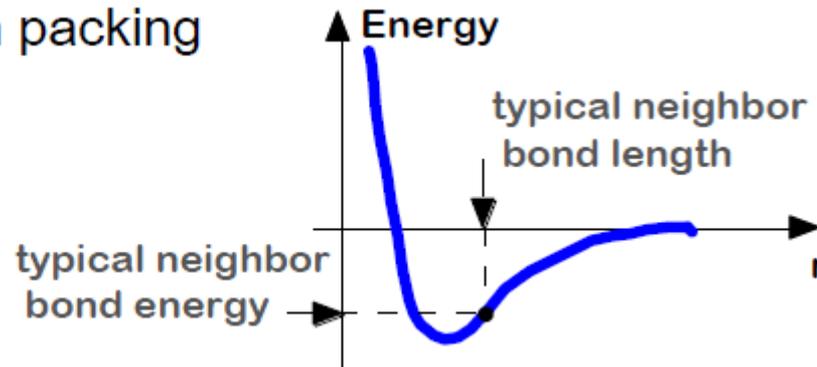
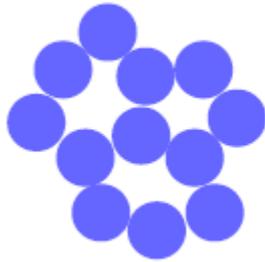


Outline:

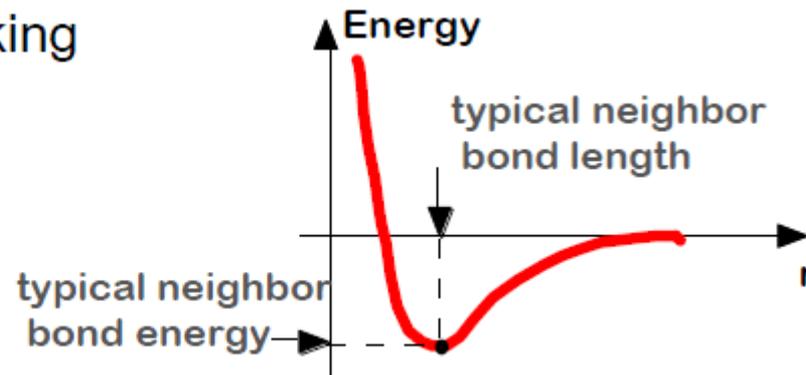
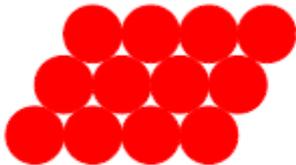
- Crystalline versus amorphous structures
- Crystal structure
 - Unit cell
 - Coordination number
 - Atomic packing factor
- Crystal systems

ENERGY AND PACKING

- Non dense, **random** packing



- Dense, **regular** packing



❖ Dense, regular-packed structures tend to have lower energy.

CRYSTAL STRUCTURES & PROPERTIES

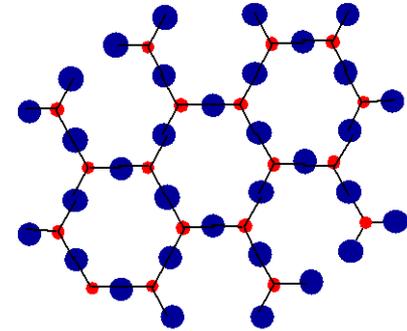
How do materials assemble?

Crystalline materials...

- atoms pack in periodic, 3D arrays
- typical of:
 - metals
 - many ceramics
 - some polymers

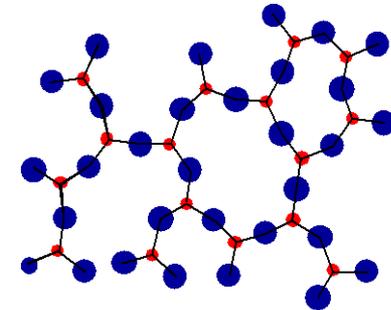
Noncrystalline (amorphous) materials...

- atoms have no periodic packing occurs for:
 - complex structures
 - rapid cooling



crystalline SiO₂

• Si • Oxygen



noncrystalline SiO₂

Crystal Structure

Motivation: Many of the properties of materials (*especially mechanical*) are determined by the arrangement of the constituent atoms.

