Condensed Matter Physics (Phy 771)

HW #4 – solution

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① problem 6.1 Marder

The free Fermi gas is described by $H\Psi = \mathcal{E}\Psi$ .... (1)

where $\Psi = \Psi(\vec{r_1}, \vec{r_2}, ..., \vec{r_N})$ is the total wave function of all fermions and

$$H = H_1 + H_2 + \cdots + H_i + \cdots H_N = \sum_{i=1}^{N} H_i = \sum_i \frac{P_i^2}{2m} = \sum_{i=1}^{N} \frac{-\hbar^2 \nabla_i^2}{2m}$$

where $P_i = -i\hbar\nabla_i$

$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2 + \cdots + \mathcal{E}_i + \cdots \mathcal{E}_N = \sum_i^N \mathcal{E}_i$$

a) let $\Psi(\vec{r_1}, \vec{r_2}, ..., \vec{r_N}) = \prod_{j=1}^{N} \psi_j(\vec{r_j}) = \psi_i(\vec{r_i}) \prod_{j\neq i}^{N} \psi_j(\vec{r_j})$

now the L.H.S of eq^n (1) reads

$$H\Psi = -\frac{\hbar^2}{2m}\sum_{i=1}^{N} \nabla_i^2 \Psi = -\frac{\hbar^2}{2m}\sum_{i=1}^{N} \nabla_i^2 \psi_i(\vec{r_i}) \prod_{\substack{j=1 \\ j\neq i}}^{N} \psi_j(\vec{r_j})$$

$$= -\frac{\hbar^2}{2m}\sum_{i=1}^{N} \prod_{\substack{j=1 \\ j\neq i}}^{N} \psi_j(\vec{r_j})\, \nabla_i^2 \psi_i(\vec{r_i}) \;;\; -\frac{\hbar^2}{2m}\nabla_i^2\psi_i = \mathcal{E}_i\psi_i$$

$$= \sum_{i=1}^{N} \prod_{\substack{j=1 \\ j\neq i}}^{N} \psi_j(\vec{r_j})\, \mathcal{E}_i \psi_i(\vec{r_i})$$

$$= \sum_{i=1}^{N} \mathcal{E}_i\, \psi_i(\vec{r_i}) \prod_{\substack{j=1 \\ j\neq i}}^{N} \psi_j(\vec{r_j}) = \sum_{i=1}^{N}\mathcal{E}_i \prod_{j=1}^{N}\psi_j(\vec{r_j}) = \mathcal{E}\Psi = \text{R.H.S}$$

b) similar to part a),

let $\psi = \frac{1}{N!} \sum_s (-1)^s \prod_{j=1}^{N} \psi_{s_j}(\vec{r_j}) = \frac{1}{N!} \sum_s (-1)^s \psi_{s_i}(\vec{r_i}) \prod_{\substack{j=1 \\ j \neq i}}^{N} \psi_{s_j}(\vec{r_j})$

so the L.H.S of eqn(1) reads

$$H\psi = -\frac{\hbar^2}{2m} \sum_{i=1}^{N} \nabla_i^2 \psi = -\frac{\hbar^2}{2m} \sum_{i=1}^{N} \nabla_i^2 \frac{1}{N!} \sum_s (-1)^s \psi_{s_i}(\vec{r_i}) \prod_{\substack{j=1 \\ j \neq i}}^{N} \psi_{s_j}(\vec{r_j})$$

$$= \frac{1}{N!} \sum_i \sum_s (-1)^s \prod_{\substack{j=1 \\ j \neq i}}^{N} \psi_{s_j}(\vec{r_j}) \underbrace{\left[-\frac{\hbar^2 \nabla_i^2}{2m} \psi_{s_i}(\vec{r_i}) \right]}_{\varepsilon_i \psi_{s_i}(\vec{r_i})}$$

$$= \frac{1}{N!} \sum_i \sum_s (-1)^s \prod_{\substack{j=1 \\ j \neq i}}^{N} \psi_{s_j}(\vec{r_j}) \, \varepsilon_i \, \psi_{s_i}(\vec{r_i})$$

$$= \sum_i \varepsilon_i \frac{1}{N!} \sum_s (-1)^s \psi_{s_i}(\vec{r_i}) \prod_{\substack{j=1 \\ j \neq i}}^{N} \psi_{s_j}(\vec{r_j})$$

