Phys 771
Condensed Matter Physics
Problem Set # 5

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1. Marder 7.2
2. Show that for real potential $U(\mathbf{r})$, $U^*_{\mathbf{K}} = U_{-\mathbf{K}}$
3. show that $\int_\Omega e^{i\mathbf{K}.\mathbf{r}}\, d^3\mathbf{r} \ = 0$ for any non-vanishing reciprocal lattice vector $\mathbf{K}$. The integral is carried out over a unit cell (Ω)
4. Consider the 1D periodic potential $U(x) = 2\ V_o\ cos^2(\frac{\pi x}{a})$, where V_o is real constant
   (a) Find all non-zero Fourier components of $U(x)$
   (b) Find the band gap at $k = \pi/a$ and at $k = 5\pi/a$
   (c) Find the eigenstates at $k = \pi/a$
5. Consider the 1D periodic potential $U(x) = 2\ V_o\ cos(\frac{2\pi x}{a})$, where V_o is constant and a is the lattice spacing. The energy dispersion relation, $E(k)$, in the first Brillouin zone $[-\pi/a, \pi/a]$ is given by
   $$E(k) = \frac{\hbar^2}{2m}\left[(k-\pi/a)^2 + \frac{\pi^2}{a^2}\right] \pm \sqrt{\left[\frac{\pi\hbar^2}{ma}(k-\pi/a)\right]^2 + V_o^2}$$
   Show that $E(k)$ is singular at $k = \pi/a$ and show that the density of states near $k = \pi/a$ exhibits Van Hove singularities and behaves like $D(E) \sim \frac{1}{\sqrt{|E-E_{edge}|}}$, where $E_{edge} = \frac{\hbar^2\pi^2}{2ma^2} \pm V_o$
6. Consider a constant 2D energy surface in k-space defined by $E(k_x, k_y) = \frac{\hbar^2}{2m}(k_x^2 + k_y^2) = C$, where C is a constant.
   (a) Find a tangent vector to the energy surface
   (b) Find $\nabla_{\mathbf{k}} E$
   (c) Show that $\nabla_{\mathbf{k}} E$ is normal to the energy surface
   (d) Identify the critical points of the energy dispersion $E(k_x, k_y)$ and classify them as either minima (E_{min}), maxima (E_{max}), or saddle points (mixed curvature). Are there Van Hove Singularities among these points
7. Consider a constant 2D energy surface in k-space defined by $E(k_x, k_y) = E_o - \frac{\hbar^2}{2m}(k_x^2 - k_y^2)$. Identify the critical points of the energy dispersion $E(k_x, k_y)$ and classify them as either minima (E_{min}), maxima (E_{max}), or saddle points (mixed curvature). Are there Van Hove singularities at these points? If so, write down the integral corresponding to the density of states and comment on how it behaves at the saddle point.
8. Marder 7.5 (1D Kronig-Penney model)