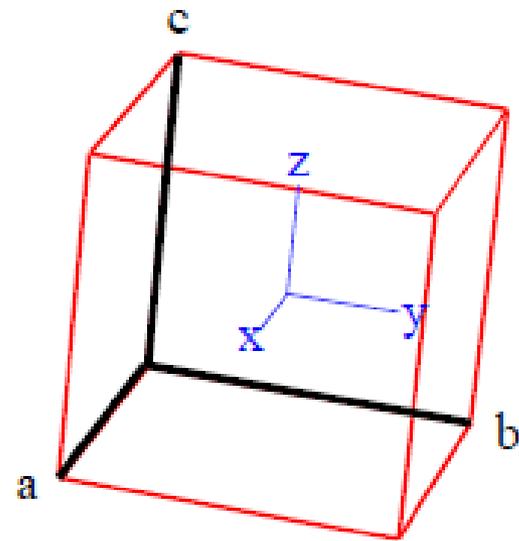
*This arrangement is called the material's **crystal structure**.*

- **Atomic structure** relates to the number of protons and neutrons in the nucleus of an atom, as well as the number and probability distributions of the constituent electrons.
- On the other hand, **crystal structure** pertains to the arrangement of atoms in the crystalline solid material.

Crystal Structure

- We need a way to specify crystallographic directions and planes.
- Let's start with the hard sphere model (in which nearest neighbour atoms “touch” each other)...

To illustrate the concept of a crystal structure and lattice systems, we first identify a coordinate system (x, y, z):



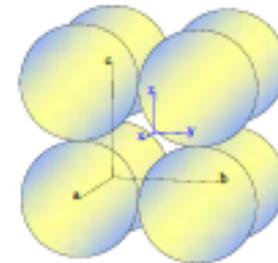
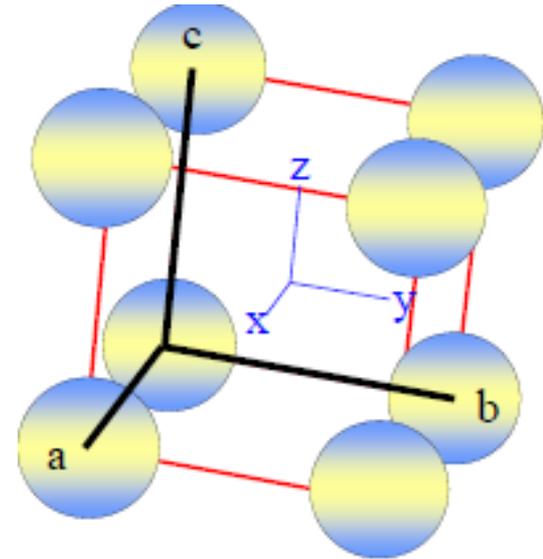
We can't specify directions or planes without knowing what the reference system is.

Crystal Structure

Now place an atom at each corner...

This represents the “hard sphere” model of a simple crystal system

- Atoms “touch” along the crystal axes
- These directions are referred to as “close-packed” in the simple cubic system



not many examples of simple cubic systems in nature, except for Po

The above diagram represents a simple cubic crystal structure

What is Unit Cell?

- A unit cell is the smallest entity that exhibits the chemical and physical properties of the material.

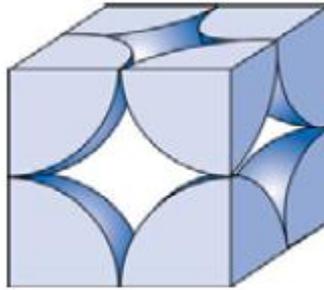
– Unit cells are the most elementary arrangement of atoms which can generate the entire crystal upon application of suitable translation, rotation, mirror, or inversion operations.

Definition:

the length of each unit cell axis is called a lattice

- In cubic systems, all three orthogonal lattice parameters are equal*
- Lattice parameters are typically on the order of a few Angstroms (or a few tenths of a nanometer)

Simple Cubic Unit Cell



How many atoms does the simple cubic unit cell contain?

You should be able to convince yourself that a simple cubic structure contains ... [atom/unit cell](#).

(Remember, a part of each atom is shared by another unit cell!)