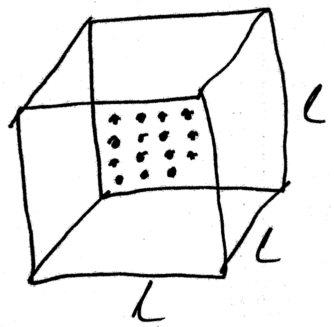
$$= \sum_i \varepsilon_i \frac{1}{N!} \sum_s (-1)^s \prod_{j=1}^{N} \psi_{s_j}(\vec{r_j})$$

$$= \sum_i \varepsilon_i \psi = \varepsilon \psi = \text{R.H.S} \checkmark$$

② problem 6.2 Marder :

Consider 15 electrons confined in a cube of side length l

Here N is small number, Hence we can not convert sum to integral, instead we will distribute them manually according to



$$\varepsilon_k = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2}{2m}(k_x^2 + k_y^2 + k_z^2) \; ; \text{ where}$$

$$\vec{k} = (k_x, k_y, k_z) = \frac{2\pi}{l}(n_x, n_y, n_z) \; ;$$

$$= \frac{\hbar^2}{2m}\frac{4\pi^2}{l^2}(n_x^2 + n_y^2 + n_z^2)$$

$n_x, n_y, n_z = 0, \pm 1, \pm 2, \cdots$

- the lowest energy level $\varepsilon_0 = 0$ corresponds to $(n_x, n_y, n_z) = (0,0,0)$. This state can accept 2 electrons $(\uparrow, \downarrow)$ with opposite spins.
- the next energy level $\varepsilon_1 = \frac{2\hbar^2\pi^2}{ml^2}$ corresponds to the combinations $(1,0,0), (0,1,0), (0,0,1), (-1,0,0), (0,-1,0), (0,0,-1)$. This level is 6-fold degenerate and can accept 12 electrons.
- The highest occupied single particle state (level) is $\varepsilon_2 = \frac{4\hbar^2\pi^2}{ml^2} = 2\varepsilon_1$, which corresponds to $(n_x, n_y, n_z) = (1,1,0)$, $(1,0,1), (0,1,1), (-1,1,0), (-1,0,1), (0,-1,1), (1,-1,0), (1,0,-1), (0,1,-1)$, $(-1,-1,0), (-1,0,-1), (0,-1,-1)$. This level is 12-fold degenerate and can accept 24 electrons. but here we have 1 electron left to this level $\Rightarrow$ the ground state energy is

ε_2 ——1e——

ε_1 ——12e——

ε_0 ——2e——

$$\varepsilon_{\text{ground state}} = 2\varepsilon_0 + 12\varepsilon_1 + 1\varepsilon_2$$

$$= 0 + 12\left(\frac{2\hbar^2\pi^2}{ml^2}\right) + \frac{4\hbar^2\pi^2}{ml^2}$$

$$= 28\,\hbar^2\pi^2/ml^2$$

③ problem 6.3 Marder:

$$\Omega = -k_B T \int_0^\infty d\varepsilon \, D(\varepsilon) \ln(1+e^{\beta(\mu-\varepsilon)})$$

in the zero temperature limit ($T \to 0$), $\beta \gg 1$, so $e^{\beta(\mu-\varepsilon)} \gg 1$

