Practice Second Exam

Real Analysis I — April 18, 2013

Student Name: .... — Serial Number:.........

*** Show your work in details, no credit will be given for *answers* without details.**

1. (5 points) Use the ϵ- definition of the limit to prove that (show all details)
   (a) $\lim\limits_{n\to\infty} \frac{(-1)^n n}{n^2+1} = 0$.
   (b) $\lim\limits_{n\to\infty} \frac{n^2-1}{2n^2+3} = \frac{1}{2}$.
   (c) $\lim\limits_{n\to\infty} \frac{n^2}{n!} = 0$.
2. (5 points) If $0 < a < b$, show that $\lim\limits_{n\to\infty} (a^n + b^n)^{1/n} = b$.
3. (5 points) **State** and **prove** the "Monotone Convergence Theorem" for the bounded increasing sequences of real numbers.
4. (5 points) Let $y_n := \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots + \frac{1}{n^2}$, for each $n \in \mathbb{N}$. Prove that the sequence $Y = \{y_n\}_{n\in\mathbb{N}}$ converges. (Hint: Show that Y is bounded monotone).
5. (5 points) Let $c > 1$, show that $\lim\limits_{n\to\infty} c^{1/n} = 1$.
6. (5 points)
   (a) State the definition of a "Cauchy sequence".
   (b) Prove that " If a sequence $X = (x_n)$ of real numbers converge, then it is a Cauchy sequence".
7. (5 points) Show that the sequence $\{\frac{n^3+2}{n^2+1}\}_{n\in\mathbb{N}}$ is properly divergent sequence.
8. (5 points) Give an example of
   (a) A bounded sequence which diverges.
   (b) A convergent sequence which is not monotone.
9. (5 points)
10. (5 points)
11. (5 points)

Good Luck