# TRIGONOMETRIC IDENTITIES

- **Reciprocal identities**

$$\sin u = \frac{1}{\csc u} \quad \cos u = \frac{1}{\sec u}$$

$$\tan u = \frac{1}{\cot u} \quad \cot u = \frac{1}{\tan u}$$

$$\csc u = \frac{1}{\sin u} \quad \sec u = \frac{1}{\cos u}$$

- **Pythagorean Identities**

$$\sin^2 u + \cos^2 u = 1$$

$$1 + \tan^2 u = \sec^2 u$$

$$1 + \cot^2 u = \csc^2 u$$

- **Quotient Identities**

$$\tan u = \frac{\sin u}{\cos u} \quad \cot u = \frac{\cos u}{\sin u}$$

- **Co-Function Identities**

$$\sin(\frac{\pi}{2} - u) = \cos u \quad \cos(\frac{\pi}{2} - u) = \sin u$$

$$\tan(\frac{\pi}{2} - u) = \cot u \quad \cot(\frac{\pi}{2} - u) = \tan u$$

$$\csc(\frac{\pi}{2} - u) = \sec u \quad \sec(\frac{\pi}{2} - u) = \csc u$$

- **Parity Identities (Even & Odd)**

$$\sin(-u) = -\sin u \quad \cos(-u) = \cos u$$
$$\tan(-u) = -\tan u \quad \cot(-u) = -\cot u$$
$$\csc(-u) = -\csc u \quad \sec(-u) = \sec u$$

- **Sum & Difference Formulas**

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$
$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$
$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

- **Double Angle Formulas**

$$\sin(2u) = 2 \sin u \cos u$$
$$\cos(2u) = \cos^2 u - \sin^2 u$$
$$= 2 \cos^2 u - 1$$
$$= 1 - 2 \sin^2 u$$
$$\tan(2u) = \frac{2 \tan u}{1 - \tan^2 u}$$

- **Power-Reducing/Half Angle Formulas**

$$\sin^2 u = \frac{1 - \cos(2u)}{2}$$

$$\cos^2 u = \frac{1 + \cos(2u)}{2}$$

$$\tan^2 u = \frac{1 - \cos(2u)}{1 + \cos(2u)}$$

- **Sum-to-Product Formulas**

$$\sin u + \sin v = 2 \sin\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)$$

$$\sin u - \sin v = 2 \cos\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right)$$

$$\cos u + \cos v = 2 \cos\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)$$

$$\cos u - \cos v = -2 \sin\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right)$$

- **Product-to-Sum Formulas**

$$\sin u \sin v = \frac{1}{2}\left[\cos(u-v) - \cos(u+v)\right]$$

$$\cos u \cos v = \frac{1}{2}\left[\cos(u-v) + \cos(u+v)\right]$$

$$\sin u \cos v = \frac{1}{2}\left[\sin(u+v) + \sin(u-v)\right]$$

$$\cos u \sin v = \frac{1}{2}\left[\sin(u+v) - \sin(u-v)\right]$$