$\Rightarrow \ln(1+e^{\beta(\mu-\varepsilon)}) \approx \ln e^{\beta(\mu-\varepsilon)} = \beta(\mu-\varepsilon) \ln e = \beta(\mu-\varepsilon)$. and

furthermore, in the zero temperature limit, $\mu \approx \varepsilon_F$ and

$$D(\varepsilon) = \begin{cases} K V \varepsilon^{1/2}, & \varepsilon < \varepsilon_F \\ 0, & \varepsilon > \varepsilon_F \end{cases}$$, so the integral becomes

$$\Omega = -k_B T \int_0^{\varepsilon_F} d\varepsilon \, K V \varepsilon^{1/2} \beta(\varepsilon_F - \varepsilon) = -KV \int_0^{\varepsilon_F} d\varepsilon \, \varepsilon^{1/2} (\varepsilon_F - \varepsilon)$$

$$= -KV \left[\varepsilon_F \int_0^{\varepsilon_F} \varepsilon^{1/2} d\varepsilon - \int_0^{\varepsilon_F} \varepsilon^{3/2} d\varepsilon \right] = -KV \left[\frac{2}{3} \varepsilon_F^{5/2} - \frac{2}{5} \varepsilon_F^{5/2} \right]$$

$$= -\frac{4}{15} K V \varepsilon_F^{5/2}$$

Now $P = -\left(\frac{\partial \Omega}{\partial V}\right)_{T,\mu} = \frac{4}{15} K \varepsilon_F^{5/2}$

$= \frac{2}{5} \underbrace{\left(\frac{2}{3} K \varepsilon_F^{3/2}\right)}_{n} \varepsilon_F$; but $N = \frac{2}{3} K V \varepsilon_F^{3/2}$

$\Rightarrow \frac{N}{V} = n = \frac{2}{3} K \varepsilon_F^{3/2}$

$= \frac{2}{5} n \varepsilon_F$; for copper and using table 6.1
$n = 8.49 \times 10^{28} / m^3$, $\varepsilon_F = 7.04 \, eV$

$\Rightarrow P_{Cu} = \frac{2}{5}(8.49 \times 10^{28})(7.04 \times 1.6 \times 10^{-19} J) = 3.8 \times 10^{10} \, J/m^3 = 3.8 \times 10^{10} \, Pa$

$= \frac{3.8 \times 10^{10} \, Pa}{1.013 \times 10^5 \, Pa/atm} = 3.7 \times 10^5$ atm !! huge pressure for electrons gas in Cu at $T = 300K$

④ problem 6.4 Marder (modified)

a) Electron gas in 2D ($d=2$)

$$D(\varepsilon) = \frac{2}{(2\pi)^2} \int d^2q\, d^2k\, \delta(\varepsilon - \varepsilon_p) \; ; \qquad \varepsilon_p = \frac{\hbar^2 k^2}{2m}$$

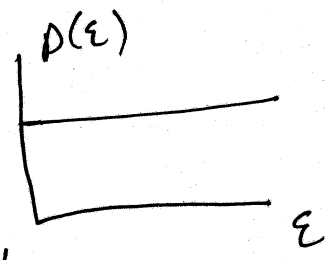
$$= \frac{2}{(2\pi)^2} \underbrace{\int d^2q}_{A} \int k\,dk\,d\phi\, \delta(\varepsilon-\varepsilon_p) \; ; \qquad 0 < k < k_F, \quad 0 < \phi < 2\pi$$

; where $d^2k = k\,dk\,d\phi$ in polar coordinates

$$= \frac{4\pi A}{4\pi^2} \int_0^\infty dk\, k\, \delta(\varepsilon - \varepsilon_p) \; ; \quad \text{and } k^2 = \frac{2m}{\hbar^2}\varepsilon_p, \quad k\,dk = \frac{m}{\hbar^2} d\varepsilon_p$$

$$= \frac{Am}{\pi\hbar^2} \underbrace{\int_0^\infty d\varepsilon_p\, \delta(\varepsilon - \varepsilon_p)}_{1} = \frac{Am}{\pi\hbar^2} = \kappa A;$$

