Practice Final Exam

Real Analysis I — May 23, 2013

Student Name: .... Serial Number:.........

*** Show your work in details, no credit will be given for *answers* without details.**

1. Use the ϵ- definition of the limit to prove that (show all details)
   (a) $\lim_{n\to\infty} \frac{\sqrt{n}}{5n+3} = 0.$
   (b) $\lim_{n\to\infty} \frac{n^2}{n!} = 0.$
2. **State** and **prove** the following theorems:
   (a) Monotone Convergence Theorem for the bounded increasing sequences of real numbers.
   (b) Boundedness Theorem (for functions).
   (c) Uniform Continuity Theorem.
   (d) Sequential Criterion for Limits of functions.
   (e) ...
   (f) ...
3. Give an example of a function $f : [0,1] \to \mathbb{R}$ that is discontinuous at every point of $[0,1]$ but such that $|f|$ is continuous on $[0,1]$.
4. Show that the function $f(x) = \frac{1}{x}$ is uniformly continuous on the set $A := [a, \infty)$, where $a > 0$.
5. Give an example of a function $f : \mathbb{R} \to \mathbb{R}$ which is nowhere continuous. Justify your answer in details.
6. Show that if a function $f : \mathbb{R} \to \mathbb{R}$ is Lipschitz on $\mathbb{R}$, then f is continuous at every point of $c \in \mathbb{R}$.
7. Prove that if a sequence $X = \{x_n\}_{n\in\mathbb{N}} \subset \mathbb{R}$ converges to a real number x, then any subsequence $X' = \{x_{n_k}\}$ of X also converges to x.
8. True or Faulse:
   (a) If $f : A \to \mathbb{R}$ is continuous function on $A \subset \mathbb{R}$, then f is uniformly continuous on A.
   (b) Any Lipschitz continuous function is uniformly continuous.
   (c) Any bounded sequence converges.
   (d) Any convergent sequence in $\mathbb{R}$ is Cauchy sequence in $\mathbb{R}$.
   (e) ...
   (f) ...
9.
10.
11.

Good Luck