- The number of atoms/unit cell is an important quantity and determines many physical properties.
- In general, the number of atoms/unit cell, N , is given by

$$N = N_i + \frac{N_f}{2} + \frac{N_c}{8}$$

Where: N_i = # interior atoms,
 N_f = # face atoms,
 N_c = # corner atoms

Simple Cubic Unit Cell

- volume of the unit cell = a^3
 - where a is the lattice parameter
- coordination # = 6
 - for simple cubic structures

✓ CN is the number of nearest-neighbor atoms

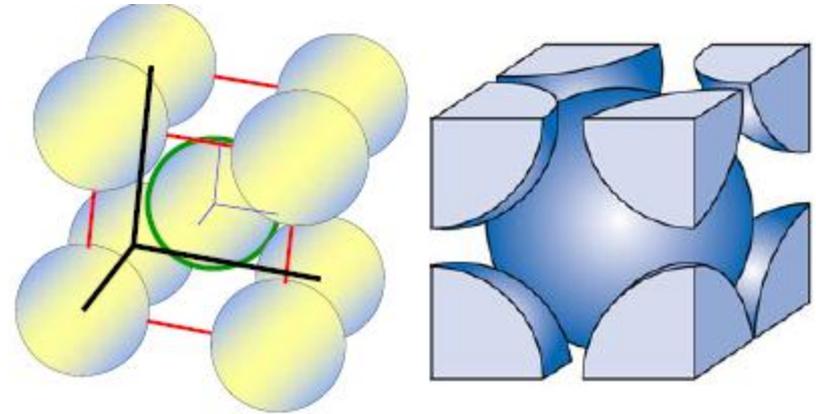
✓ Coordination number is important in determining the structure of crystalline materials.

✓ Large atoms tend to have large CN, small atoms usually have small CN

it's easier to surround a big atom with lots of atoms than a smaller one.

BODY CENTERED CUBIC STRUCTURE (BCC)

Now, suppose we add another atom at the center of the cube



We no longer have a simple cubic structure but instead, a
..... (BCC) structure

Examples of BCC systems: Cr, W, Mo, Ta, Fe (α Fe T_m below 912°C)

Notice that in the BCC structure, atoms touch along the body diagonals. These are the close-packed directions in the body-centered cubic structure.

BCC unit cells have a CN = ..8...

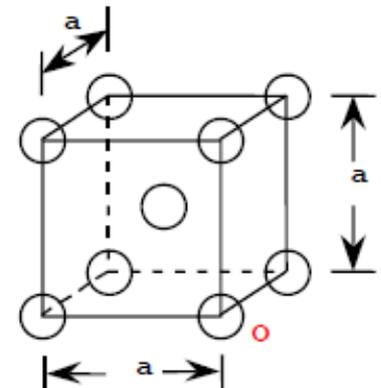
2 atoms in unit cell

Atomic Packing Factor

$$\text{Atomic Packing Factor (APF or APE)} = \frac{\text{total "sphere" volume}}{\text{total cell volume}}$$

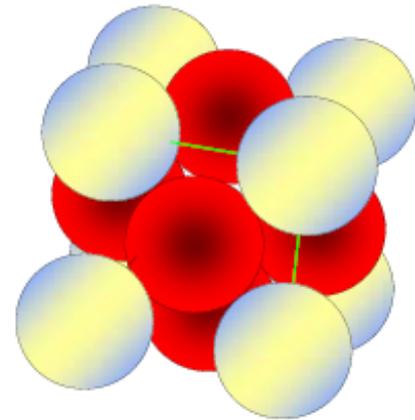
“total sphere volume” is just the volume per atomic “sphere” multiplied by the number of atoms in the unit cell

Example: Calculate the APF for a BCC unit cell:



Face Centered Cubic Unit Cell

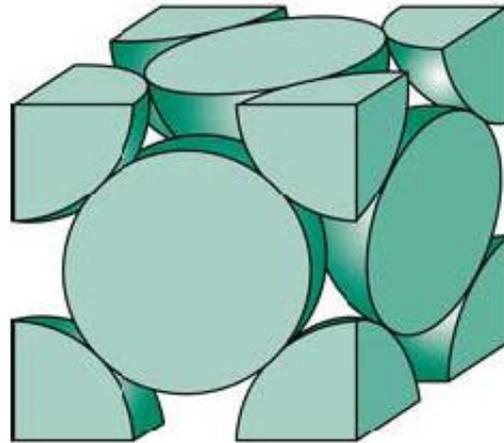
Now suppose we place equivalent atoms at the corners of the unit cell, AND in the center of each face:



This is a (FCC) crystal structure

- Examples of FCC metals: **Cu, Ni, Au, Ag, Fe** (T_m above 912°C)
- Close-packed directions in FCC metals are along face diagonals

Face Centered Cubic Unit Cell



- **Q:** How many atoms per unit cell in the FCC structure?