$= \text{constant}$ where $\kappa = A/\pi\hbar^2$



- now from $N = \int_0^{\varepsilon_F} d\varepsilon\, D(\varepsilon) f(\varepsilon) \; ; \; f(\varepsilon) = 1$

$$\Rightarrow N = \kappa A \int_0^{\varepsilon_F} d\varepsilon = \kappa A \varepsilon_F \Rightarrow \varepsilon_F = \frac{1}{\kappa}\underbrace{\left(\frac{N}{A}\right)}_{n} = \frac{\pi\hbar^2 n}{m} \checkmark$$

and
$$E = \int_0^{\varepsilon_F} d\varepsilon\, \varepsilon\, D(\varepsilon) f(\varepsilon) = \kappa A \int_0^{\varepsilon_F} \varepsilon\, d\varepsilon = \kappa A \frac{\varepsilon_F^2}{2} = \frac{Am}{\pi\hbar^2}\frac{\varepsilon_F^2}{2}$$

$$= \frac{Am}{2\pi\hbar^2}\varepsilon_F \varepsilon_F = \frac{Am}{2\pi\hbar^2}\cdot \frac{n\pi\hbar^2}{m}\varepsilon_F = \frac{1}{2} A n \varepsilon_F = \frac{1}{2} A \frac{N}{A}\varepsilon_F$$

$$= \frac{1}{2} N \varepsilon_F \Rightarrow \boxed{\frac{E}{N} = \frac{1}{2}\varepsilon_F} \checkmark$$

- equation of state reads $PA = \frac{2}{3}E \Rightarrow P = \frac{2}{3}\frac{E}{A}$

$$\Rightarrow P = \frac{2}{3}\cdot \frac{\frac{1}{2} A n \varepsilon_F}{A} = \frac{1}{3} n \varepsilon_F$$

b) Electron gas in 1 D ($d=1$)

$$D(\varepsilon) = \frac{2}{(2\pi)} \cdot 2 \cdot \int dq \, dk \, \delta(\varepsilon - \varepsilon_p) \; ; \quad \varepsilon_p = \frac{\hbar^2 k^2}{2m} \text{ as before}$$

from spin

$$\Rightarrow k^2 = \frac{2m}{\hbar^2} \varepsilon_p$$

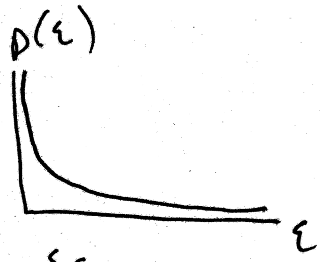
This factor comes from the fact that in 1 D, k can take both positive and negative values (two-fold degenerate) in comparision with 2 D and 3 D cases, where k is always positive, so

$$k = \left(\frac{2m}{\hbar^2}\right)^{1/2} \varepsilon_p^{1/2}$$

$$dk = \frac{(2m)^{1/2}}{\hbar} \cdot \frac{1}{2} \varepsilon_p^{-1/2} d\varepsilon_p$$

$$\Rightarrow D(\varepsilon) = \frac{2}{\pi} \underbrace{\int dL}_{L} \int_0^\infty dk \, \delta(\varepsilon - \varepsilon_p) = \frac{2L}{\pi} \frac{(2m)^{1/2}}{2\hbar} \underbrace{\int_0^\infty \varepsilon_p^{-1/2} d\varepsilon_p \, \delta(\varepsilon - \varepsilon_p)}_{\varepsilon^{-1/2}}$$

$$= \frac{L}{\pi \hbar} \sqrt{\frac{2m}{\varepsilon}} \Rightarrow$$



$$\Rightarrow N = \int_0^{\varepsilon_F} d\varepsilon \, D(\varepsilon) f(\varepsilon) = \frac{L}{\pi \hbar} \sqrt{2m} \int_0^{\varepsilon_F} \varepsilon^{-1/2} d\varepsilon = \frac{2L}{\pi \hbar} \sqrt{2m} \, \varepsilon_F^{1/2}$$

$$\Rightarrow \frac{N}{L} = n = \frac{2}{\pi \hbar} \sqrt{2m} \, \varepsilon_F^{1/2} \Rightarrow \varepsilon_F = \frac{n^2 \pi^2 \hbar^2}{8m} \quad ; \; f(\varepsilon) = 1$$

and
$$E = \int_0^{\varepsilon_F} d\varepsilon \, D(\varepsilon) \, \varepsilon \, f(\varepsilon) = \frac{L}{\pi \hbar} \sqrt{2m} \int_0^{\varepsilon_F} \varepsilon^{1/2} d\varepsilon = \frac{2}{3} \frac{L}{\pi \hbar} \sqrt{2m} \, \varepsilon_F^{3/2}$$

$$= \frac{L}{3} \underbrace{\frac{2}{\pi \hbar} \sqrt{2m} \, \varepsilon_F^{1/2}}_{N/L} \varepsilon_F = \frac{1}{3} N \varepsilon_F \Rightarrow \boxed{\frac{E}{N} = \frac{1}{3} \varepsilon_F}$$

