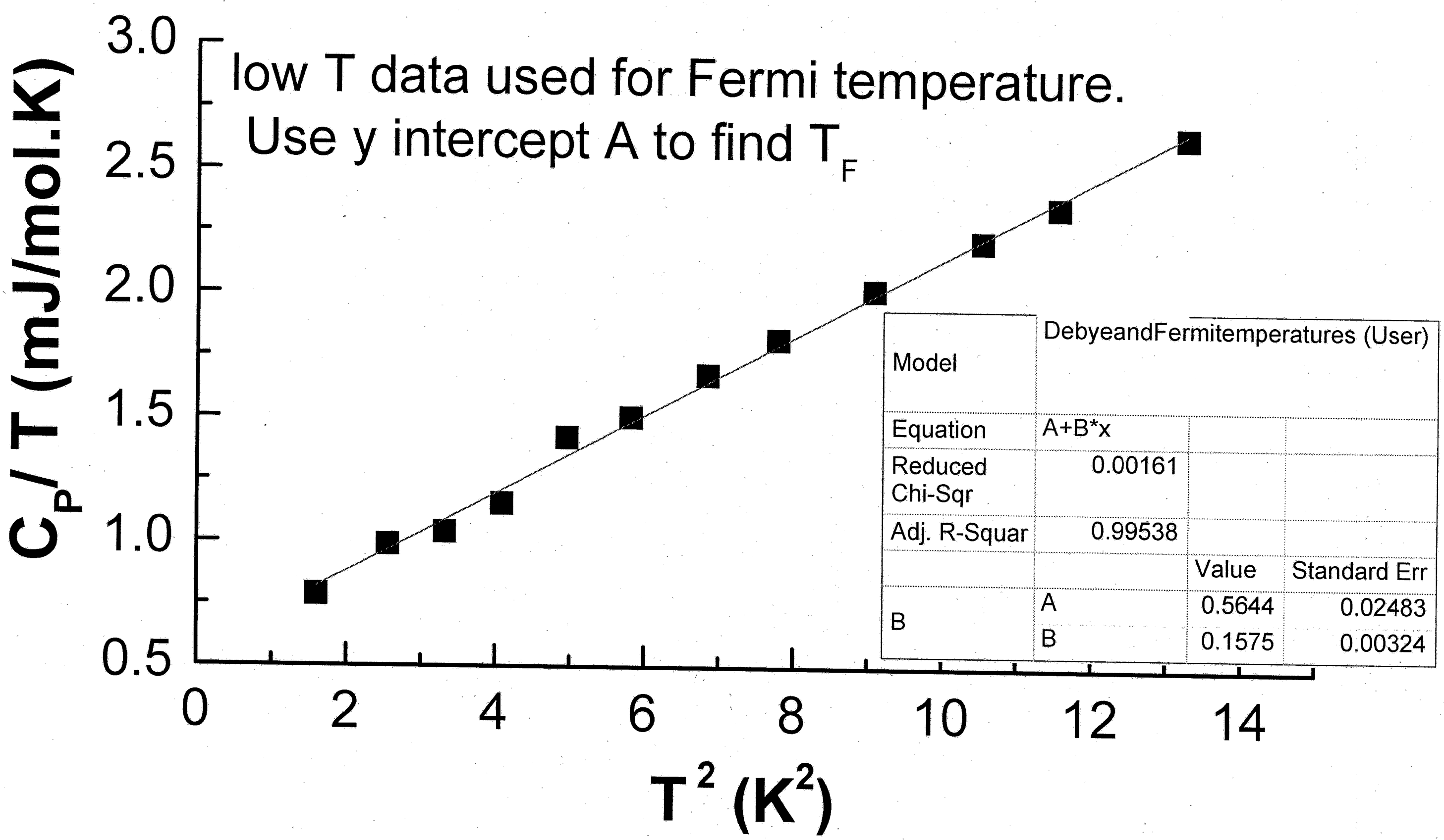
consistent with $T_D = 227\,K$ found in table 13.1 for Ag

$\underline{\underline{T_F:}}$ $y\text{-intercept} = A = 0.564\times10^{-3} = \frac{\pi^2}{2}\frac{R}{T_F}$

$$\Rightarrow T_F = \frac{\pi^2 R}{2\times0.564\times10^{-3}} = \frac{\pi^2\times 8.314}{2\times0.564\times10^{-3}} \approx 73{,}000\,K$$

which is consistent with $T_F \approx 64{,}000\,K$ listed in table 6.1 for Ag


Model
DebyeandFermitemperatures (User)
Equation
A+B*x
Reduced Chi-Sqr
0.23387
Adj. R-Squar
0.9982
Value
Standard Err
B
A
-0.9416
0.31237
B
0.19582
0.00277
high T data used for Debye temperature. Use slope B to find $T_D$
$C_P/T$ (mJ/mol.K)
$T^2$ ($K^2$)
0
10
20
30
40
40
80
120
160
200


low T data used for Fermi temperature. Use y intercept A to find $T_F$
Model
DebyeandFermitemperatures (User)
Equation
A+B*x
Reduced Chi-Sqr
0.00161
Adj. R-Squar
0.99538
Value
Standard Err
B
A
0.5644
0.02483
B
0.1575
0.00324
$C_P/T$ (mJ/mol.K)
$T^2$ ($K^2$)
0.5
1.0
1.5
2.0
2.5
3.0
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2
4
6
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12
14