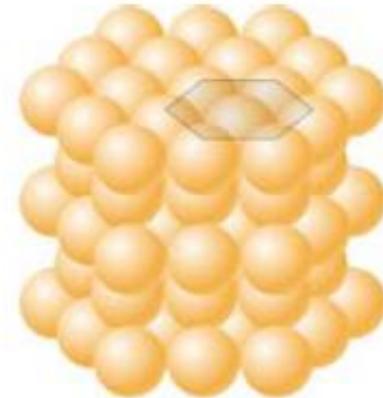
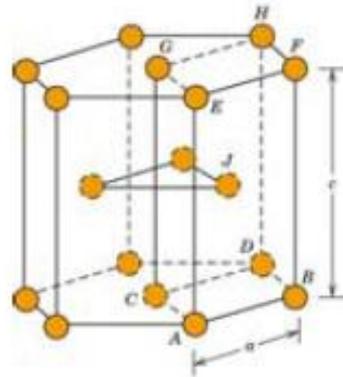
$$\text{recall, } N = N_i + \frac{N_f}{2} + \frac{N_c}{8}$$

- FCC unit cells have a CN = 12 and an APF = 0.75
 - *maximum packing efficiency for monosized spheres*

Hexagonal Unit Cell

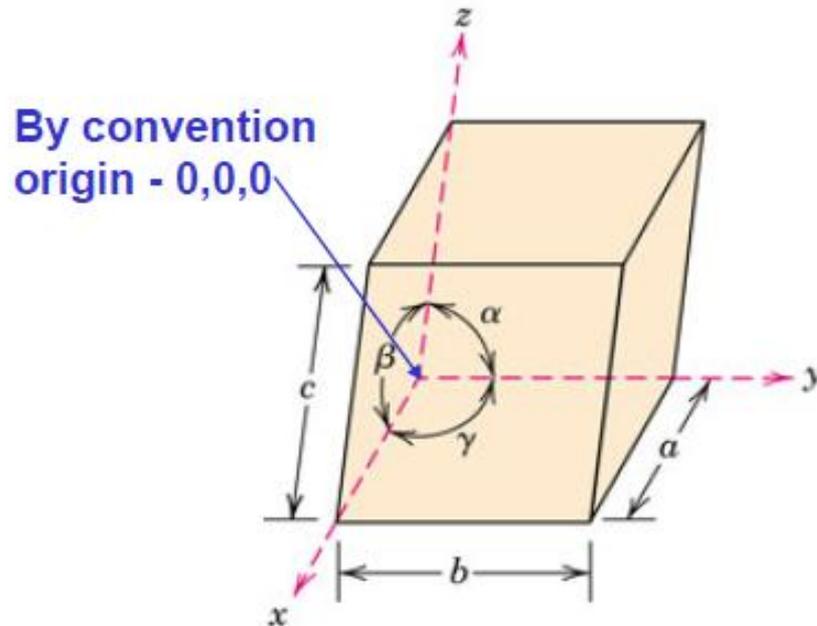
- There are other ways in which atoms can be arranged to form unit cells:
 - For example...

This represents an HCP (.....*packed*.....) structure



- Examples of HCP systems (*Mg, Co, Ti, Zn, Zr, RE*)
- The HCP unit cell consists of **6 atoms** forming the corners of a hexagon in the basal planes + 1 atom in the center. In addition, there are 3 interior atoms midway between basal planes along the c-axis.
 - # atoms/unit cell = $(1/6) * 12$ (corner atoms) + $(1/2) * 2$ (center face atoms) + 3 (interior atoms) =
- Coordination # = ..**12**.

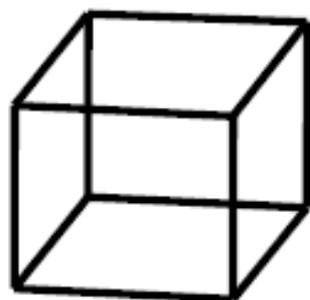
General convention for unit cell axis and angle notation:



In total, there are 7 distinct and unique crystal systems:

cubic represents only one of the 7

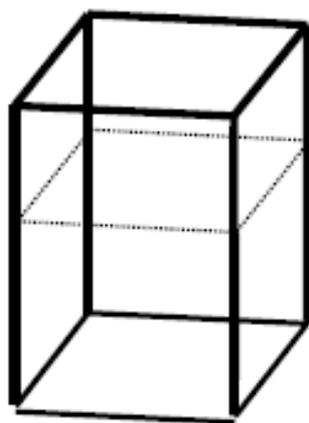
Crystal Systems



Cubic

$$a=b=c$$

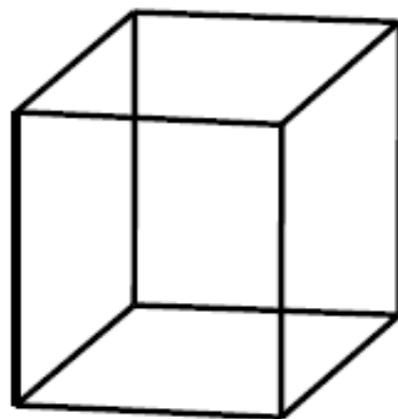
$$\alpha=\beta=\gamma=90$$



Tetragonal

$$a=b\neq c$$

$$\alpha=\beta=\gamma=90$$

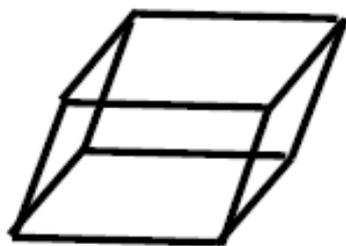


Orthorhombic

$$a\neq b\neq c$$

$$\alpha=\beta=\gamma=90$$

Crystal Systems

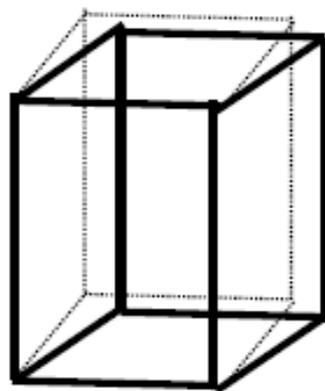


Rhombohedral

$$a=b=c$$

$$\alpha=\beta=\gamma \neq 90$$

“Pushed over”
cube



Hexagonal

$$a=b \neq c$$

$$\alpha=\beta=90 \quad \gamma=120$$

“Squished”
tetragonal

Monoclinic

$$a \neq b \neq c$$

$$\alpha=\gamma=90, \beta \neq 90$$

“Pushed over”
orthorhombic
(in one direction)

Triclinic

$$a \neq b \neq c$$

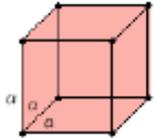
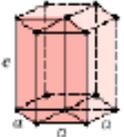
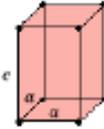
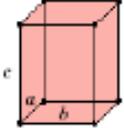
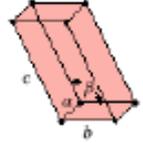
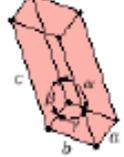
$$\alpha \neq \beta \neq \gamma \neq 90$$

“Pushed over”
orthorhombic
(in two directions)

Crystal Systems

Note that these 7 crystal systems do not account for all the possible lattice types

for example, the cubic system contains SC, FCC, and BCC as subsets

<i>Crystal System</i>	<i>Axial Relationships</i>	<i>Interaxial Angles</i>	<i>Unit Cell Geometry</i>
Cubic	$a = b = c$	$\alpha = \beta = \gamma = 90^\circ$	
Hexagonal	$a = b \neq c$	$\alpha = \beta = 90^\circ, \gamma = 120^\circ$	
Tetragonal	$a = b \neq c$	$\alpha = \beta = \gamma = 90^\circ$	
Rhombohedral	$a = b = c$	$\alpha = \beta = \gamma \neq 90^\circ$	
Orthorhombic	$a \neq b \neq c$	$\alpha = \beta = \gamma = 90^\circ$	
Monoclinic	$a \neq b \neq c$	$\alpha = \gamma = 90^\circ \neq \beta$	
Triclinic	$a \neq b \neq c$	$\alpha \neq \beta \neq \gamma \neq 90^\circ$	



Crystal Systems

There are 14 unique lattice types from this framework of 7 crystal systems (called Bravais lattices):

Crystal system	Types of possible lattice arrangements
Cubic	SC, BCC, FCC
Hexagonal	HCP
Tetragonal	Simple, body-centered
Orthorhombic	Simple, base-centered, BC, FC
Rhombohedral	Simple
Monoclin	Simple, base-centered
Triclinic	Simple

We will mainly be concerned with cubic and hexagonal systems in this class. But you need to realize that many other types of symmetries exist!