– equation of state reads $pL = \frac{2}{3} E \Rightarrow p = \frac{2}{3} \frac{E}{L}$

$$\Rightarrow p = \frac{2}{3} \cdot \frac{1}{3} n \varepsilon_F = \frac{2}{6} n \varepsilon_F = \frac{1}{3} n \varepsilon_F \Rightarrow p = \begin{cases} \frac{2}{5} n \varepsilon_F, & 3D \\ \frac{1}{3} n \varepsilon_F, & 2D, 1D \end{cases}$$

(5) Problem 6.8 Marder

a) $N = KV\int_0^\infty \frac{\varepsilon^{1/2}d\varepsilon}{e^{\beta(\varepsilon-\mu)}+1}$; where $K = \frac{(2m)^{3/2}}{2\pi^2\hbar^3} = 1.062\times10^{56}$ $m^{-3}\cdot J^{-3/2}$

$\frac{N}{V} = n = K\int_0^\infty \frac{\varepsilon^{1/2}d\varepsilon}{e^{(\beta\varepsilon-\beta\mu)}+1}$ $m = 9.11\times10^{-31}$ kg, $\hbar = 1.054\times10^{-34}$ J.s

$\Rightarrow \frac{n}{K} = \int_0^\infty \frac{\varepsilon^{1/2}d\varepsilon}{e^{(\beta\varepsilon-\beta\mu)}+1}$, let $\beta\varepsilon = x \Rightarrow \varepsilon = \frac{x}{\beta}$, $d\varepsilon = \frac{dx}{\beta}$

$\Rightarrow \frac{n}{K} = \frac{1}{\beta^{3/2}}\int_0^\infty \frac{x^{1/2}dx}{e^{(x-\beta\mu)}+1} \Rightarrow \frac{n\beta^{3/2}}{K} = \int_0^\infty \frac{x^{1/2}dx}{e^{(x-\beta\mu)}+1}$

$\Rightarrow \frac{n}{K(k_BT)^{3/2}} = \int_0^\infty \frac{x^{1/2}dx}{e^{(x-\beta\mu)}+1}$

for Aluminum (Al), $n = 18.1\times10^{28}/m^3$ (Table 6.1) $\Rightarrow$
and $k_B = 1.38\times10^{-23}$ J/K $\Rightarrow$

$\frac{n}{K(k_B)^{3/2}}\frac{1}{T^{3/2}} = \int_0^\infty \frac{x^{1/2}dx}{e^{(x-\beta\mu)}+1}$

$\frac{18.1\times10^{28}}{1.062\times10^{56}\times(1.38\times10^{-23})^{3/2}}\frac{1}{T^{3/2}} = \int_0^\infty \frac{\sqrt{x}\,dx}{e^{(x-\beta\mu)}+1}$

$\frac{3.32\times10^{7}}{T^{3/2}} = \int_0^\infty \frac{\sqrt{x}\,dx}{e^{(x-\beta\mu)}+1}$ ----- (1)

in Kelvin

The last integral equation can be solved numerically for (T > 0 K) to find the roots ($\beta\mu$) and hence the chemical potential μ. At T=0 K, $\mu = \varepsilon_F = \underline{11.668\ eV}$ for Aluminum (see table 6.1)

To find the roots of the integral, use the online WolframAlpha (https://www.wolframalpha.com/) as shown below

i) T=300 K, The L.H.S of equation (1) reads 6392 and letting $b = \beta\mu$, we obtain

WolframAlpha

FindRoot[NIntegrate[Sqrt(x)/(Exp[x-b]+1), {x, 0, Infinity}]==6392, {b,1}

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initial value

An attempt was made to fix mismatched parentheses, brackets, or braces.

