The Hashemite University | Real Analysis I, First Exam | November 3, 2013

1^{st} semester 2013/2014 | Time: One Hour

Student Name: .... | Serial Number:.........

*** Show your work in details, no credit will be given for *answers* without details.**

1. (8 points) **State** the following results:
   - (a) The Density Theorem of the rational numbers.
   - (b) The Completeness Property of the Real numbers.
   - (c) Archimedean Property.
   - (d) Bernoulli's Inequality.

2. (5 points) Let S and T be a bounded nonempty subsets of $\mathbb{R}$, and define
$$S - T = \{s - t : s \in S, t \in T\}.$$
Show that $\sup(S - T) = \sup(S) - \inf(T)$.

3. (3 points) For any $\alpha > 0$, $\beta > 0$, there exist $n \in \mathbb{N}$ such that $\frac{\alpha}{n} < \beta$.

4. (9 points) Determine whether these statements are true or false, **and justify (shortly) why a statement is false**.
   1. $(T\ F)$ If $a \in \mathbb{R}$ such that $0 < a < \epsilon$ for every $\epsilon > 0$, then $a = 0$.
   2. $(T\ F)$ If $0 \leq a < b$, then $a^2 < ab < b$.
   3. $(T\ F)$ The set $S = \cup_{n=1}^{\infty}(n, n + \frac{1}{2})$ is bounded below by 1.
   4. $(T\ F)$ If A is a nonempty subset of $\mathbb{R}$ that is bounded below, then $\inf(A) = -\sup\{-a : a \in A\}$.
   5. $(T\ F)$ Between any two real numbers there is a unique rational number.
   6. $(T\ F)$ Let $f : \mathbb{R} \to \mathbb{R}$ be bounded. Then $\sup\{f(x^2) : x \in \mathbb{R}\} = \sup\{f(x) : x \in \mathbb{R}\}$.

Good Luck