Input

$$\text{FindRoot}\left[\text{NIntegrate}\left[\frac{\sqrt{x}}{\exp(x-b)+1}, \{x, 0, \infty\}\right] = 6392, \{b, 1\}\right]$$

Result

$\{b \to 450.589\}$ $\Rightarrow \beta\mu = 450.589 \Rightarrow \dfrac{\mu}{k_B T} = 450.589$

$$\Rightarrow \mu = (k_B T)(450.589) = (8.617\times10^{-5}\ \tfrac{eV}{K} \times 300K)(450.589) = \underline{11.648 eV}$$

ii) T=10,000 K, The L.H.S of equation (1) reads 33.21 and letting $b = \beta\mu$, we obtain

WolframAlpha

FindRoot[NIntegrate[Sqrt(x)/(Exp[x-b]+1), {x, 0, Infinity}]==33.21, {b,1}

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An attempt was made to fix mismatched parentheses, brackets, or braces.

Input

$$\text{FindRoot}\left[\text{NIntegrate}\left[\frac{\sqrt{x}}{\exp(x-b)+1}, \{x, 0, \infty\}\right] = 33.21, \{b, 1\}\right]$$

Result

$\{b \to 13.4773\}$

$$\mu = (k_B T)(13.4773) = (8.617\times10^{-5} \times 10000)(13.4773) = \underline{11.613 eV}$$

b) μ can also be calculated using sommerfeld expansion formula

$$\mu(T) = \varepsilon_F - \delta\mu = \varepsilon_F - \frac{\pi^2}{12}\frac{(k_B T)^2}{\varepsilon_F} \; ; \quad \varepsilon_F = 11.668\ \text{eV}$$

- at 300K $\Rightarrow k_B T = 0.026$ eV

$\Rightarrow \mu(300K) = 11.668 - 4.7\times10^{-5} \approx 11.668$ same as $\mu(T=0K)$

- at 10,000K $\Rightarrow (k_B T) = 0.862$ eV

$\Rightarrow \mu(10{,}000K) = 11.668 - 0.052 = 11.616$ eV

c) $f(\varepsilon) = \dfrac{1}{e^{\beta\varepsilon - \beta\mu} + 1}$; at 300K, $f(\varepsilon) = \dfrac{1}{e^{\left(\frac{\varepsilon}{0.026} - 450.589\right)} + 1}$

and at 10,000K, $f(\varepsilon) = \dfrac{1}{e^{\left(\frac{\varepsilon}{0.862} - 13.477\right)} + 1}$

WolframAlpha

Plot [{1/(Exp (x/0.026-450.589)+1), 1/(Exp (x/0.862-13.4773)+1)}, {x,0,20}]

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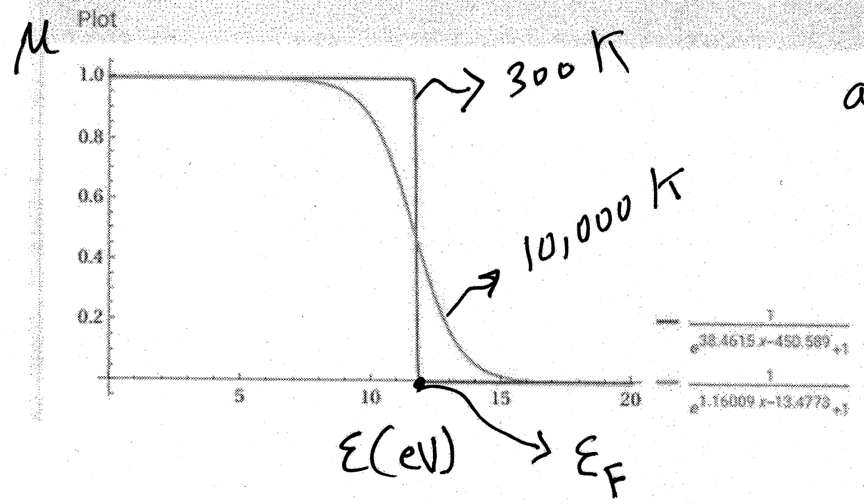
Input interpretation

plot	$\dfrac{1}{\exp\left(\frac{x}{0.026} - 450.589\right) + 1}$	$x = 0$ to 20
	$\dfrac{1}{\exp\left(\frac{x}{0.862} - 13.4773\right) + 1}$	

Plot   Plain Text



at 0K, $f(\varepsilon)$ is a step function similar to $f(\varepsilon)$ at 300K

⑥ for 2D electron gas, we found in problem 6.4 that

$$D(\varepsilon) = \frac{Am}{\pi\hbar^2} \quad \text{and} \quad \varepsilon_F = \frac{\pi\hbar^2 N}{mA} \Rightarrow D(\varepsilon) = \frac{N}{\varepsilon_F}, \text{ so}$$

$$N = \int_0^\infty d\varepsilon\, D(\varepsilon) f(\varepsilon) = \frac{N}{\varepsilon_F}\int_0^\infty \frac{d\varepsilon}{e^{\beta(\varepsilon-\mu)}+1}$$

$$\Rightarrow \varepsilon_F = \int_0^\infty \frac{d\varepsilon}{e^{\beta(\varepsilon-\mu)}+1} = \int_0^\infty \frac{d\varepsilon\, e^{-\beta(\varepsilon-\mu)}}{1+e^{-\beta(\varepsilon-\mu)}} = -\frac{1}{\beta}\int_0^\infty \frac{-\beta e^{-\beta(\varepsilon-\mu)}\, d\varepsilon}{1+e^{-\beta(\varepsilon-\mu)}}$$

$$= -k_BT \ln[1+e^{-\beta(\varepsilon-\mu)}]\Big|_0^\infty = -k_BT\left[0 - \ln(1+e^{\beta\mu})\right]$$

$$\varepsilon_F = k_BT\ln(1+e^{\beta\mu}) \Rightarrow \frac{\varepsilon_F}{k_BT} = \ln(1+e^{\beta\mu})$$

$$\Rightarrow e^{\varepsilon_F/k_BT} = 1+e^{\beta\mu} \Rightarrow e^{\beta\mu} = e^{\varepsilon_F/k_BT}-1$$

$$\Rightarrow \beta\mu = \ln[e^{\varepsilon_F/k_BT}-1] \Rightarrow \boxed{\mu = k_BT\ln[e^{\varepsilon_F/k_BT}-1]}$$

- low T limit ($k_BT \ll \varepsilon_F$) $\Rightarrow e^{\varepsilon_F/k_BT} \gg 1$

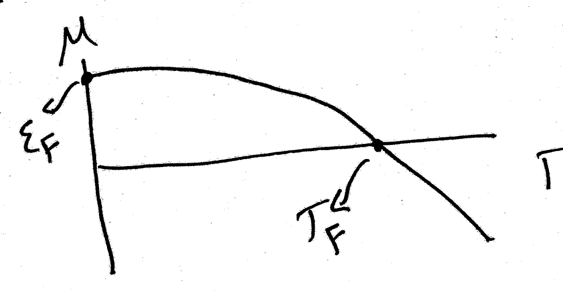
$$\Rightarrow \mu = \cancel{k_BT}\,\frac{\varepsilon_F}{\cancel{k_BT}} = \varepsilon_F$$

- high T limit ($k_BT \gg \varepsilon_F$): $e^{\varepsilon_F/k_BT} = 1 + \frac{\varepsilon_F}{k_BT}$

where I used $e^x \approx 1+x$ for $x \ll 1$

$$\Rightarrow \mu = k_BT\ln\left[1+\frac{\varepsilon_F}{k_BT}-1\right]$$

$$= k_BT\ln\frac{\varepsilon_F}{k_BT}$$



Note that $\mu = 0$ when $\ln\frac{\varepsilon_F}{k_BT} = 0 \Rightarrow \frac{\varepsilon_F}{k_BT} = 1 \Rightarrow T = T_F = \frac{\varepsilon_F}{k_B}$